

Constraints on regular black holes with nonlinear electromagnetic fields

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XVIII BLACK HOLES WORKSHOP, LISBON
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in collaboration with

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Intro
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No-go
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REGULAR BLACK HOLES?

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are singular branches absent?

NON-SINGULAR GENERAL RELATIVISTIC GRAVITATIONAL
COLLAPSE.

James M. Bardeen

University of Washington, USA

$$ds^2 = \phi(r) dt^2 - \phi(r)^{-1} dr^2 - r^2(d\theta^2 + \sin^2\theta d\varphi^2)$$

$$\phi = 1 - 2mr^2(r^2 + r_0^2)^{-3/2}$$

REGULAR BHs WITH NLE

- ★ E. Ayón-Beato and E. García
PLB 493 (2000) 149–152

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- ★ NonLinear Electromagnetism
- ★ proliferation of NLE theories 2000–2025

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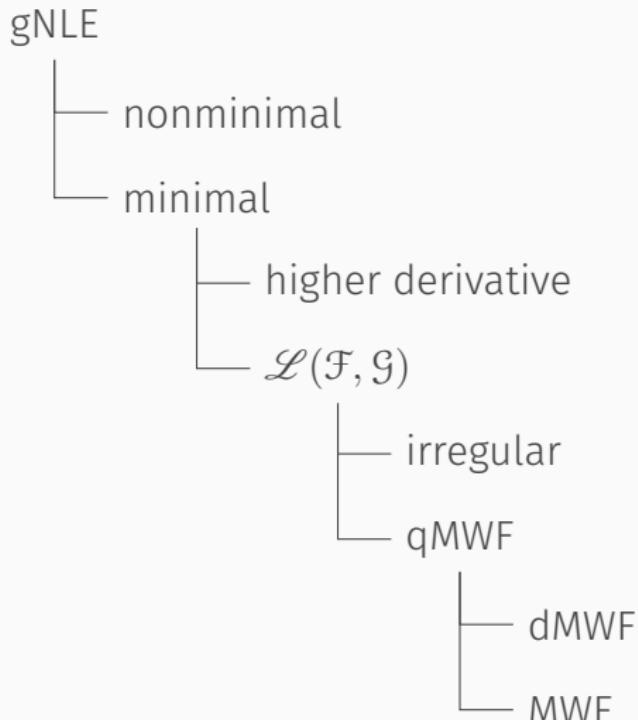
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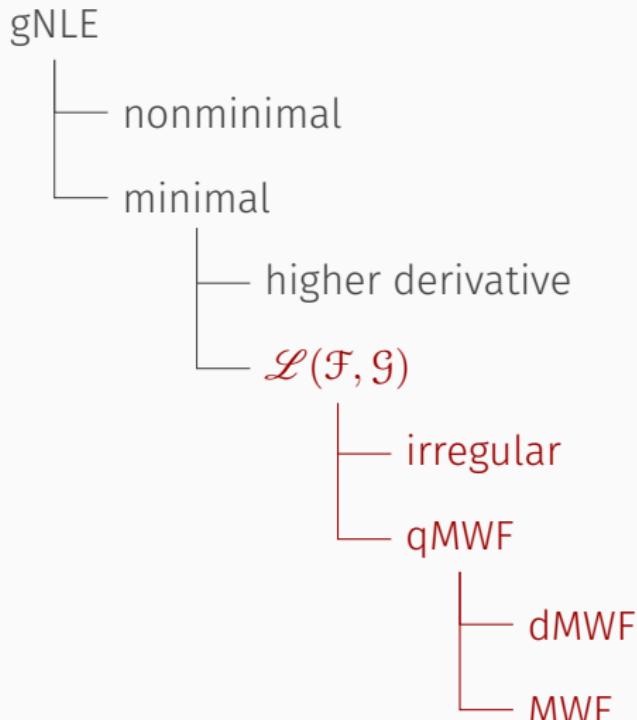
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- ★ more general family of NLE Lag's $\mathcal{L}(\mathcal{F}, \mathcal{G})$
- ★ Maxwellian weak field (**MWF**) limit if
 $\mathcal{L}_{\mathcal{F}}(\mathcal{F}, \mathcal{G}) = -1/4 + O(\mathcal{H}), \quad \mathcal{L}_{\mathcal{G}}(\mathcal{F}, \mathcal{G}) = O(\mathcal{H})$
as $\mathcal{H} \rightarrow 0$, where $\mathcal{H} := \sqrt{\mathcal{F}^2 + \mathcal{G}^2}$





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Necessity of black hole mass–charge constraint

WEAK FIELD LIMIT CONSTRAINTS

ARXIV: 2206.07064 | PRD **106** (2022) 064020

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→ inequalities $R^2 \leq 4R_{ab}R^{ab} \leq 6R_{abcd}R^{abcd}$

Eur. Phys. J. C, **85** (2025) 818 [arXiv: 2502.12242]

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→ bounding via field equations

$$R, R_{ab}R^{ab} \leftrightarrow g^{ab}T_{ab}, T_{ab}T^{ab} \leftrightarrow \mathcal{L}_{\mathcal{F}}\mathcal{F}, \mathcal{L}_{\mathcal{F}}\mathcal{G}$$

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 - bounding via field equations
 $R, R_{ab}R^{ab} \leftrightarrow g^{ab}T_{ab}, T_{ab}T^{ab} \leftrightarrow \mathcal{L}_F \mathcal{F}, \mathcal{L}_F \mathcal{G}$
 - investigate details of the generalized Maxwell's equations
 $\mathcal{L}_F E(r) - \mathcal{L}_G B(r) = -Q/(4r^2)$ and $B(r) = P/r^2$

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$$\mathcal{L}(\mathcal{F}, \mathcal{G}) \quad \text{qMWF} \quad Q \neq 0, P = 0$$

$$\mathcal{L}(\mathcal{F}) \quad Q \neq 0 \neq P$$

$$-\mathcal{F}/4 + h(\mathcal{G}) \quad P \neq 0$$

$$-\mathcal{F}/4 + a\mathcal{F}^r\mathcal{G}^s \quad \text{MWF} \quad Q \neq 0 \neq P$$

$$-\mathcal{F}/4 + a\mathcal{F}^2 + b\mathcal{F}\mathcal{G} + c\mathcal{G}^2 \quad \text{MWF} \quad P \neq 0$$

$$(b, c) \neq (0, 0) \text{ if } Q = 0$$

MASS-CHARGE CONSTRAINTS

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$$(r(f(r) - 1))' = 2r^2 \mathcal{L}(2P^2/r^4, 0)$$

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$$\frac{f(r; M, P) - 1}{r^2} = \frac{-2M + |P|^{\frac{3}{2}} h(P/r^2)}{r^3}$$

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$$I := \lim_{r \rightarrow 0^+} h(P/r^2) = -2^{-\frac{1}{4}} \int_0^\infty \mathcal{L}(u, 0) u^{-\frac{7}{4}} \, du.$$

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★ limit $r \rightarrow 0^+$ implies $M = (I/2)|P|^{\frac{3}{2}}$

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- ★ Which **higher derivative** EM theories admit regular black holes?
- ★ What about **rotating** regular black holes?

Thank you for the attention!

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