XVIII BLACK HOLES WORKSHOP

18 - 19 DEC 2025

Anfiteatro Abreu Faro
IST, Lisboa, Portugal

GRAVITY WITH HIGHER-CURVATURE TERMS & SECOND-ORDER FIELD EQUATIONS

$f(\mathcal{R})$ MEETS GAUSS-BONNET

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Based on: arXiv:2510.17965 (PRD in press)

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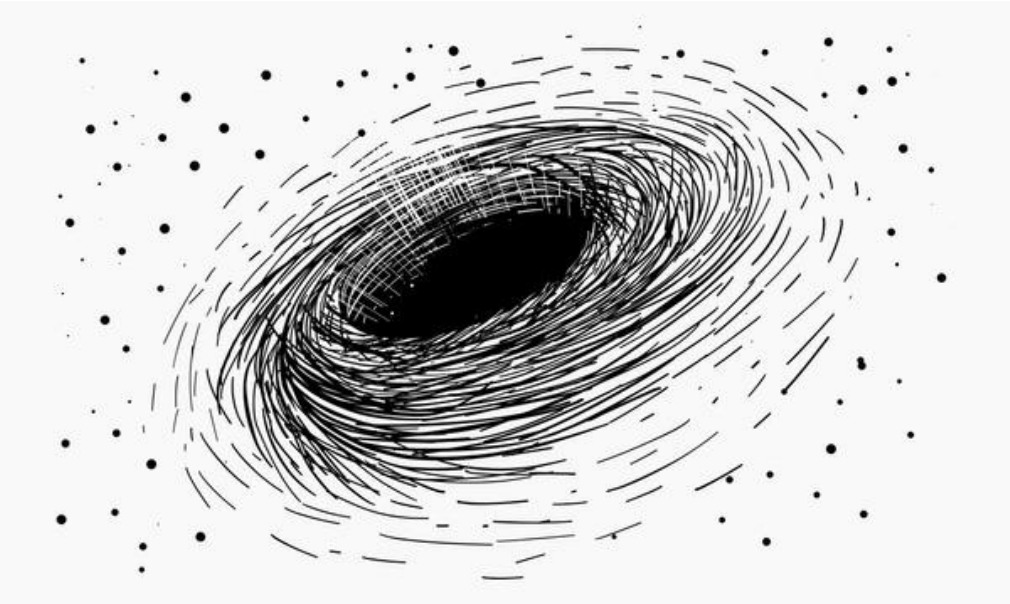
Motivations

GR breaks down in the UV (high curvature regime)

↓

Consider higher-curvature terms? [Stelle (1977)…]



Constructing healthy theories implies having second-order differential equations

$\boldsymbol{f(\mathcal{R})}$

Einstein-dilaton-Gauss-Bonnet

EdGB: nuts and bolts

$$\mathcal{G} = \mathcal{R}^2 - 4\mathcal{R}_{\mu\nu}\mathcal{R}^{\mu\nu} + \mathcal{R}_{\mu\nu\rho\sigma}\mathcal{R}^{\mu\nu\rho\sigma}$$

$$S = \frac{1}{16\pi}\int d^4x\,\sqrt{-g}\,[\mathcal{R} - (\nabla\phi)^2 + 2\eta(\phi)\,\mathcal{G}]$$

Dilatonic coupling

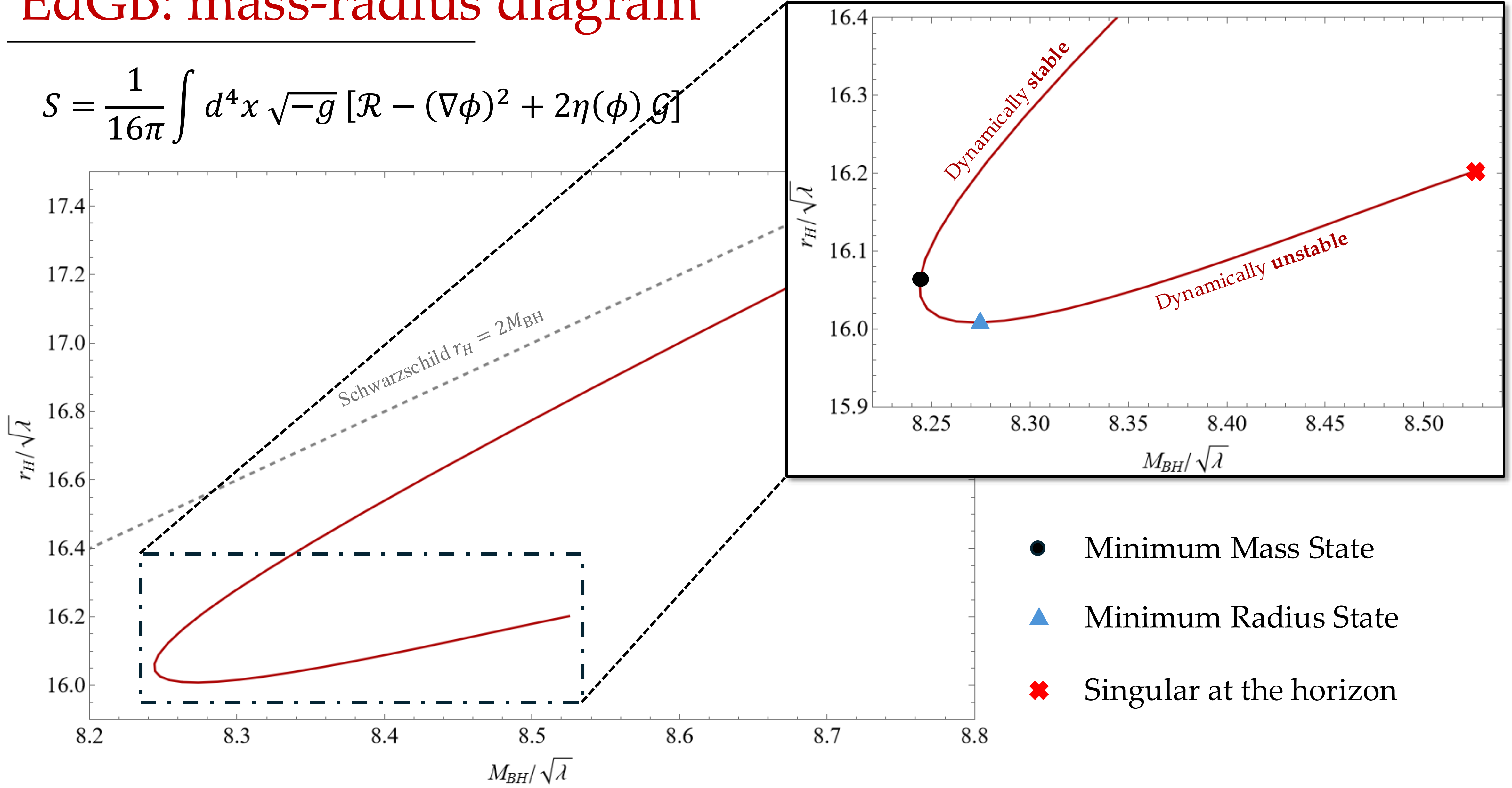
$$\eta(\phi) = \lambda e^{-\gamma\phi}$$

