

Evolving Casimir Wormholes Remo Garattini

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The traversable wormhole metric

M. S. Morris and K. S. Thorne, Am. J. Phys. 56, 395 (1988).

$$ds^2 = -\exp(2\Phi(r)) dt^2 + \frac{dr^2}{1 - \frac{b(r)}{r}} + r^2 d\theta^2 + r^2 \sin^2(\theta) d\phi^2$$

$b(r)$ is the shape function $r \in [r_0, +\infty)$ Condition $b_{\pm}(r_0) = r_0$
 $\Phi(r)$ is the redshift function $b_{\pm}(r) < r$

$$b'(r) = 8\pi G \rho r^2$$

$$\Phi'(r) = \frac{b + 8\pi G p_r r^3}{2r^2(1 - b(r)/r)}$$

$$p'_r(r) = \frac{2}{r}(p_t(r) - p_r(r)) - (\rho(r) + p_r(r))\Phi'(r)$$

Exotic Energy $\rho(r) + p_r(r) < 0 \quad r \in [r_0, r_0 + \varepsilon]$

Candidate



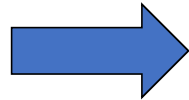
Casimir Energy

Casimir Wormholes 1

$d \rightarrow r$

R.G. Eur.Phys.J.C 79 (2019) 11, 951 ArXiv: 1907.03623 [gr-qc].

When $\omega = \frac{r_0^2}{r_1^2}$

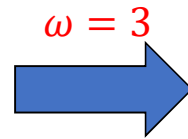


Traversable Wormhole

$$r_1^2 = \frac{\pi^3 l_p^2}{90}$$

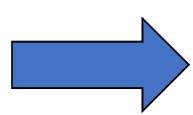
$$p_r(r) = \omega \rho(r)$$

$$\Phi(r) = \frac{1}{2}(\omega - 1) \ln \left(\frac{r(\omega + 1)}{(\omega r + r_0)} \right)$$

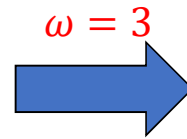


$$\Phi(r) = \ln \left(\frac{4r}{3r + r_0} \right)$$

Planckian



$$b(r) = \left(1 - \frac{1}{\omega} \right) r_0 + \frac{r_0^2}{\omega r}$$



$$b(r) = \frac{2}{3} r_0 + \frac{r_0^2}{3r}$$

$$SET \quad T_{\mu\nu} = \left(\frac{r_0^2}{3\kappa r^4} \right) \left[\text{diag}(-1, -3, 1, 1) + \left(\frac{6r}{3r + r_0} \right) \text{diag}(0, 0, 1, 1) \right]$$

Alternatively

$$\tau_\rho = 0; \quad \tau_r = 0; \quad \tau_t(r) = \frac{2r_0^2}{\kappa r^3 (3r + r_0)}.$$

Casimir Wormholes 2

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$



$$G_{\mu\nu} = \frac{8\pi G}{c^4} \langle T_{\mu\nu} \rangle^{Ren}$$

$$\rho(d) = -\frac{\hbar c \pi^2}{720 d^4} = -\rho_0 \quad p_r(d) = -3 \frac{\hbar c \pi^2}{720 d^4} \quad p_t(d) = \frac{\hbar c \pi^2}{720 d^4}$$

The Casimir Tensor is traceless and divergenceless

$$b(r) = r_0 - \frac{\kappa \rho_0}{3} (r^3 - r_0^3) = r_0 - \frac{\pi^3}{270 d^4} \left(\frac{\hbar G}{c^3} \right) (r^3 - r_0^3)$$

$$\kappa = \frac{8\pi G}{c^4}$$

$$\omega(r) = -\frac{b(r)}{r b'(r)} = -\frac{r_0 - \frac{\kappa \rho_0}{3} (r^3 - r_0^3)}{r(-\kappa \rho_0 r^2)}$$

$$\omega(r_0) = \frac{1}{\kappa \rho_0 r_0^2} = 3$$

$$\frac{p_r(r)}{\rho(r)} = 3$$

$$b(\bar{r}) = 0$$



$$\bar{r} = r_0 \sqrt[3]{1 + \frac{3}{\kappa \rho_0 r_0^2}}$$

$$\bar{r} = \sqrt[3]{10} r_0.$$

$$b(r) = r_0 - \frac{\kappa \rho_0}{3} (r^3 - r_0^3) \quad 0 \leq r \leq \bar{r}$$

$$b(r) = 0 \quad r > \bar{r}$$

$$\Phi(r) = 0 \quad r \in [r_0, +\infty)$$

M.S. Morris, K.S. Thorne, U. Yurtsever.
1988 Phys.Rev.Lett. 61 (1988) 1446-1449

M. Visser, Lorentzian Wormholes: From Einstein to Hawking
(American Institute of Physics, New York), 1995.

Other Profiles *Generalized Absurdly Benign TW*

R.G. *Eur.Phys.J.C* 80 (2020) 12, 1172 ArXiv: 2008.05901 [gr-qc]

$$b(r) = r_0(1 - \mu(r - r_0))^\alpha \quad \Phi(r) = 0 \quad r \in [r_0, r_0 + 1/\mu] \quad \text{For } \alpha = 2 \text{ we have the absurdly benign TW}$$

$$b(r) = 0 \quad \Phi(r) = 0 \quad r \in [r_0 + 1/\mu, \infty] \quad \img alt="blue arrow" data-bbox="445 345 525 405"/> \text{Minkowski}$$

