Backreaction effects in a recursive Penrose process in Reissner-Nordström-AdS black hole spacetimes

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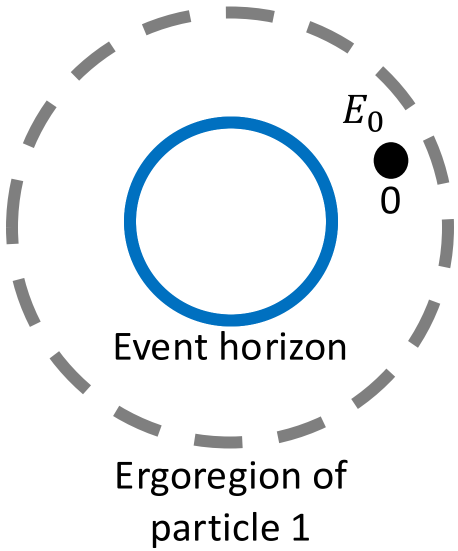
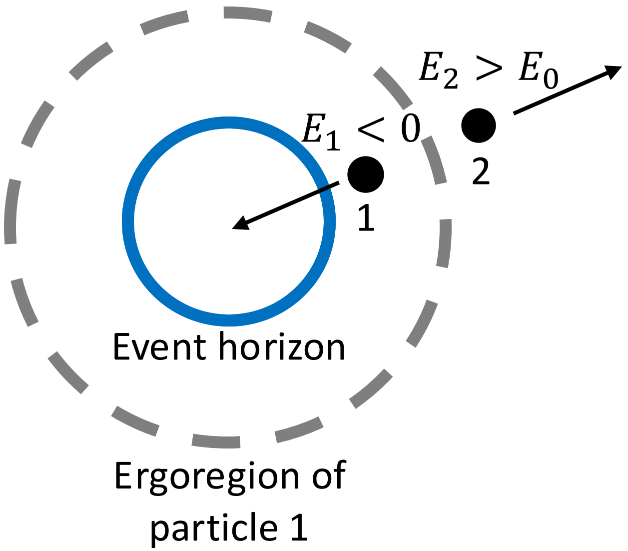
Motivation

Feiteira, Lemos, Zaslavskii; Phys.Rev.D 109 (2024) 6, 064065

- **Penrose process:**





- Particle 2 is reflected: **recursive Penrose process**.

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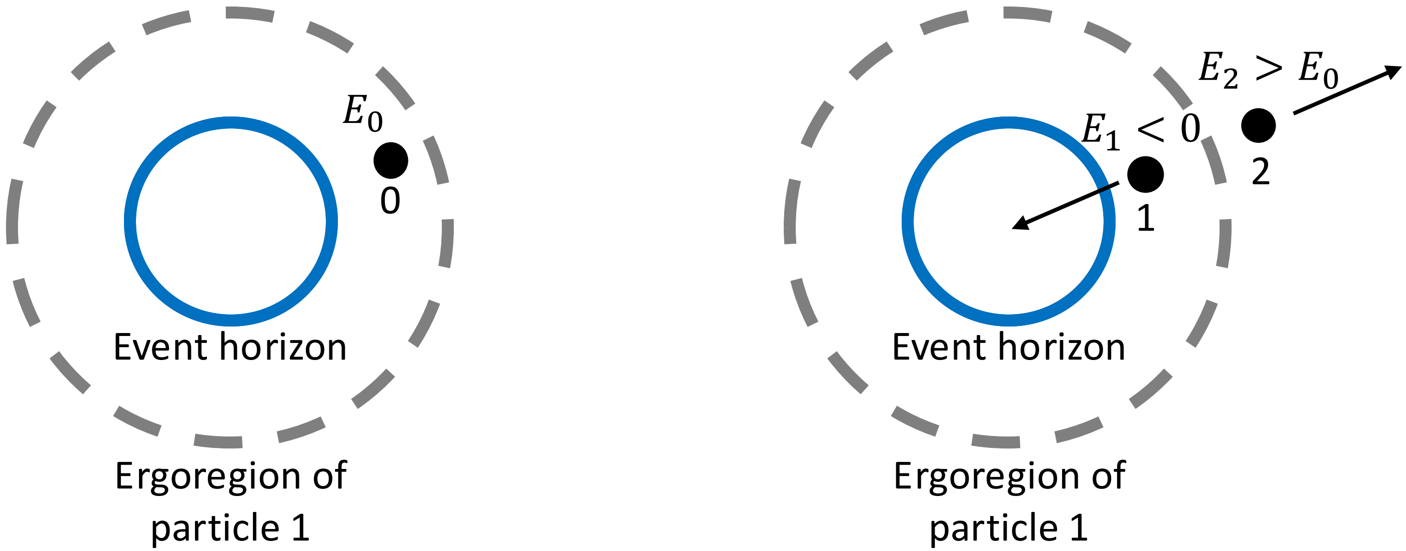

