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Evolving Casimir Wormholes

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The traversable wormhole metric

M. S. Morris and K. S. Thorne, Am. J. Phys. 56, 395 (1988).

$$ds^2 = -exp\big(2\Phi(r)\big)\,dt^2 + \frac{dr^2}{1-\frac{b(r)}{r}} + r^2 d\theta^2 + r^2 sin^2(\theta) d\phi^2$$

$\boldsymbol{b(r)}$ is the shape function

$\phi\boldsymbol{(r)}$ is the redshift function

$r \in \left[r_0 , +\infty \right)$

Condition $b_\pm\left(r_0\right) = r_0$

$b_\pm\left(r\right) < r$

Exotic Energy $\rho(r) + p_r(r) < 0 \quad r \in [r_0, r_0 + \varepsilon]$

$$b'(r) = 8\pi G \rho r^2$$

$$\Phi'(r) = \frac{b + 8\pi G p_r r^3}{2r^2(1 - b(r)/r)}$$

$$p'_r(r) = \frac{2}{r}\big(p_t(r) - p_r(r)\big) - \big(\rho(r) + p_r(r)\big)\Phi'(r)$$

Candidate

Casimir Energy

Casimir Wormholes 1

d → r

R.G. Eur.Phys.J.C 79 (2019) 11, 951 ArXiV: 1907.03623 [gr-qc].

When $\omega = \frac{r_0^2}{r_1^2}$ → Traversable Wormhole

$$r_1^2 = \frac{\pi^3 l_p^2}{90} \qquad p_r(r) = \omega\rho(r)$$

$$\Phi(r) = \frac{1}{2}(\omega - 1)ln\left(\frac{r(\omega+1)}{(\omega r + r_0)}\right) \xrightarrow{\omega = 3} \Phi(r) = ln\left(\frac{4r}{3r + r_0}\right)$$

Planckian

$$\rightarrow b(r) = \left(1 - \frac{1}{\omega}\right) r_0 + \frac{r_0^2}{\omega r} \xrightarrow{\omega = 3} b(r) = \frac{2}{3} r_0 + \frac{r_0^2}{3r}$$

$$SET \quad T_{\mu\nu} = \left(\frac{r_0^2}{3\kappa r^4}\right)\left[diag(-1,-3,1,1) + \left(\frac{6r}{3r + r_0}\right) diag(0,0,1,1)\right]$$

Alternatively $$\tau_\rho = 0; \qquad \tau_r = 0; \qquad \tau_t(r) = \frac{2r_0^2}{\kappa r^3 (3r + r_0)}.$$

Casimir Wormholes 2

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

$$\rho(d) = -\frac{\hbar c \pi^2}{720 d^4} = -\rho_0 \quad p_r(d) = -3\frac{\hbar c \pi^2}{720 d^4} \quad p_t(d) = \frac{\hbar c \pi^2}{720 d^4}$$

The Casimir Tensor is traceless and divergenceless

$$G_{\mu\nu} = \frac{8\pi G}{c^4} \langle T_{\mu\nu} \rangle^{Ren}$$

$$b(r) = r_0 - \frac{\kappa \rho_0}{3}\left(r^3 - r_0^3\right) = r_0 - \frac{\pi^3}{270 d^4}\underbrace{\left(\frac{\hbar G}{c^3}\right)}_{l_p^2}\left(r^3 - r_0^3\right)$$

$$\kappa = \frac{8\pi G}{c^4}$$

$$b(\bar{r}) = 0 \Longleftrightarrow \bar{r} = r_0 \sqrt[3]{1 + \frac{3}{\kappa \rho_0 r_0^2}}$$

$$\bar{r} = \sqrt[3]{10}\, r_0 \,.$$

$$\omega(r) = -\frac{b(r)}{r b'(r)} = -\frac{r_0 - \frac{\kappa \rho_0}{3}\left(r^3 - r_0^3\right)}{r\left(-\kappa \rho_0 r^2\right)}$$

$$\frac{p_r(r)}{\rho(r)} = 3$$

$$b(r) = r_0 - \frac{\kappa \rho_0}{3}\left(r^3 - r_0^3\right) \qquad 0 \le r \le \bar{r}$$

$$b(r) = 0 \qquad r > \bar{r}$$

$$\omega(r_0) = \frac{1}{\kappa \rho_0 r_0^2} = 3$$

$$\Phi(r) = 0$$

$$r \in [r_0, +\infty)$$

M.S. Morris, K.S. Thorne, U. Yurtsever. 1988 Phys.Rev.Lett. 61 (1988) 1446-1449

M. Visser, Lorentzian Wormholes: From Einstein to Hawking (American Institute of Physics, New York), 1995.

Other Profiles → *Generalized Absurdly Benign TW*

R.G. *Eur.Phys.J.C* 80 (2020) 12, 1172ArXiV: 2008.05901 [gr-qc]

$b(r) = r_0\left(1-\mu(r-r_0)\right)^{\alpha}$ $\Phi(r) = 0$ $r \in \left[r_0, r_0 + {}^{1}/_{\mu}\right]$ For $\alpha = 2$ we have the absurdly benign TW

$b(r) = 0$ $\Phi(r) = 0$ $r \in \left[r_0 + {}^{1}/_{\mu}, \infty\right]$ → Minkowski