Low-energy truncation of string theory [Gross & Sloan (1987)]

$$[\lambda] = \ell^2$$

EdGB: mass-radius diagram

$$S = \frac{1}{16\pi}\int d^4x\,\sqrt{-g}\,[\mathcal{R} - (\nabla\phi)^2 + 2\eta(\phi)\,\mathcal{G}]$$

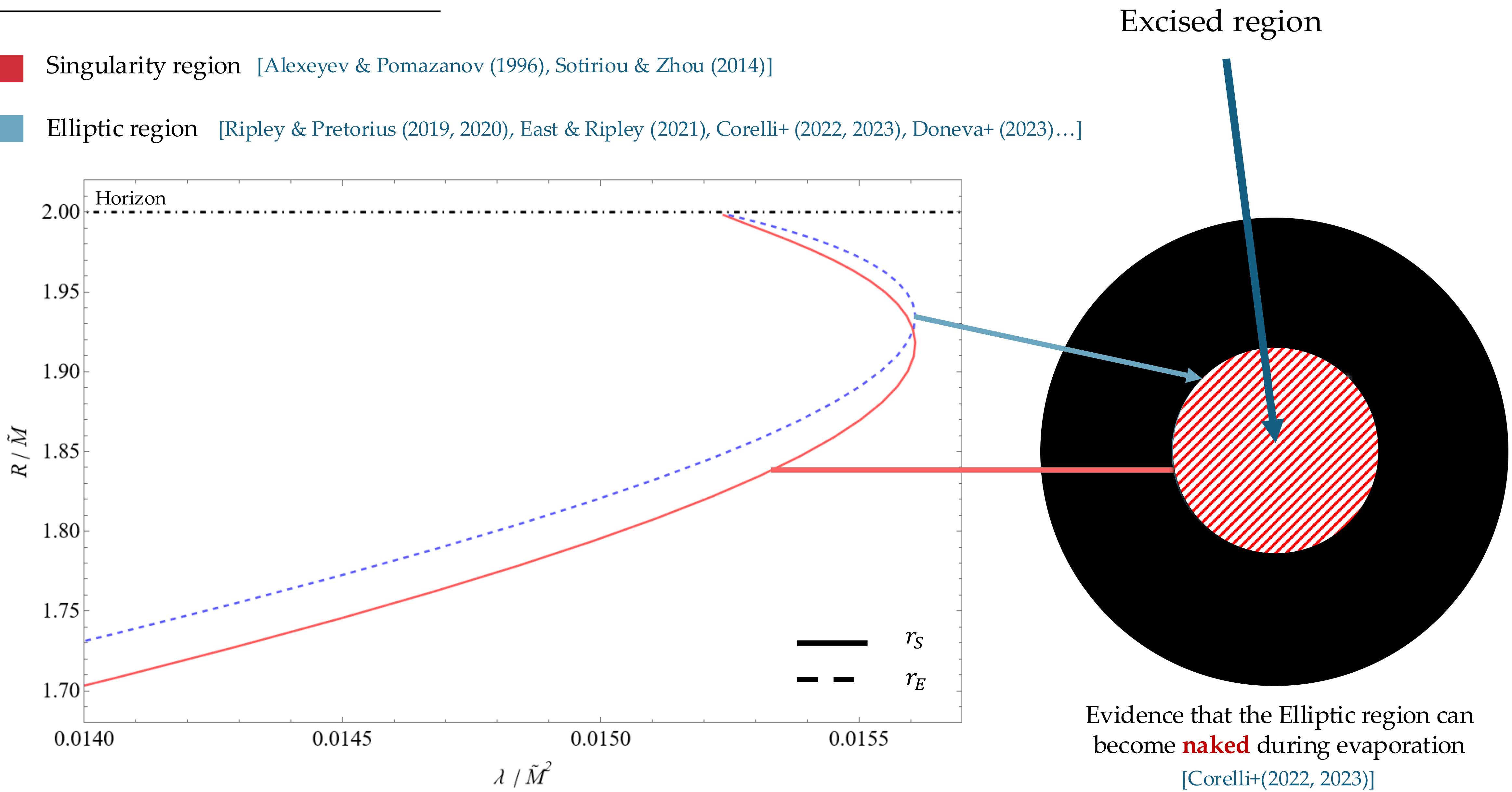


[Kanti+(1996), Torii+(1996), Torii & Maeda(1998), Guo+(2008), Ripley & Pretorius(2020), Corelli+(2022, 2023)]

EdGB: black-hole interior structure

Singularity region [Alexeyev & Pomazanov (1996), Sotiriou & Zhou (2014)]

Elliptic region [Ripley & Pretorius (2019, 2020), East & Ripley (2021), Corelli+ (2022, 2023), Doneva+ (2023)…]



Evidence that the Elliptic region can become **naked** during evaporation

[Corelli+(2022, 2023)]

Some approaches try to address the problem…

Linear coupling between the scalar field and the Ricci scalar [Antoniou+(2021), Thaalba+(2023, 2024)]

$$S = \frac{1}{16\pi}\int d^4x\,\sqrt{-g}\,\left[\mathcal{R} - \frac{1}{2}(\nabla\phi)^2 - \frac{1}{2}\left(\frac{\beta}{2}\mathcal{R} - \alpha\,\mathcal{G}\right)\phi^2\right]$$

