Constraints on regular black holes with nonlinear electromagnetic fields

Ivica Smolić

University of Zagreb

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in collaboration with

Ana Bokulić

University of Aveiro

Tajron Jurić

Institute Ruđer Bošković, Zagreb

Regular black holes?

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  energy conditions?

  are singular branches absent?

NON-SINGULAR GENERAL RELATIVISTIC GRAVITATIONAL COLLAPSE.

James M.Bardeen
University of Washington, USA

$$ds^2=\phi(r)dt^2-\phi(r)^{-1}dr^2-r^2(d\theta^2+\sin^2\theta\, d\varphi^2)$$

$$\phi=1-2mr^2(r^2+r_0^2)^{-3/2}$$

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- ⋆ E. Ayón-Beato and E. García
  PLB **493** (2000) 149–152

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- ⋆ proliferation of NLE theories 2000–2025

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- ⋆ more general family of NLE Lag's $\mathscr{L}(\mathcal{F},\mathcal{G})$
- ⋆ Maxwellian weak field (**MWF**) limit if

  $\mathscr{L}_\mathcal{F}(\mathcal{F},\mathcal{G}) = -1/4 + O(\mathcal{H})$, $\mathscr{L}_\mathcal{G}(\mathcal{F},\mathcal{G}) = O(\mathcal{H})$

  as $\mathcal{H} \to 0$, where $\mathcal{H} := \sqrt{\mathcal{F}^2 + \mathcal{G}^2}$

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    - higher derivative
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Necessity of black hole mass–charge constraint

Weak field limit constraints

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→ inequalities $R^2 \leq 4R_{ab}R^{ab} \leq 6R_{abcd}R^{abcd}$

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→ bounding via field equations

$R,\ R_{ab}R^{ab} \ \leftrightarrow\ g^{ab}T_{ab},\ T_{ab}T^{ab} \ \leftrightarrow\ \mathscr{L}_{\mathcal{F}}\mathcal{F},\ \mathscr{L}_{\mathcal{F}}\mathcal{G}$

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→ investigate details of the generalized Maxwell's equations
$\mathscr{L}_{\mathcal{F}}E(r) - \mathscr{L}_{\mathcal{G}}B(r) = -Q/(4r^2)$ and $B(r) = P/r^2$

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$\mathscr{L}(\mathfrak{F}, \mathfrak{G})$	qMWF	$Q \neq 0$, $P = 0$
$\mathscr{L}(\mathfrak{F})$		$Q \neq 0 \neq P$
$-\mathfrak{F}/4 + h(\mathfrak{G})$		$P \neq 0$
$-\mathfrak{F}/4 + a\mathfrak{F}^r \mathfrak{G}^s$	MWF	$Q \neq 0 \neq P$
$-\mathfrak{F}/4 + a\mathfrak{F}^2 + b\mathfrak{F}\mathfrak{G} + c\mathfrak{G}^2$	MWF	$P \neq 0$
$(b, c) \neq (0, 0)$ if $Q = 0$		

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$$I := \lim_{r\to 0^+} h(P/r^2) = -2^{-\frac{1}{4}}\int_0^\infty \mathscr{L}(u,0)u^{-\frac{7}{4}}\,\mathrm{d}u.$$

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- ⋆ limit $r \to 0^+$ implies $M = (I/2)|P|^{\frac{3}{2}}$

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- ⋆ Which **higher derivative** EM theories admit regular black holes?
- ⋆ What about **rotating** regular black holes?

Thank you for the attention!

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