- Particle 2 is reflected: **recursive Penrose process**.

- A recursive Penrose process has been studied in Reissner-Nordström-AdS black hole spacetimes **neglecting backreaction effects**.

- Two possible situations: Black hole **energy factories** and black hole **bombs**.

- A infinitely long chain of decays is expected to **change significantly the charge and mass of the black hole**.

Line element and equations of motion

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- Line element of RN-AdS black hole spacetime:

$$ds^2 = -f(r)\,dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2, \qquad f(r) = \frac{r^2}{l^2} + 1 - \frac{2M}{r} + \frac{Q^2}{r^2}$$

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- Equations of motion for particles with mass m and charge e:

$$p^t \equiv m\,\dot{t} = \frac{X}{f}, \qquad p^r \equiv m\dot{r} = \sigma P$$

$$X = E - \frac{eQ}{r}, \qquad P = \sqrt{X^2 - m^2 f}$$

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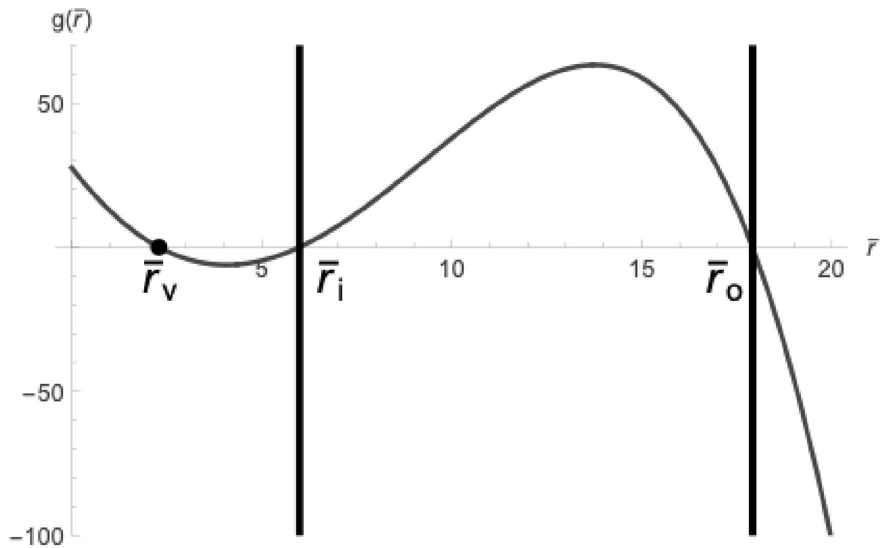

Conditions for the decay and backreaction

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- Just after the $(n+1)$-th decay and right before particle $2n+1$ falls into the black hole:

$$M_n = M_0 + E_1 + E_3 + \cdots + E_{2n+1} = M_0 + \sum_{k=0}^{n-1} E_{2k+1}$$

$$Q_n = Q_0 + e_1 + e_3 + \cdots + e_{2n+1} = Q_0 + \sum_{k=0}^{n-1} e_{2k+1}$$

$$f_n(r) = \frac{r^2}{l^2} + 1 - \frac{2M_n}{r} + \frac{Q_n^2}{r^2}$$

Energy, mass, and charge of the particles

- Assuming particles are at rest immediately after each decay, $P = 0$:

$$E_{2n} = m_{2n}\sqrt{f_n(r_\mathrm{i})} + \frac{e_{2n}Q_n}{r_\mathrm{i}}, \quad E_{2n+1} = m_{2n+1}\sqrt{f_n(r_\mathrm{i})} + \frac{e_{2n+1}Q_n}{r_\mathrm{i}}, \quad E_{2n+2} = m_{2n+2}\sqrt{f_n(r_\mathrm{i})} + \frac{e_{2n+2}Q_n}{r_\mathrm{i}}$$

