UNIVERSITÄT HEIDELBERG
ZUKUNFT SEIT 1386

STRUCTURES
CLUSTER OF EXCELLENCE

Regular black holes without mass-inflation instability and gravastars from modified gravity

Pedro G. S. Fernandes

2508.00686 with Astrid Eichhorn

XVIII Black Holes Workshop, Instituto Superior Técnico

18-12-2025

Singularities and regular black holes

- Vacuum black holes in GR have singularities

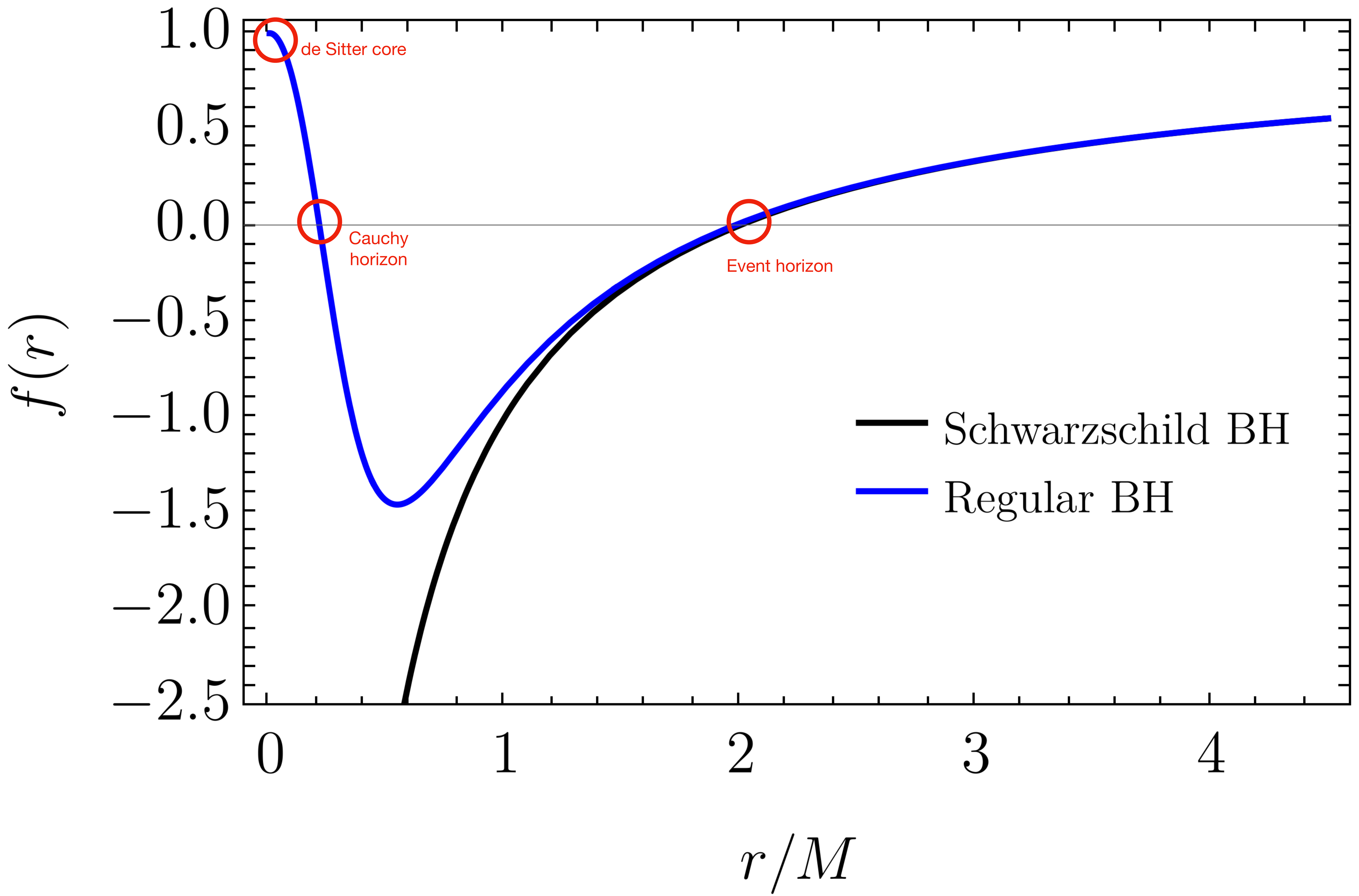
$$ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2 d\Omega^2, \qquad f(r) = 1 - \frac{2M}{r}$$

$$R_{\mu\alpha\nu\beta}R^{\mu\alpha\nu\beta} = \frac{48M^2}{r^6}, \qquad \text{diverges as} \quad r \to 0$$