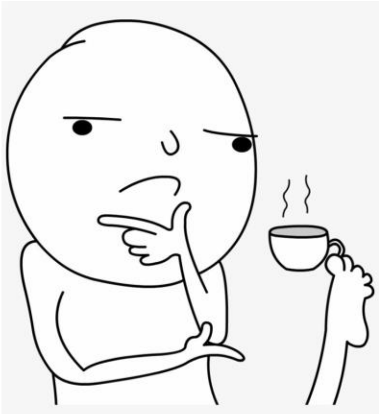
For some specific values of the constants/couplings, the naked elliptic region can be avoided in simulations…

…but the elliptic region is not eliminated entirely

Add regularising higher-order differential operators [Figueras+(2024, 2025)] EFT approach

$f(\mathcal{R})$ –Dilaton-Gauss-Bonnet theory

Corelli, Pani, Sanna, arXiv:2510.17965 (PRD in press)



$f(\mathcal{R})$ –dGB: the theory

$$f(\mathcal{R})$$

$$S = \frac{1}{16\pi} \int d^4x \sqrt{-g} \, [\ \mathcal{R} \quad - (\nabla\phi)^2 + 2\eta(\phi)\, \mathcal{G}]$$

$f(\mathcal{R})$ –dGB: the theory

$$S = \frac{1}{16\pi}\int d^4x\,\sqrt{-g}\,[f(\mathcal{R}) - (\nabla\phi)^2 + 2\eta(\phi)\,\mathcal{G}]$$

Reframe it as a scalar-tensor theory ⟶ Identify the Ricci scalar as a new self-interacting scalar field χ

[Sotiriou & Faraoni (2010), Jaime+(2011)]

Two scalar fields involved

$$S = \frac{1}{16\pi}\int d^4x\,\sqrt{-g}\,[f'(\chi)\mathcal{R} - \mathcal{V}(\chi) - (\nabla\phi)^2 + 2\eta(\phi)\,\mathcal{G}]$$

$$\mathcal{V}(\chi) = f'(\chi)\chi - f(\chi)$$

$f(\mathcal{R})$ –dGB: the theory

$$S = \frac{1}{16\pi}\int d^4x\,\sqrt{-g}\,[f'(\chi)\mathcal{R} - \mathcal{V}(\chi) - (\nabla\phi)^2 + 2\eta(\phi)\,\mathcal{G}]$$

$$f(\mathcal{R}) = \mathcal{R} + \kappa\, R^n$$

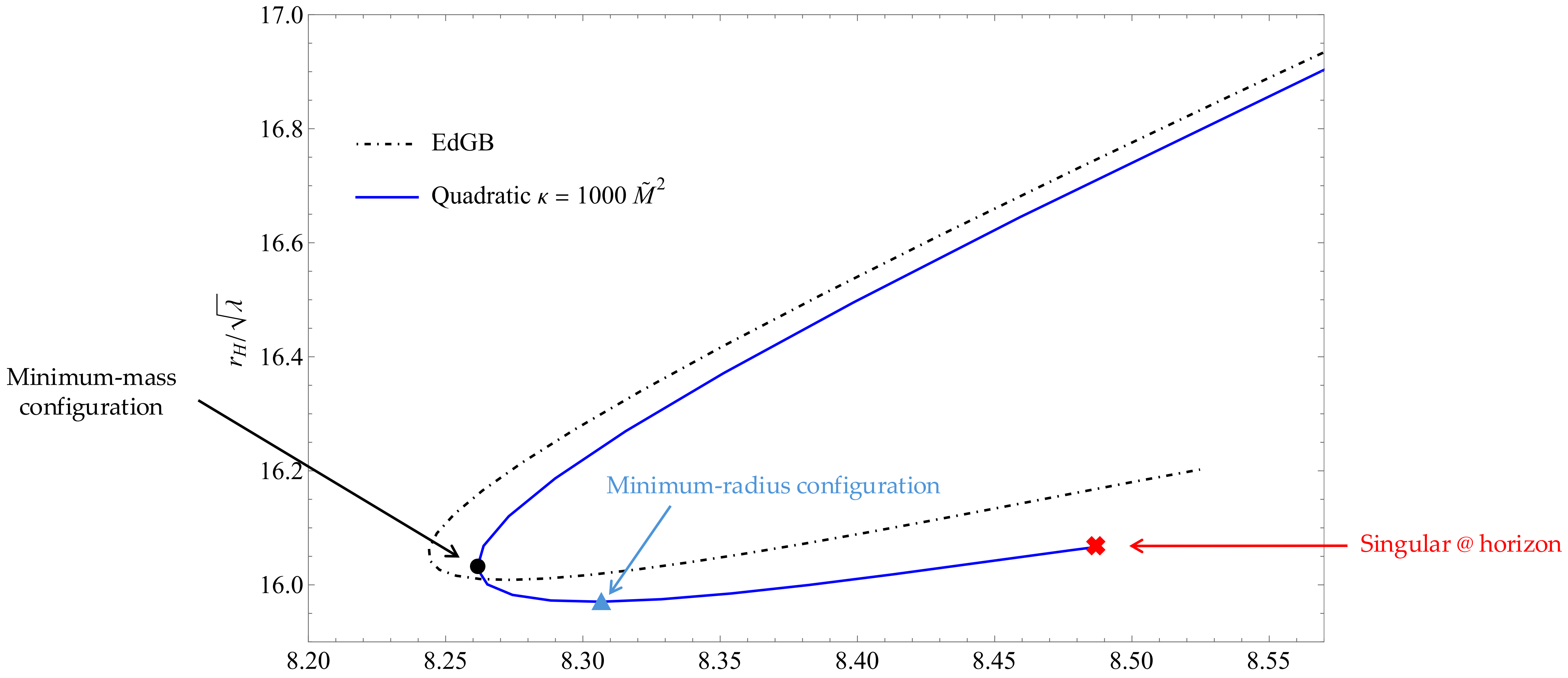
$$\eta(\phi) = \lambda e^{-\gamma\phi}$$

$$[\kappa] = \ell^{2(n-1)}$$

$n = 2$ «Quadratic Theory»