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- Mass of the particles:

$$m_{2n+1} = \alpha_1 m_{2n}, \qquad m_{2n+2} = \alpha_2 m_{2n}$$

- Mass conservation at each decay: $\alpha_1 + \alpha_2 = 1$

$$m_{2n+1} = \alpha_1 (1-\alpha_1)^n \, m_0, \qquad m_{2n+2} = \alpha_2^{n+1} \, m_0$$

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- Charge of the particles:

$$e_{2n+1} = \beta_1 e_{2n}, \qquad e_{2n+2} = \beta_2 e_{2n}$$

- Charge conservation at each decay: $\beta_1 + \beta_2 = 1$

- Odd particles must have negative energy: $\beta_1 < 0$ (assuming $Q_0 > 0$)

$$e_{2n+1} = \beta_1(1-\beta_1)^n \, e_0, \qquad e_{2n+2} = \beta_2^{n+1} \, e_0$$

Charge of the black hole and index n_c

- Charge of the black hole:

$$\left.\begin{aligned} Q_n &= Q_0 + \sum_{k=0}^{n-1} e_{2k+1} \\ e_{2n+1} &= \beta_1 (1-\beta_1)^n \, e_0 \end{aligned}\right\} \quad Q_n = Q_0 - e_0\left((1-\beta_1)^n - 1\right)$$

Charge of the black hole and index n_c

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- Definition of the index n_c: $Q_{n_c} = 0$

$$n_c = \frac{\ln\left(1 + \frac{Q_0}{e_0}\right)}{\ln \beta_2}$$

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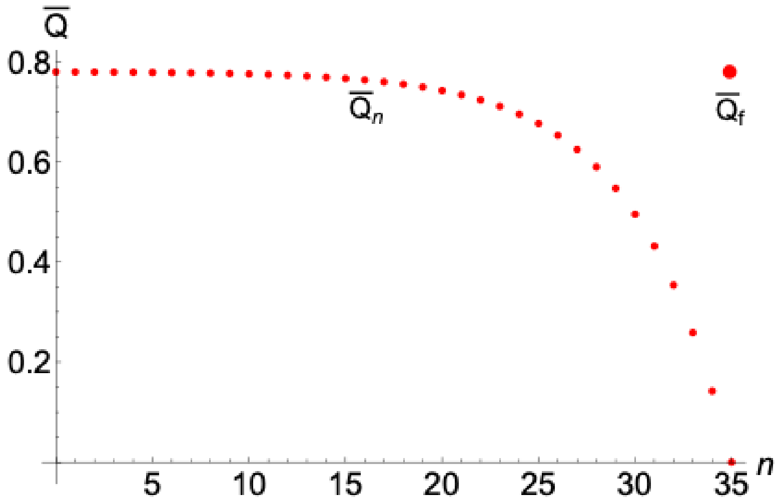
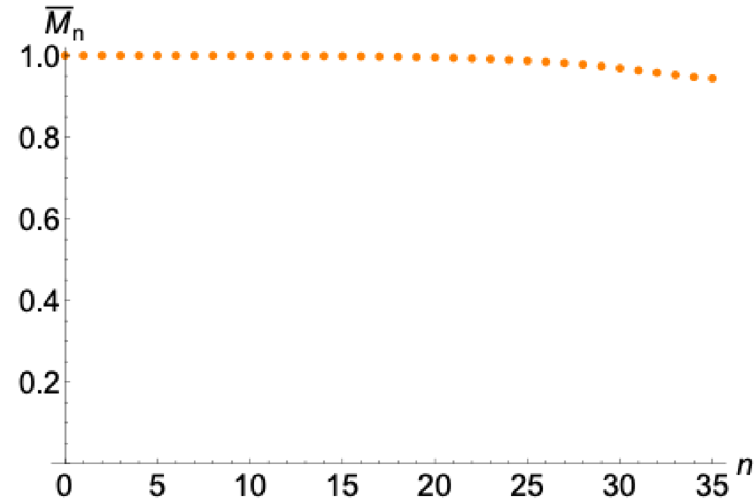
- **Case 1, integer $\boldsymbol{n_c}$:** the charge of the black hole reaches exactly zero. The recursive Penrose process terminates naturally.