Other Profiles → *Yukawa-Casimir Wormholes*

Eur.Phys.J.C 81,824 (2021) arXiv:2107.09276[gr-qc]

Other Profiles → *Hot Casimir Wormholes*

R.G. and Mir Faizal arXiv:2403.15174[gr-qc] *JCAP* 01 (2025) 081, 081

Other Profiles → *Casimir Wormholes+Scalar Fields*

R.G. and A.G. Tzikas JCAP 12,019 (2024) arXiv:2312.16736[gr-qc]

Other Profiles → Casimir+Electromagnetic

Eur.Phys.J.C 83, 824 (2023)arXiv:2302.04043 [gr-qc]

$$\rho(r) + p_r(r) = -\frac{4\hbar c\pi^2}{720 r^4} < 0 \qquad r_0 = \sqrt{3r_1^2 + r_2^2} \qquad r_1^2 = \frac{\pi^3 l_p^2}{90} \qquad r_2^2 = \frac{GQ^2}{4\pi c^4 \varepsilon_0}$$

Other Profiles → Rotating Casimir Wormholes

R.G. and A.Tzikas Eur.Phys.J.C 85 (2025) 3, 336 ArXiv: 2502.19995 [gr-qc]

R.G. and A.Tzikas 2509.15887 [gr-qc] (Charged Rotating Casimir Wormholes)

Other Profiles → Physical Sources

R.G. and A.Tzikas arXiv:2507.19134[gr-qc] accepted in CQG

Time Dependent Casimir Wormholes

Work in Progress

Model I

$$ds^2 = -e^{2\Phi(r,t)}dt^2 + \frac{dr^2}{1 - \frac{b(r,t)}{r}} + r^2\left(d\theta^2 + sin^2(\theta)d\phi^2\right)$$

$$\frac{\dot{b}(r,t)}{1 - \frac{b(r,t)}{r}} = \kappa T_{01}(r,t) \qquad \frac{1}{r^2}\frac{\partial}{\partial r}b'(r,t) = \kappa\rho(r,t)$$

$$If \;\; \rho(r,t) = \rho(r)T(t) \quad then \quad b(r,t) = b(r)T(t)$$

$\rho(r) \rightarrow \rho(r,t)$ $\quad p_r(r) \rightarrow p_r(r,t)$ $\quad p_t(r) \rightarrow p_t(r,t)$ $\quad f(r,t) \longleftrightarrow$ Energy flux or heat flux

$$\rho(d) = -\frac{\hbar c\pi^2}{720d^4} \quad p_r(d) = -3\frac{\hbar c\pi^2}{720d^4} \quad p_t(d) = \frac{\hbar c\pi^2}{720d^4}$$

d → r

Time Dependent Casimir Wormholes

Work in Progress see also papers of S.Kar, S.W. Kim, Anchordoqui et al, Cataldo et al.,Kuhfittig and T. Roman

Model II

$$ds^2 = -e^{2\Phi(r,t)}dt^2 + R^2(t)\left(\frac{dr^2}{1-\frac{b(r)}{r}} + r^2\left(d\theta^2 + sin^2(\theta)d\phi^2\right)\right)$$

$$2\Phi'(r)\frac{\dot{R}(t)}{R(t)} = \kappa T_{01}(r,t) \qquad \frac{b'(r)}{r^2} + 3\frac{\dot{R}^2(t)}{R^2(t)}e^{-2\Phi(r,t)} = \kappa\rho(r,t)$$

Model III

$$ds^2 = \Omega^2(t,r)\left(-e^{2\Phi(r)}dt^2 + \frac{dr^2}{1-\frac{b(r)}{r}} + r^2\left(d\theta^2 + sin^2(\theta)d\phi^2\right)\right)$$

$$2\Phi'(r)\frac{\dot{\Omega}(t)}{\Omega(t)} = \kappa T_{01}(r,t) \qquad \frac{b'(r)}{r^2\Omega^2(t)} + 3\frac{\dot{\Omega}^2(t)}{\Omega^4(t)}e^{-2\Phi(r)} = \kappa\rho(r,t)$$

$\rho(r) \to \rho(r,t)$ $\quad p_r(r) \to p_r(r,t)$ $\quad p_t(r) \to p_t(r,t)$ $\quad f(r,t) \longleftrightarrow$ Energy flux or heat flux

Conclusions and Perspectives

- Casimir energy is the only source of exotic matter that can be generated in laboratory.
- Traversable wormholes can be sustained by Casimir Energy and the related profile can be computed. The wormhole is traversable in principle but not in practice.
- Generalized Absurdly Benign Traversable Wormholes seem to have the right properties for traversability together with Yukawa–Casimir wormholes.
- Combining the original Casimir source with extra sources like a Scalar Field and an Electromagnetic Field seems to be promising!!! (R.G. and A.G.Tzikas in preparation)
- Adding Temperature → more realistic case!! The traversability is not destroyed!!
- Rotations do not destroy the Casimir Wormhole properties (even with an electric field → Work in Progress).
- The time dependent traversable wormhole metric has a strong dependence on the energy flux.
- A time dependent metric offers the possibility of investigating the topology change issue. (See for example A.DeBenedictis, R.G. and F.S.N. Lobo PRD 78 104003 (2008))

Thank You for Your Attention