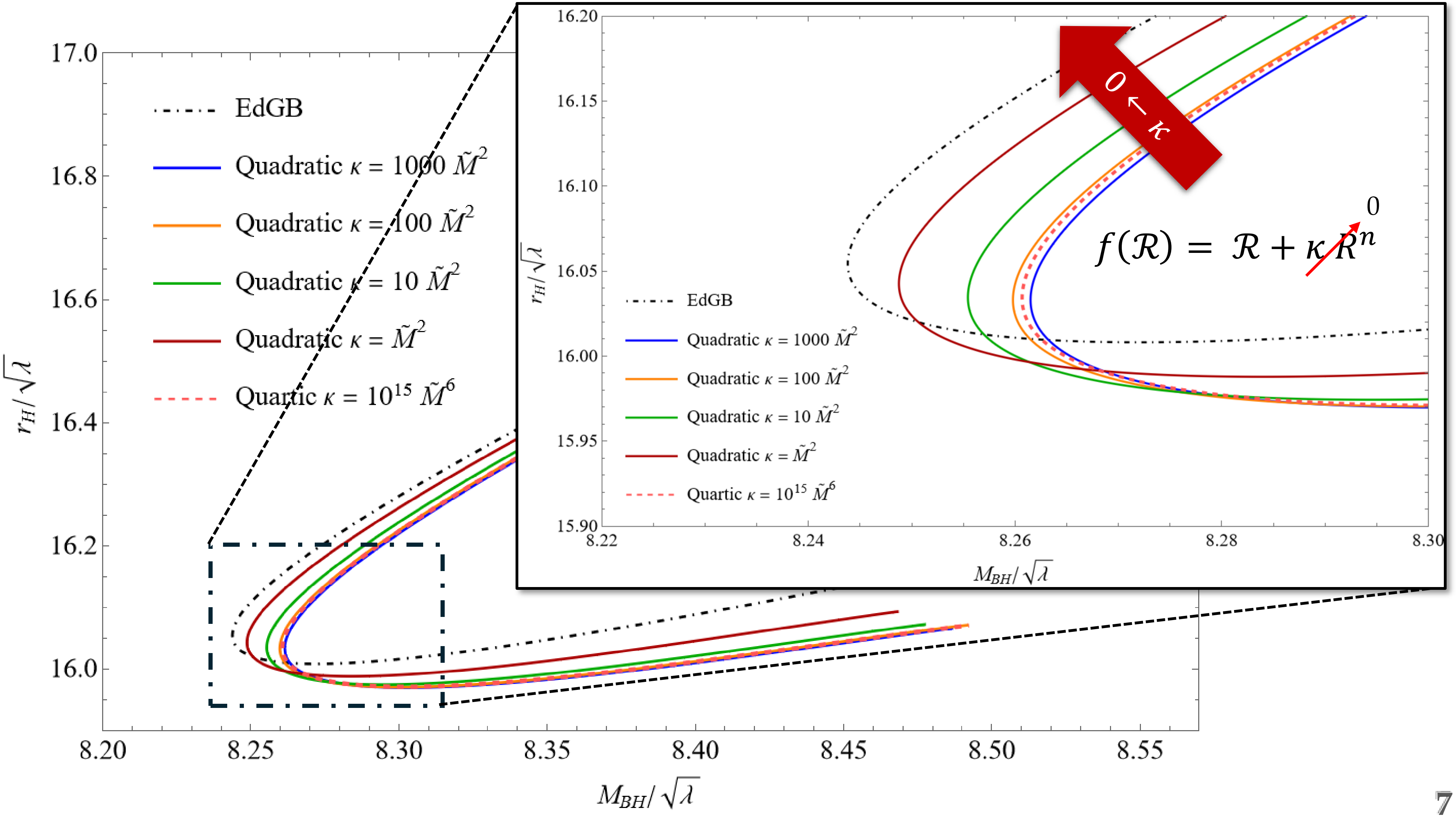
$n = 4$ «Quartic Theory»

$f(\mathcal{R})$ −dGB: mass-radius diagrams



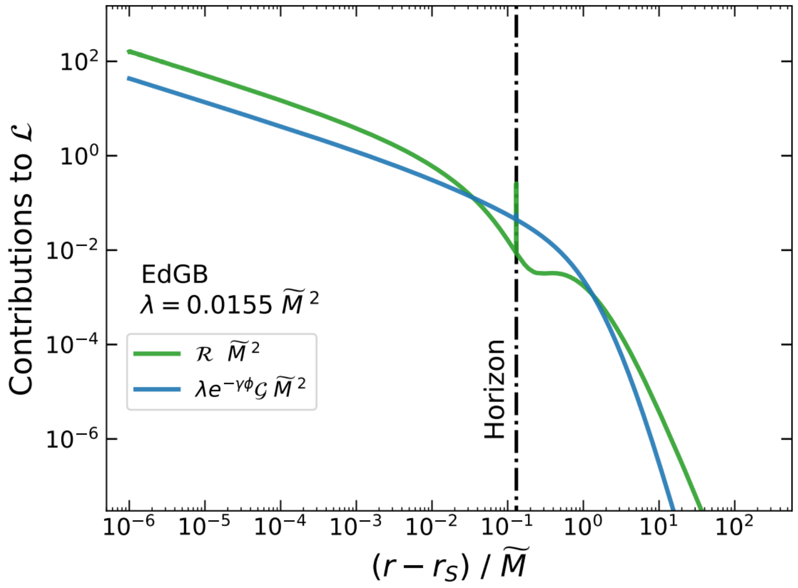
Nonperturbative features of EdGB preserved!