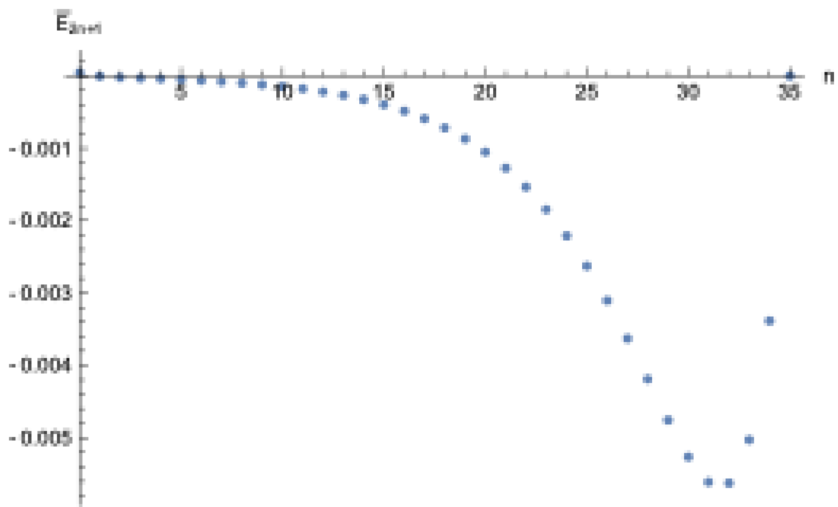
- **Case 2, non-integer $\boldsymbol{n_c}$:** the charge of the black hole approaches but does not reach zero. The last viable decay occurs for the closest integer lower than $\boldsymbol{n_c}$. After that, spherical symmetry is no longer a valid approximation.

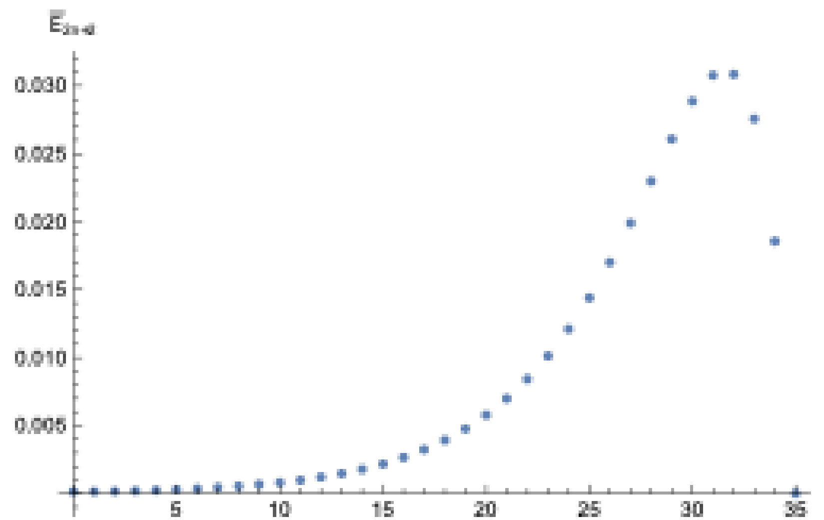
Case 1, integer n_c

- **Result of the process:**
  Black hole with charge $Q_0 + e_0$;
  Finite energy gain – black hole energy factory;
  No black hole bomb as obtained without backreaction effects.