- Regular black holes are a proposed solution to this problem
- A metric description remains valid in the core, and is non-singular everywhere
- Example: Hayward metric [Hayward, 2006]

$$f(r) = 1 - \frac{2Mr^2}{r^3 + 2M\ell^2}, \qquad f(r) \sim 1 + \mathcal{O}(r^2) \quad \text{near} \quad r = 0$$

Length scale associated with new physics



1.0
0.5
0.0
−0.5
−1.0
−1.5
−2.0
−2.5
0
1
2
3
4
$f(r)$
r/M
de Sitter core
Cauchy horizon
Event horizon
Schwarzschild BH
Regular BH

There are challenges with regular black holes

The main ones

1. Regular BHs of generic mass are ***rarely*** found as solutions to the field equations of a theory [Challenge 1]

2. The Cauchy horizon is **unstable**: if its surface gravity is non-zero, perturbations grow exponentially in a phenomenon known as mass-inflation [Challenge 2]

Here we discuss a framework where both challenges are addressed

The theory

Motivation and brief derivation

- In four-dimensions, under reasonable assumptions, GR is unique (Lovelock's theorem)

- In higher-dimensions the first non-trivial correction to GR comes in the form of a Gauss-Bonnet term, $\mathcal{G} = R_{\mu\alpha\nu\beta}R^{\mu\alpha\nu\beta} - 4R_{\mu\nu}R^{\mu\nu} + R^2$

- In 4D the GB term is topological/trivial

- Much work has been done in recent years to "dimensionally regularize" the GB term to get a 4D-GB theory of gravity [see the review PF et. al, 2022]

$$\sim \lim_{D\to 4} \int \mathrm{d}^D x \sqrt{-g}\, \frac{\mathcal{G} - \text{counterterms}}{D-4}$$

The theory

Motivation and brief derivation

- In performing one of such procedures, by considering different connections with Weyl vectors, we arrive at the following 4D-GB theory

Length scale associated with new-physics

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[R + \ell^2\big(\mathscr{L}[A] - \mathscr{L}[B]\big)\right]$$

where $$\mathscr{L}[X] = 4G^{\mu\nu}X_\mu X_\nu + 8X_\mu X^\mu \nabla_\nu X^\nu + 6(X_\mu X^\mu)^2$$

- It is a 4D theory, with two Proca fields, and second-order equations of motion
- For details see [Charmousis et. al 2025, Eichhorn and PF 2025]

Black hole solutions and regularity

$$A_\mu \mathrm{d}x^\mu = a(r)\left(\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{f(r)}\right) \qquad B_\mu \mathrm{d}x^\mu = b(r)\left(\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{f(r)}\right)$$

Primary hair　　　　Primary hair

$$a(r) = \frac{r - rf(r) - q_a/2}{2r^2}, \qquad b(r) = \frac{r - rf(r) - q_b/2}{2r^2}$$