$f(\mathcal{R})$ –dGB: mass-radius diagrams

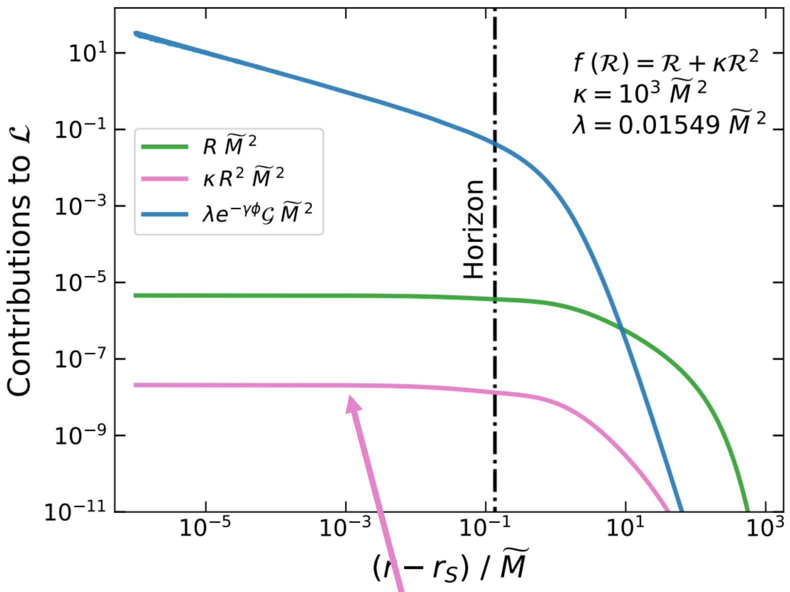


$f(\mathcal{R})$ –dGB : why so similar to EdGB?

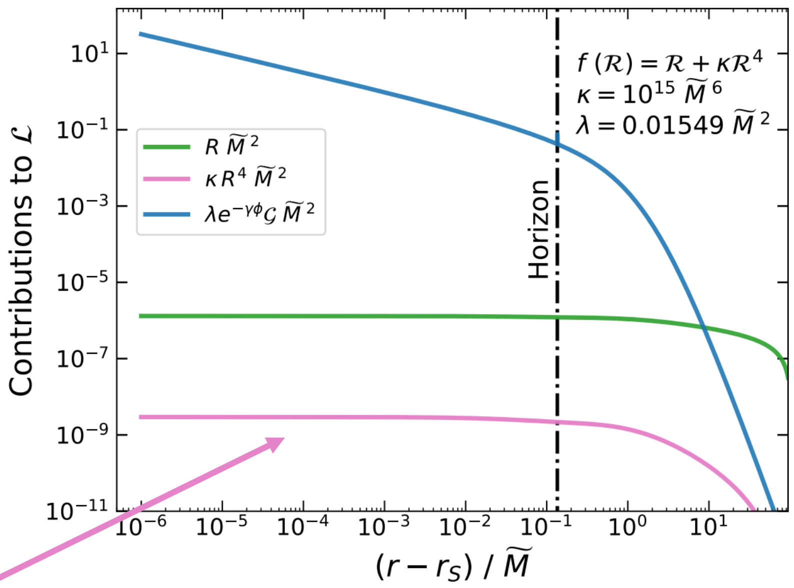
EdGB



$f(\mathcal{R})$-dGB Quadratic



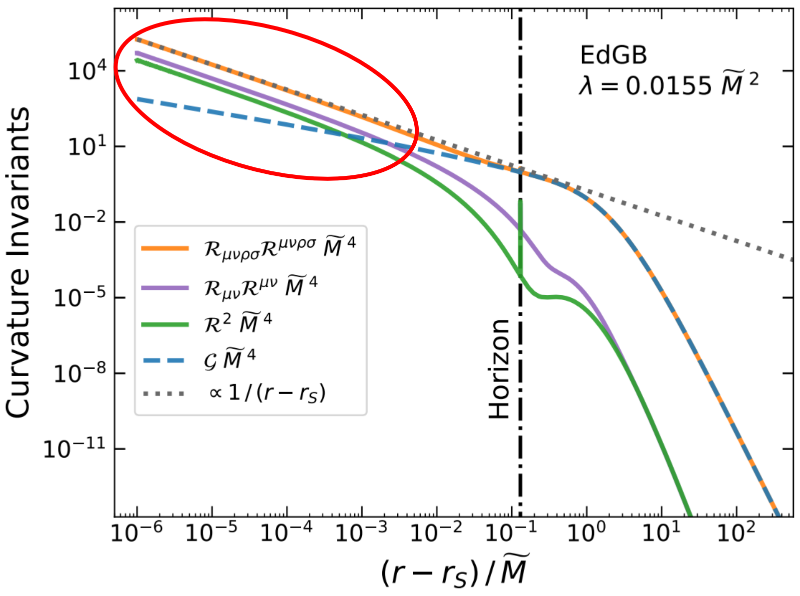
$f(\mathcal{R})$-dGB Quartic



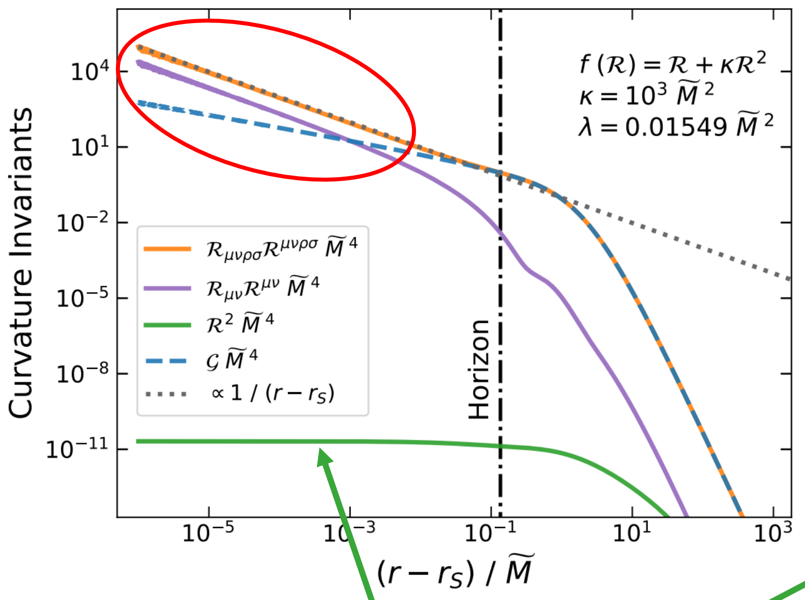
Hihger-curvature corrections always **suppressed**