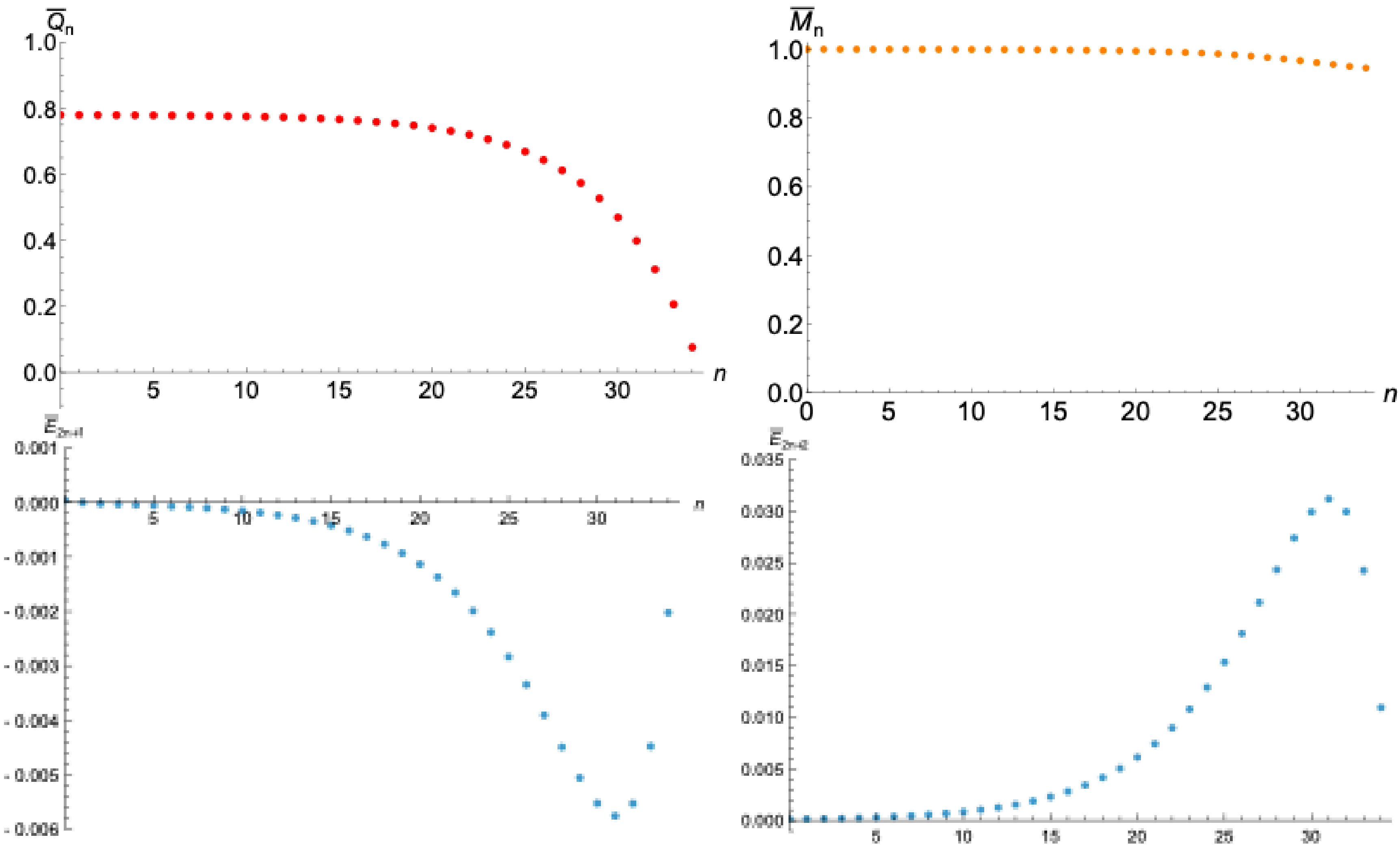






Case 2, non-integer n_c

- **Result of the process:**
  Black hole with arbitrary small charge and particle with large charge;
  No spherical symmetry – two body problem in GR;
  Electric repulsion should make the two bodies fall apart.



Conclusions

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- Backreaction effects turn black hole bombs into **black hole energy factories**.
- Particle confinement can be ensured due to the **negative cosmological constant of AdS** spacetime.
- The AdS turning point is totally **equivalent to a reflective mirror**.

Conclusions

- Backreaction effects turn black hole bombs into **black hole energy factories**.

- Particle confinement can be ensured due to the **negative cosmological constant of AdS** spacetime.

- The AdS turning point is totally **equivalent to a reflective mirror**.

- Depending on the initial charges of the black hole and decaying particles, one can either get at the end of the process a **black hole with very large or very small charge**.

- For **non-integer index n_c**, one cannot resolve the full process within a spherical symmetry and test particle approximation: **2-body problem in GR**.

Thank you!

Oulanka National Park, Finland