$$f(r) = 1 - \frac{2Mr^2 + \ell^2(q_a^2 - q_b^2)/(4r)}{r^3 + (q_a - qb)\ell^2}$$

Regularity is achieved if $q_b = -\, q_a$

$$f(r) = 1 - \frac{2Mr^2}{r^3 + 2q_a\ell^2}$$

Usual Hayward metric is recovered when $q_a = M$

Challenge 1

We have a theory where black holes of all masses can be regular

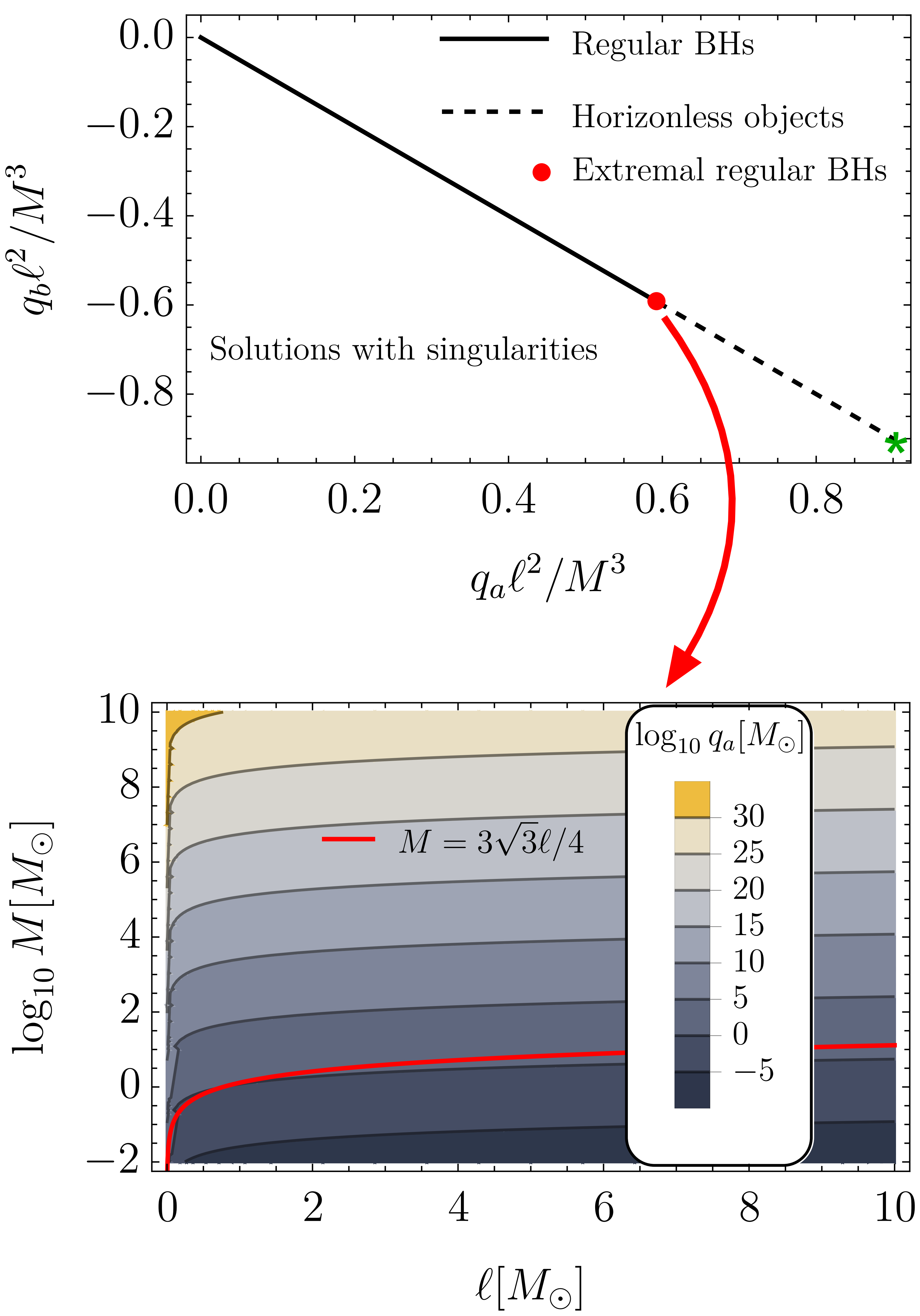
What about Challenge 2? (Stability against mass-inflation instability)

Mass-inflation instability

- A black hole with a non-extremal Cauchy horizon generally suffers from mass-inflation instability
- No instability only if the Cauchy horizon is extremal (zero surface gravity) [Carballo-Rubio et. al, 2022]
- Regular extremal black holes of all masses are possible in our theory if

$$q_a = \frac{16M^3}{27\ell^2}$$

$$f(r) = 1 - \frac{2Mr^2}{r^3 + \frac{32}{27}M^3}$$



0.0
−0.2
−0.4
−0.6
−0.8
$q_b\ell^2/M^3$
Regular BHs
Horizonless objects
Extremal regular BHs
Solutions with singularities
0.0
0.2
0.4
0.6
0.8
$q_a\ell^2/M^3$
10
8
6
4
2
0
−2
$\log_{10} M[M_\odot]$
$M = 3\sqrt{3}\ell/4$
$\log_{10} q_a[M_\odot]$
30
25
20
15
10
5
0
−5
0
2
4
6
8
10
$\ell[M_\odot]$

Challenge 2

We have a theory where black holes of all masses can be regular and free from mass-inflation instability

Bonus: Gravastars

- Gravastars are proposed alternatives to regular black holes [Mazur and Mottola 2001]
- Regular BHs smoothly interpolate between a de Sitter core and a Schwarzschild exterior
- Gravastars are defined by a sharp transition

$$f(r) = \begin{cases} 1 - \frac{\Lambda_{\text{eff}}}{3} r^2, & r \leq r_{\text{trans}} \\ 1 - \frac{2M}{r}, & r > r_{\text{trans}} \end{cases}$$