$f(\mathcal{R})$ –dGB : black-hole interior

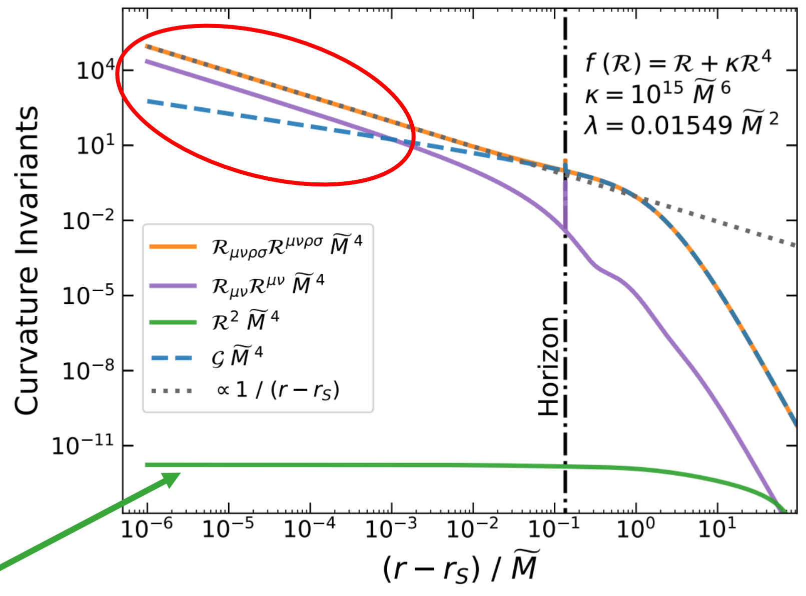
EdGB



$f(\mathcal{R})$-dGB Quadratic

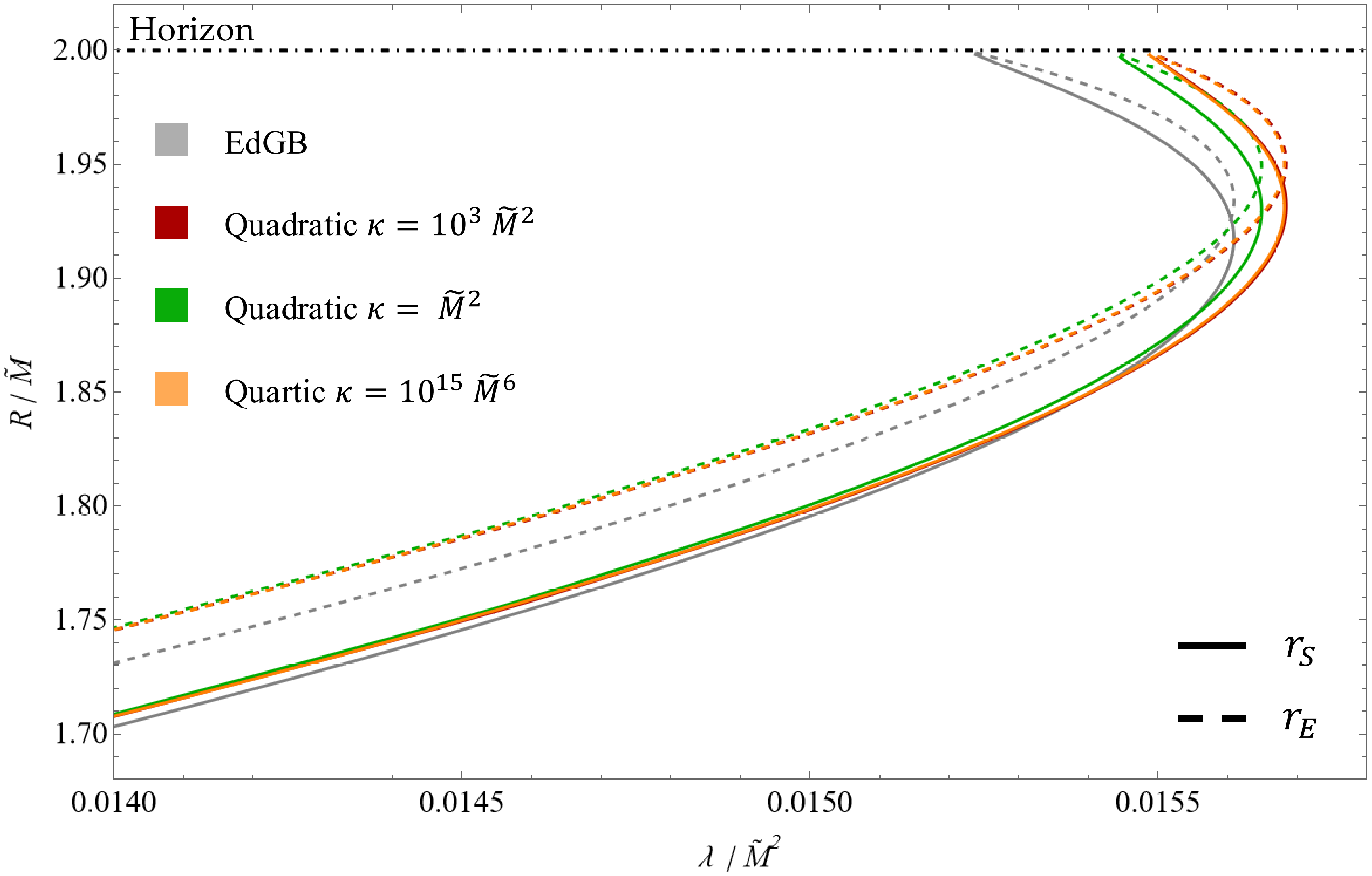


$f(\mathcal{R})$-dGB Quartic



Dynamical mechanism suppressing the Ricci scalar

$f(\mathcal{R})$ −dGB: singularity and elliptic regions



Concluding remarks

The inclusion of individual higher-curvature corrections in EdGB reveals qualitatively similar features …and issues

An infinite tower of higher-curvature terms might be needed [Bueno+ (2025)]

What could be done next? If you are brave enough…

- Description of the formation of these black holes
- Description of Hawking evaporation
- Other numerical simulations (maybe?...)
- Analyze the loss of hyperbolicity in a gauge-invariant way [Reall (2021)]

Corelli, Pani, Sanna, arXiv:2510.17965

Thanks for the attention!