Other Profiles *Yukawa-Casimir Wormholes*

Eur.Phys.J.C 81,824 (2021) arXiv:2107.09276[gr-qc]

Other Profiles *Hot Casimir Wormholes*

R.G. and Mir Faizal arXiv:2403.15174[gr-qc] *JCAP* 01 (2025) 081, 081

Other Profiles *Casimir Wormholes+Scalar Fields*

R.G. and A.G. Tzikas *JCAP* 12,019 (2024) arXiv:2312.16736[gr-qc]

Other Profiles *Casimir+Electromagnetic*

Eur.Phys.J.C 83, 824 (2023) arXiv:2302.04043 [gr-qc]

$$\rho(r) + p_r(r) = -\frac{4\hbar c\pi^2}{720r^4} < 0 \quad r_0 = \sqrt{3r_1^2 + r_2^2} \quad r_1^2 = \frac{\pi^3 l_p^2}{90} \quad r_2^2 = \frac{GQ^2}{4\pi c^4 \epsilon_0}$$

Other Profiles *Rotating Casimir Wormholes*

R.G. and A.Tzikas Eur.Phys.J.C 85 (2025) 3, 336 ArXiv: 2502.19995 [gr-qc]

R.G. and A.Tzikas 2509.15887 [gr-qc] (Charged Rotating Casimir Wormholes)

Other Profiles *Physical Sources*

R.G. and A.Tzikas arXiv:2507.19134[gr-qc] accepted in CQG

Time Dependent Casimir Wormholes

Work in Progress

Model I

$$ds^2 = -e^{2\Phi(r,t)} dt^2 + \frac{dr^2}{1 - \frac{b(r,t)}{r}} + r^2(d\theta^2 + \sin^2(\theta)d\phi^2)$$

$$\frac{\dot{b}(r,t)}{1 - \frac{b(r,t)}{r}} = \kappa T_{01}(r,t) \qquad \frac{1}{r^2} \frac{\partial}{\partial r} b'(r,t) = \kappa \rho(r,t)$$

If $\rho(r,t) = \rho(r)T(t)$ then $b(r,t) = b(r)T(t)$

$\rho(r) \rightarrow \rho(r,t)$ $p_r(r) \rightarrow p_r(r,t)$ $p_t(r) \rightarrow p_t(r,t)$ $f(r,t)$  Energy flux or heat flux

$$\rho(d) = -\frac{\hbar c \pi^2}{720 d^4} \quad p_r(d) = -3 \frac{\hbar c \pi^2}{720 d^4} \quad p_t(d) = \frac{\hbar c \pi^2}{720 d^4} \qquad d \rightarrow r$$

Time Dependent Casimir Wormholes

Work in Progress see also papers of S.Kar, S.W. Kim, Anchordoqui et al, Cataldo et al., Kuhfittig and T. Roman

Model II

$$ds^2 = -e^{2\Phi(r,t)} dt^2 + R^2(t) \left(\frac{dr^2}{1 - \frac{b(r)}{r}} + r^2 (d\theta^2 + \sin^2(\theta) d\phi^2) \right)$$

$$2\Phi'(r) \frac{\dot{R}(t)}{R(t)} = \kappa T_{01}(r, t) \qquad \frac{b'(r)}{r^2} + 3 \frac{\dot{R}^2(t)}{R^2(t)} e^{-2\Phi(r,t)} = \kappa \rho(r, t)$$

Model III

$$ds^2 = \Omega^2(t, r) \left(-e^{2\Phi(r)} dt^2 + \frac{dr^2}{1 - \frac{b(r)}{r}} + r^2 (d\theta^2 + \sin^2(\theta) d\phi^2) \right)$$

$$2\Phi'(r) \frac{\dot{\Omega}(t)}{\Omega(t)} = \kappa T_{01}(r, t) \qquad \frac{b'(r)}{r^2 \Omega^2(t)} + 3 \frac{\dot{\Omega}^2(t)}{\Omega^4(t)} e^{-2\Phi(r)} = \kappa \rho(r, t)$$

$\rho(r) \rightarrow \rho(r, t)$ $p_r(r) \rightarrow p_r(r, t)$ $p_t(r) \rightarrow p_t(r, t)$ $f(r, t) \longleftrightarrow$ Energy flux or heat flux

Conclusions and Perspectives

- Casimir energy is the only source of exotic matter that can be generated in laboratory.
- Traversable wormholes can be sustained by Casimir Energy and the related profile can be computed. The wormhole is traversable in principle but not in practice.
- Generalized Absurdly Benign Traversable Wormholes seem to have the right properties for traversability together with Yukawa–Casimir wormholes.
- Combining the original Casimir source with extra sources like a Scalar Field and an Electromagnetic Field seems to be promising!!! (R.G. and A.G.Tzikas in preparation)
- Adding Temperature → more realistic case!! The traversability is not destroyed!!
- Rotations do not destroy the Casimir Wormhole properties (even with an electric field → Work in Progress).
- The time dependent traversable wormhole metric has a strong dependence on the energy flux.
- A time dependent metric offers the possibility of investigating the topology change issue. (See for example A.DeBenedictis, R.G. and F.S.N. Lobo PRD 78 104003 (2008))

Thank You for Your Attention