Canonical ensemble of a self-gravitating matter thin shell in an AdS cavity

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TÉCNICO LISBOA

centra

grit gravitation in técnico

1. Why this problem?

Standard statistical mechanics $\neq$ **Gravitational thermodynamics**

Standard statistical mechanics

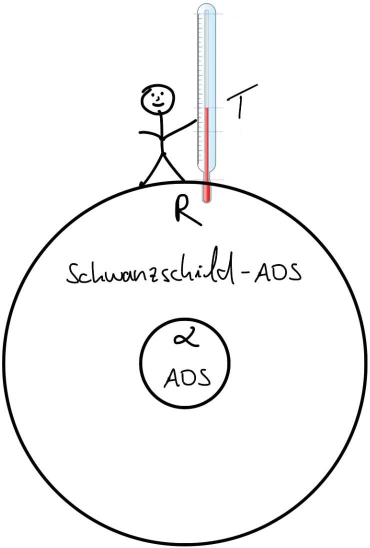
- Short-range interactions
- Additive energy and entropy
- Uniform temperature at equilibrium
- Equivalence of ensembles

Gravitational thermodynamics

- Long-range interactions
- Non-additive energy
- Redshifted (non-uniform) temperature
- Microcanonical is privileged

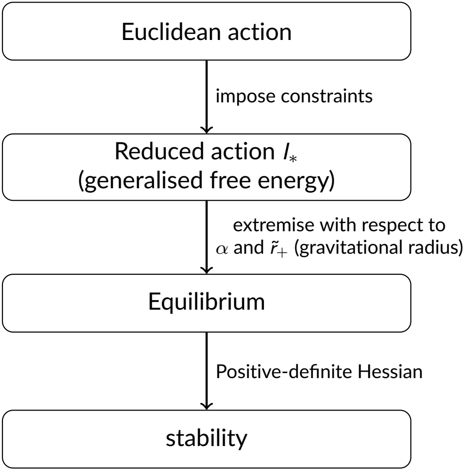
- AdS (Fernandes, Gandum, Lemos, 2025) and finite cavities can restore thermodynamic control

2. Setup



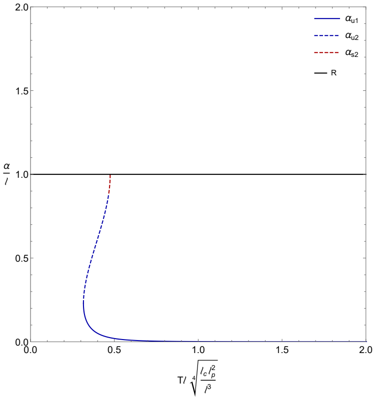
- Static and spherically symmetric spacetime
- Barotropic equation of state: $p_{\rm m} = \frac{1}{3}\,\frac{m}{4\pi\alpha^2}$
- Temperature power law: $T_{\rm m} = \frac{4}{3}\left(\frac{m}{4\pi\alpha^2 l_{\rm c}}\right)^{\frac{1}{4}}$
- Canonical ensemble imposed via boundary conditions
- Self-gravitating matter instead of horizon

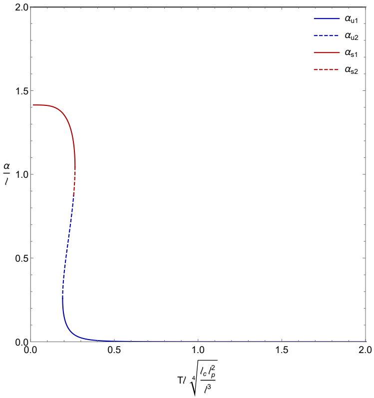
3. Method



- $E = \frac{R}{l_{\mathrm{p}}^2}\left(\sqrt{1+\frac{r^2}{\ell^2}} - \sqrt{1+\frac{r^2}{\ell^2} - \frac{\tilde{r}_+(1+\tilde{r}_+^2/\ell^2)}{r}}\right), \quad S = S_{\mathrm{m}}, \quad F = TI_* = E - TS$
- Formalism unifies thermodynamics and mechanics

5.1 Results: Hot thin shell phase





- Mechanically unstable solutions: dashed = thermodynamically stable; solid = thermodynamically unstable
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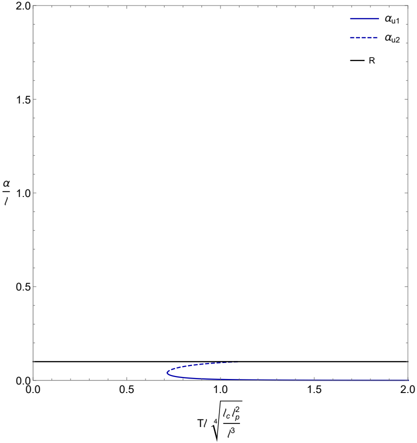
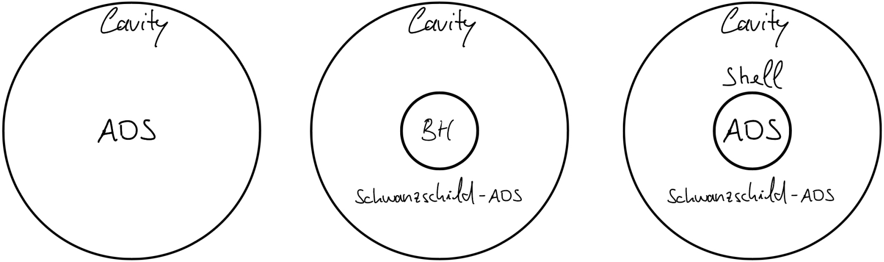
- The unique fully stable phase (solid red line) exists only if $R > l$

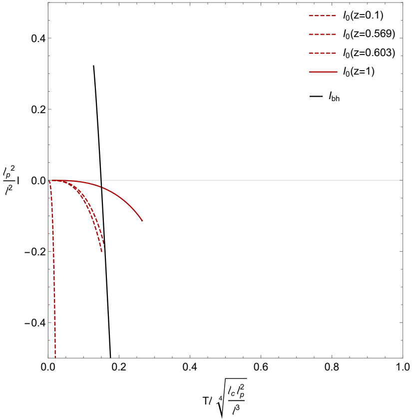


- Mechanically unstable solutions: dashed = thermodynamically stable; solid = thermodynamically unstable

5.2 Results: Phase transitions



- Three allowed phases in the semiclassical approximation: pure AdS, hot thin shell, and black hole
- Pure AdS has a null action
- The shell action depends on the fixed ensemble quantities R and T, but also on the natural scales $z = \left(\frac{l}{l_{\mathrm{p}}}\right)^{\frac{1}{2}} \left(\frac{l}{l_{\mathrm{c}}}\right)^{\frac{1}{4}}$



- Hot thin shell: dashed ($z < 0.6$) = no phase transition; solid ($z > 0.6$) = phase transition to the black hole (dark line)
- At low T, shell dominates ensemble; at high T, black hole is favored ($R > l$ and physically reasonable z)
- Hot thin shell models hot AdS

6. Takeaway & outlook

- Thin shells in an AdS cavity provide a clean setting to study canonical ensembles of self-gravitating matter without horizons
- Results reduce to the ones obtained without a cavity in AdS (Fernandes, Gandum, Lemos, 2025)
- In the semiclassical regime, the canonical ensemble exists only in a restricted region of parameter space, with multiple competing equilibrium solutions
- The hot thin shell offers a controlled realisation of hot AdS and admits a phase transition to the black hole phase at high temperatures
- Next step: account for hot AdS