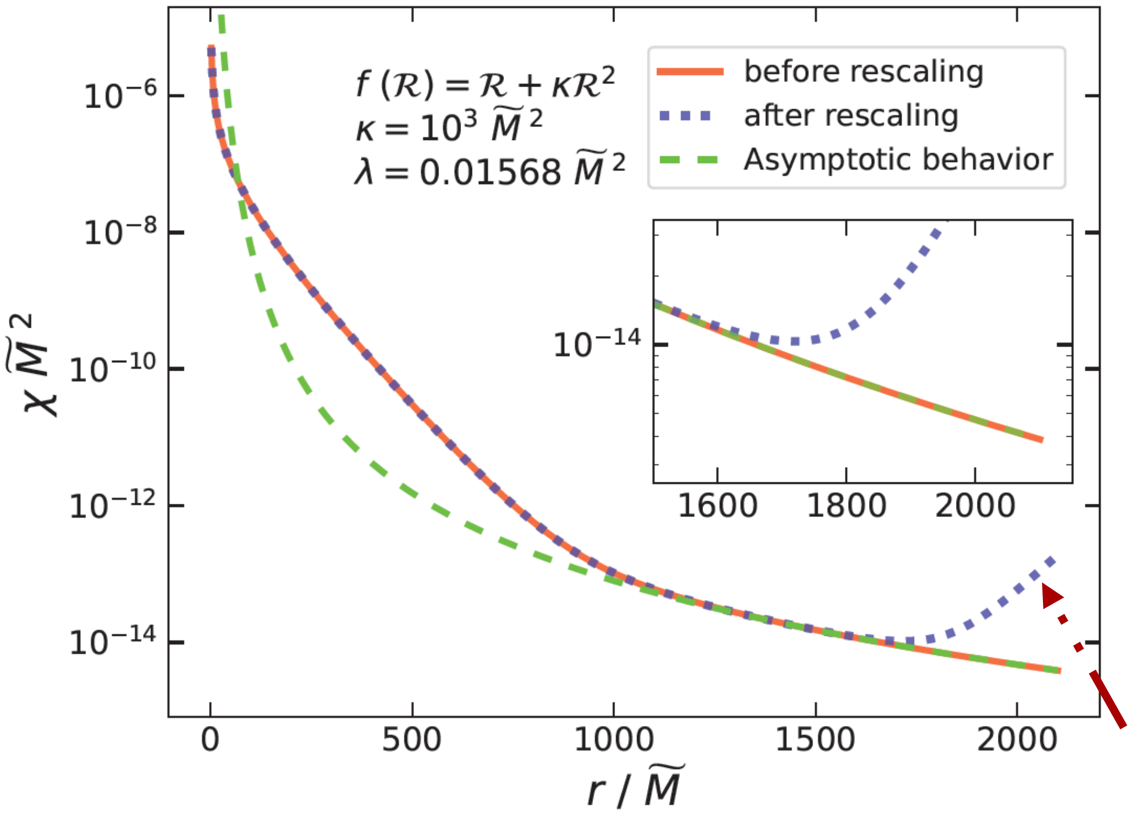
Back-up slides

$f(\mathcal{R})$ −dGB: static asymptotically-flat black holes (exterior)

$$\phi \to \phi + C \qquad \lambda \to \lambda\, e^{\gamma C}$$

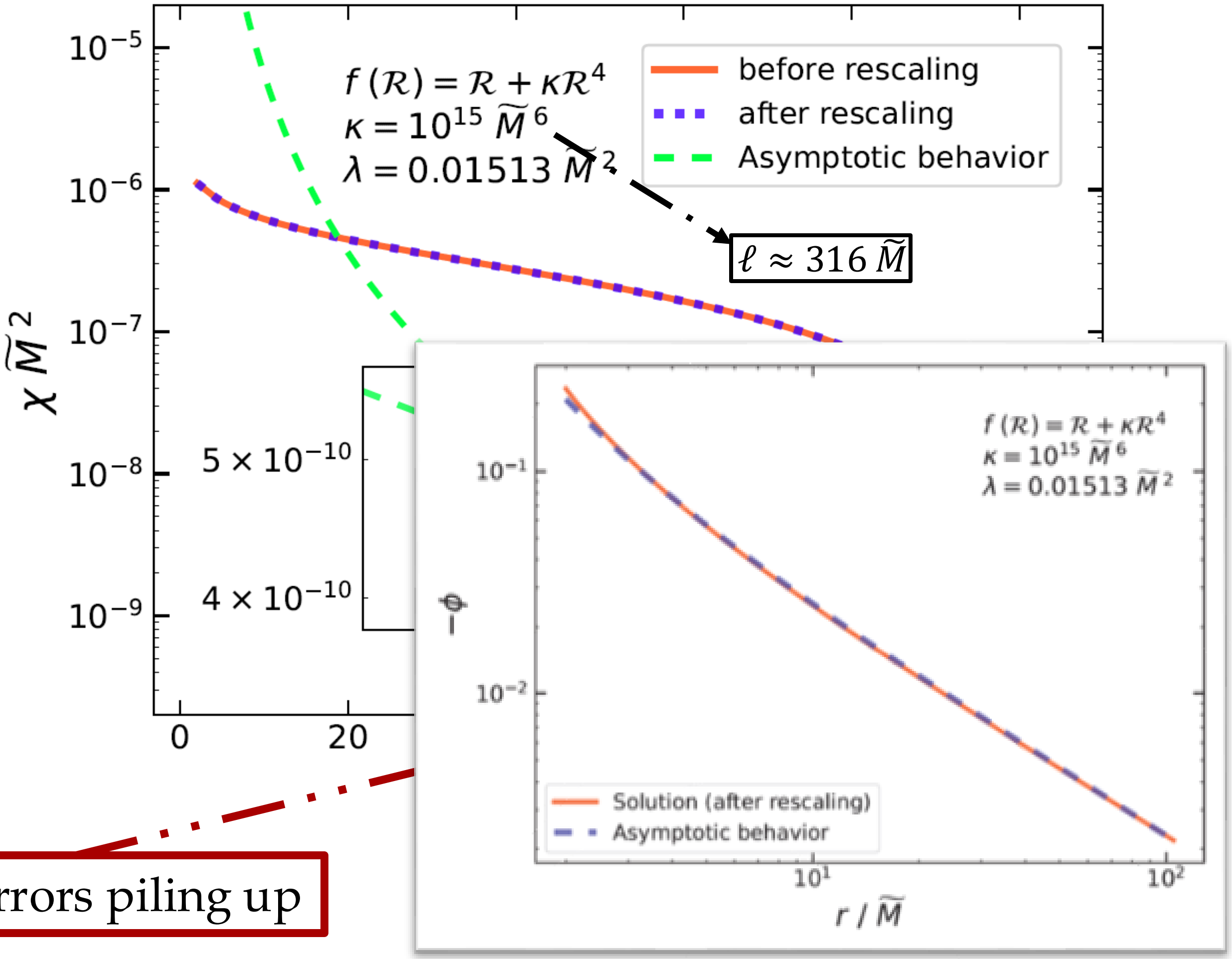
Quadratic case

Accuracy: 35 significant figures



Quartic case

Accuracy: 55 significant figures



Numerical errors piling up

EdGB: loss of hyperbolicity

Starting from initial regular data, does the evolution depend continously on the initial data?