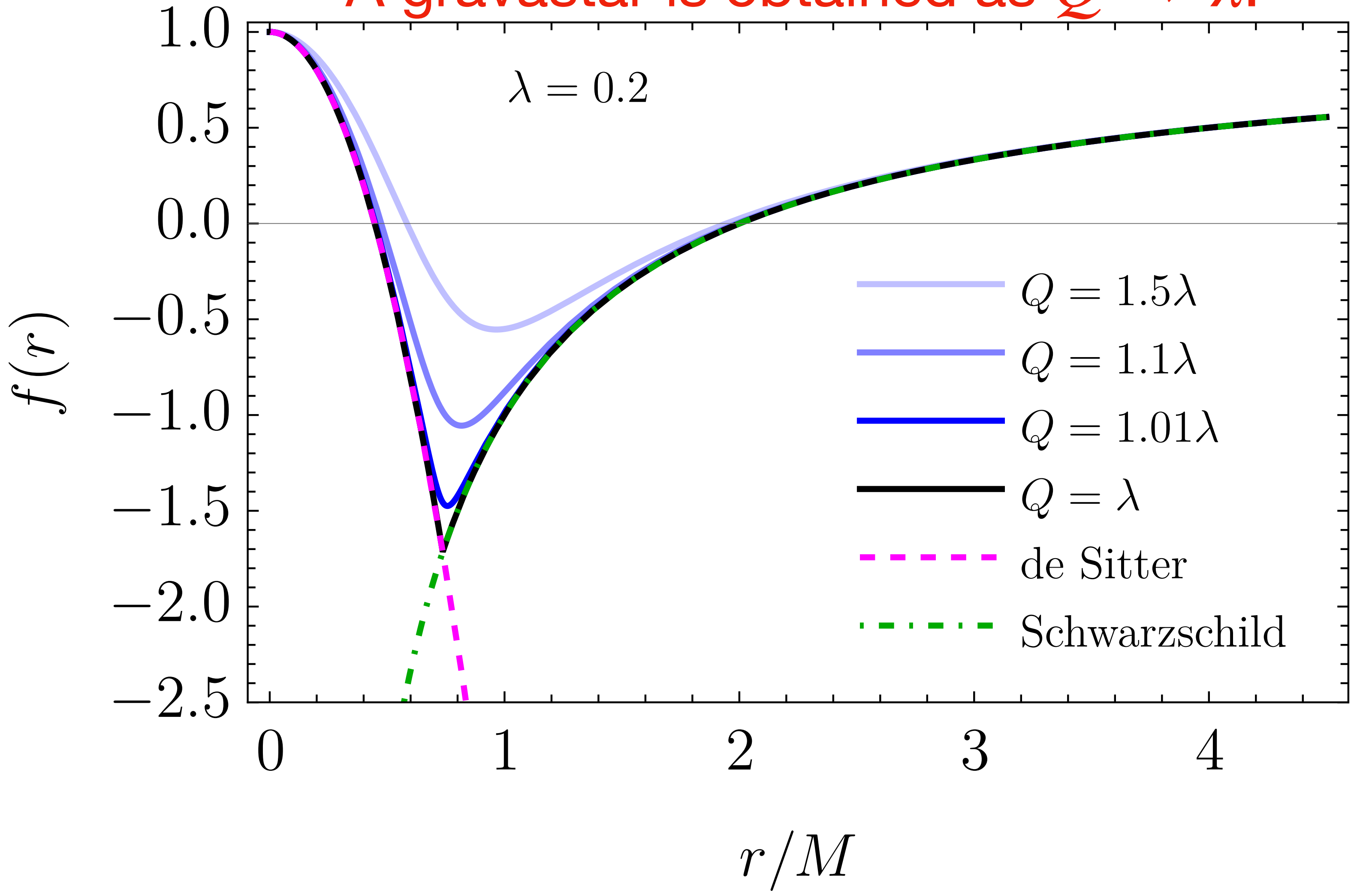
- Thin shell is needed
- No known theories where gravastars arise as solutions

