

# Constraints on regular black holes with nonlinear electromagnetic fields

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XVIII BLACK HOLES WORKSHOP, LISBON

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in collaboration with

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are singular branches absent?

NON-SINGULAR GENERAL RELATIVISTIC GRAVITATIONAL  
COLLAPSE.

James M. Bardeen  
University of Washington, USA

$$ds^2 = \phi(r) dt^2 - \phi(r)^{-1} dr^2 - r^2 (d\theta^2 + \sin^2 \theta d\varphi^2)$$

$$\phi = 1 - 2m r^2 (r^2 + r_0^2)^{-3/2}$$

## REGULAR BHS WITH NLE

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PLB **493** (2000) 149–152

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- ★ NonLinear Electromagnetism
- ★ proliferation of NLE theories 2000–2025

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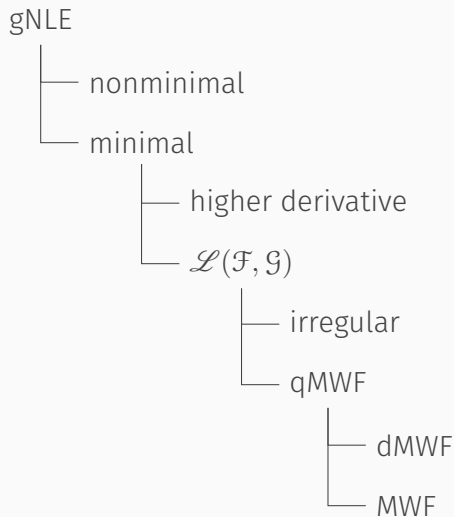
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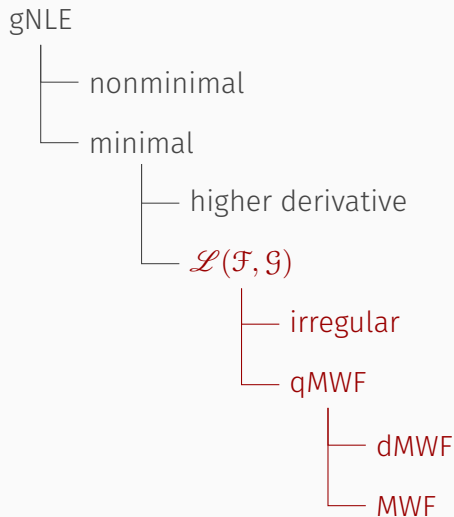
★ more general family of NLE Lag's  $\mathcal{L}(\mathcal{F}, \mathcal{G})$

★ Maxwellian weak field (**MWF**) limit if

$$\mathcal{L}_{\mathcal{F}}(\mathcal{F}, \mathcal{G}) = -1/4 + O(\mathcal{H}), \quad \mathcal{L}_{\mathcal{G}}(\mathcal{F}, \mathcal{G}) = O(\mathcal{H})$$

as  $\mathcal{H} \rightarrow 0$ , where  $\mathcal{H} := \sqrt{\mathcal{F}^2 + \mathcal{G}^2}$





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Necessity of black hole mass-charge constraint

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$$R, R_{ab}R^{ab} \leftrightarrow g^{ab}T_{ab}, T_{ab}T^{ab} \leftrightarrow \mathcal{L}_{\mathcal{F}}\mathcal{F}, \mathcal{L}_{\mathcal{F}}\mathcal{G}$$

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→ investigate details of the generalized Maxwell's equations

$$\mathcal{L}_{\mathcal{F}}E(r) - \mathcal{L}_{\mathcal{G}}B(r) = -Q/(4r^2) \quad \text{and} \quad B(r) = P/r^2$$

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$\mathcal{L}(\mathcal{F}, \mathcal{G})$	qMWF	$Q \neq 0, P = 0$
$\mathcal{L}(\mathcal{F})$		$Q \neq 0 \neq P$
$-\mathcal{F}/4 + h(\mathcal{G})$		$P \neq 0$
$-\mathcal{F}/4 + a\mathcal{F}^r\mathcal{G}^s$	MWF	$Q \neq 0 \neq P$
$-\mathcal{F}/4 + a\mathcal{F}^2 + b\mathcal{F}\mathcal{G} + c\mathcal{G}^2$	MWF	$P \neq 0$
$(b, c) \neq (0, 0)$ if $Q = 0$		

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$$I := \lim_{r \rightarrow 0^+} h(P/r^2) = -2^{-\frac{1}{4}} \int_0^\infty \mathcal{L}(u, 0) u^{-\frac{7}{4}} du.$$

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- ★ limit  $r \rightarrow 0^+$  implies  $M = (I/2)|P|^{\frac{3}{2}}$

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- ★ Which **higher derivative** EM theories admit regular black holes?
- ★ What about **rotating** regular black holes?

# Thank you for the attention!

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