The system of equations must be strongly hyperbolic

[Sarbach & Tiglio (2012), Hilditch (2013)]

Time-dependent case $[\alpha(r), \zeta(r), \phi(r)] \;\rightarrow\; [\alpha(r,t), \zeta(r,t), \phi(r,t)]$

Auxiliary field variable + conjugate momentum

$$Q \equiv \partial_r \phi$$
$$P \equiv \frac{1}{\alpha}\partial_t \phi - \zeta Q$$

$\longrightarrow$ Evolution equations + 2 constraints for α & ζ

Principal symbol $\mathcal{P}_{IJ}(\eta_\mu) = \dfrac{\delta E_{v^I}}{\delta\, \partial_\mu v^J}\eta_\mu \qquad v^I = (\phi, Q, P, \alpha, \zeta)$

Strong hyperbolicity $\equiv$ Complete set of η_μ's satisfying $\det \mathcal{P}(\eta_\mu) = 0$

EdGB: loss of hyperbolicity

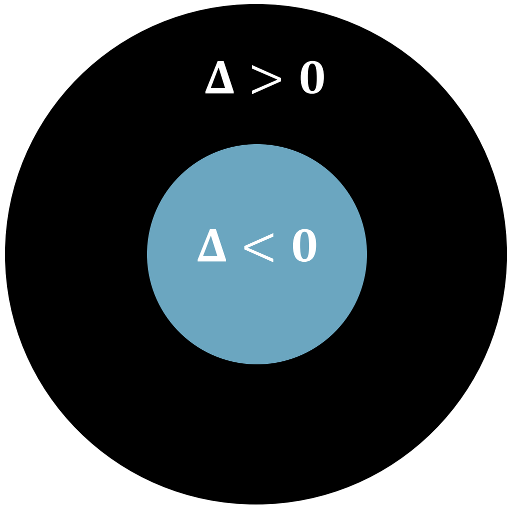
Strong hyperbolicity $\equiv$ Complete set of η_μ's satisfying $\det \mathcal{P}(\eta_\mu) = 0$

$$\det \mathcal{P} \propto \eta_t\, \eta_r^2 \left[\mathfrak{a} \left(\frac{\eta_t}{\eta_r}\right)^2 + \mathfrak{b}\left(\frac{\eta_t}{\eta_r}\right) + \mathfrak{c}\right] = 0$$

$\eta_t = 0$ Redundancy of the equation for $\partial_t \phi$

$\eta_r = 0$ 2 constraints for α and ζ

Real distinct solutions if $\Delta = \mathfrak{b}^2 - 4\mathfrak{a}\mathfrak{c} > 0$



Where $\Delta < 0$, the system is elliptic ⟶ Breakdown of predictability

$f(\mathcal{R})$ −dGB: loss of hyperbolicity

Time-dependent case $[\alpha(r), \zeta(r), \phi(r)] \rightarrow [\alpha(r,t), \zeta(r,t), \phi(r,t)]$

Auxiliary field variables + conjugate momenta

$$
\begin{aligned}
Q &\equiv \partial_r \phi \\
P &\equiv \frac{1}{\alpha}\partial_t \phi - \zeta Q \\
\Theta &\equiv \partial_r \chi \\
\Pi &\equiv \frac{1}{\alpha}\partial_t \chi - \zeta \Theta
\end{aligned}
$$

$\longrightarrow$ Evolution equations + 2 constraints for α & ζ

Principal symbol $\mathcal{P}_{IJ}(\eta_\mu) = \dfrac{\delta E_{v^I}}{\delta\, \partial_\mu v^J}\eta_\mu$ $\qquad v^I = (\phi, Q, \chi, \Theta, P, \Pi, \alpha, \zeta)$

$$
\det \mathcal{P} \propto \eta_t^2\, \eta_r^2 \left[\mathfrak{a}\left(\frac{\eta_t}{\eta_r}\right)^4 + \mathfrak{b}\left(\frac{\eta_t}{\eta_r}\right)^3 + \mathfrak{c}\left(\frac{\eta_t}{\eta_r}\right)^2 + \mathfrak{d}\left(\frac{\eta_t}{\eta_r}\right) + \mathfrak{e}\right] = 0
$$

$f(\mathcal{R})$ −dGB: loss of hyperbolicity

$$\det \mathcal{P} \propto \eta_t^2\, \eta_r^2 \left[\mathfrak{a}\left(\frac{\eta_t}{\eta_r}\right)^4 + \mathfrak{b}\left(\frac{\eta_t}{\eta_r}\right)^3 + \mathfrak{c}\left(\frac{\eta_t}{\eta_r}\right)^2 + \mathfrak{d}\left(\frac{\eta_t}{\eta_r}\right) + \mathfrak{e}\right] = 0$$

$\eta_t = 0$ Redundancy of the equation for $\partial_t \phi$ and $\partial_t \chi$

$\eta_r = 0$ 2 constraints for α and ζ

$$\Delta > 0$$

Real distinct solutions if

$$64\mathfrak{a}^3\mathfrak{e} - 16\mathfrak{a}^2\mathfrak{c}^2 + 16\mathfrak{a}\mathfrak{b}^2\mathfrak{c} - 16\mathfrak{a}^2\mathfrak{b}\mathfrak{d} - 3^4 < 0$$

$$8\mathfrak{a}\mathfrak{c} - 3\mathfrak{b}^2 < 0$$