Bonus: Gravastars

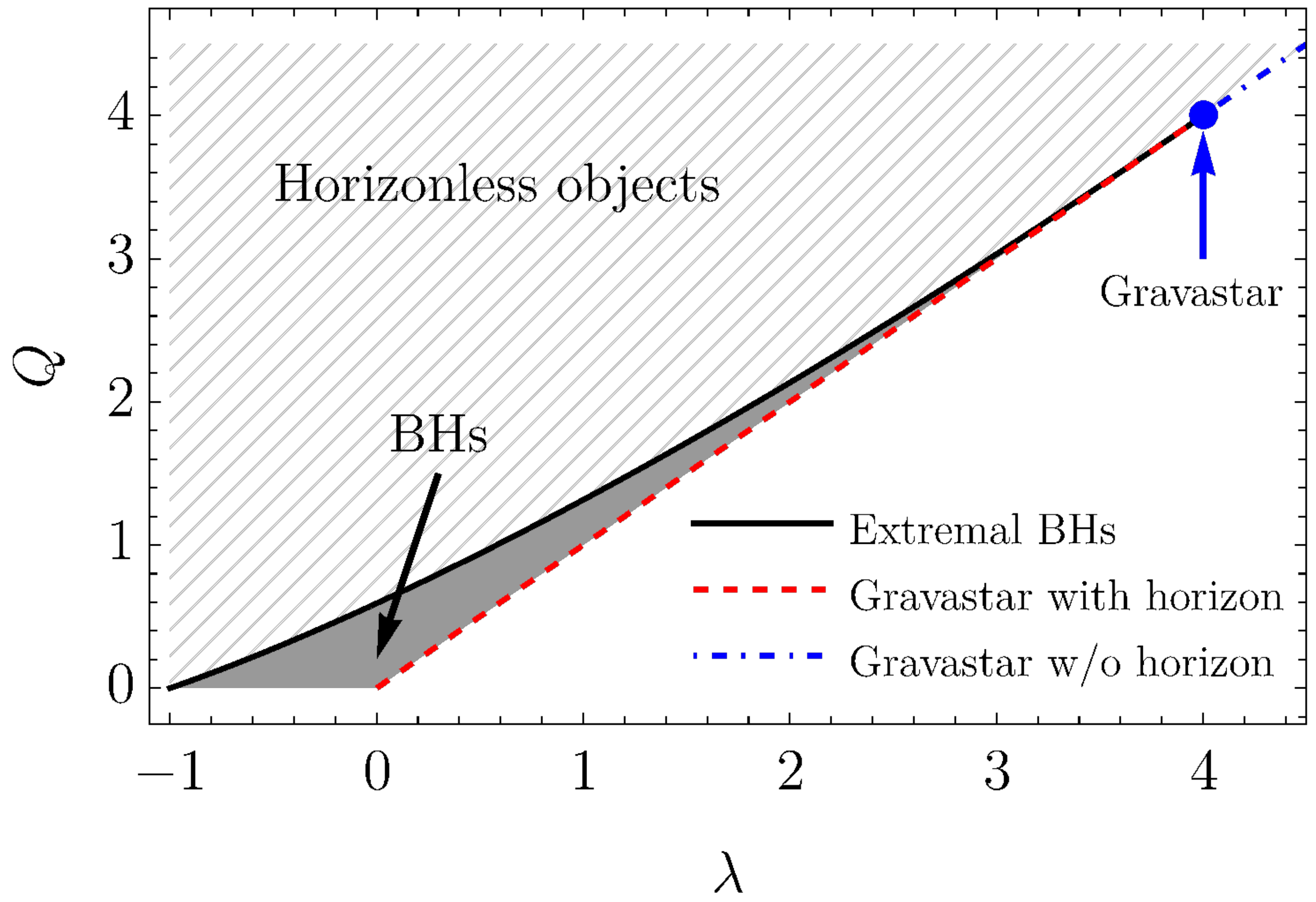
$$S = \int \mathrm{d}^4x\sqrt{-g}\left[R + \ell^2\left(\mathscr{L}[A] - \kappa\mathscr{L}[B]\right)\right]$$

$$f(r) = 1 - \frac{MQ}{r\lambda} - \frac{r^2}{2\lambda M^2}\left(1 - \sqrt{\left(1 + \frac{2M^3Q}{r^3}\right)^2 - \frac{8\lambda M^3}{r^3}}\right), \qquad \text{where} \qquad Q = \frac{q_a\ell^2(1+\sqrt{\kappa})}{2M^3}, \qquad \lambda = \frac{(1-\kappa)\ell^2}{M^2}$$

A gravastar is obtained as $Q \to \lambda$!



$r_{\text{trans}} = M(2\lambda)^{1/3}$



Open questions

- Are the theories healthy? No canonical kinetic term for the Proca fields. Is this an issue?
- Can the values of q_a that eliminate mass-inflation be achieved dynamically? Numerical simulations needed
- Are these regular BHs good dark matter candidates? If BHs of all masses are extremal, do we solve the information paradox?
- Are the horizonless compact objects (obtained for large q_a) stable against the light-ring instability?
- Sizeable deviations from GR at all masses. Observational signatures?
- …

Thank you for your attention!