Two Kerr Black Holes in Equilibrium with Self-interacting Scalar Hair

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Introduction

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Horizonless

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  (mini-boson stars)

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  (mini-boson stars)
- Self-interactions
  - Quartic
    $\mu^2|\Psi|^2+\lambda|\Psi|^4$
  - Q-ball type
    $\mu^2|\Psi|^2-\lambda|\Psi|^4+\beta|\Psi|^6$
  - ……

N. Sanchis-Gual et al. Class.Quant.Grav. 39 (2022)

M. Brito et al. Phys.Rev.D 107 (2023)

P. Ildefonso et al. *Phys.Rev.D* 108 (2023)

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Single hairy BHs

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Scalar environment - DsBSs

Dipolar spinning boson stars (DsBSs)

Action $S = \int d^4x\sqrt{-g}\left(\frac{R}{16\pi G} - g^{ab}\Psi^*_{,a}\Psi_{,b} - U(|\Psi|^2)\right)$ with $U(|\Psi|^2) = \mu^2|\Psi|^2 + \lambda|\Psi|^4$

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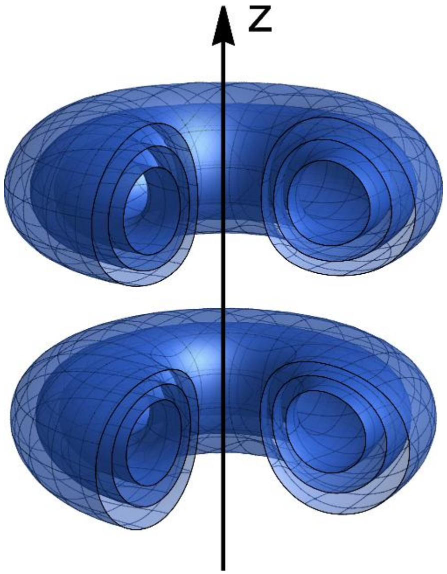

Surfaces of constant energy density $-T_t^t$

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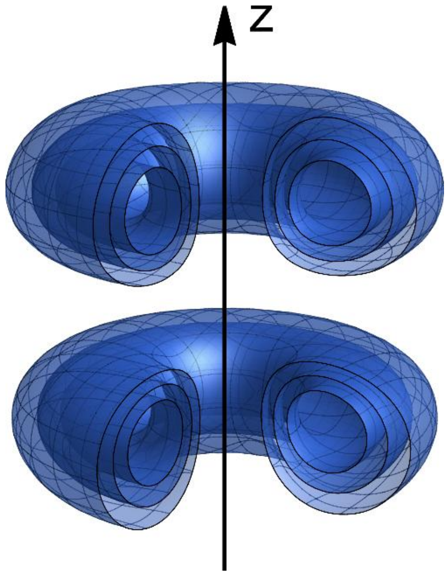

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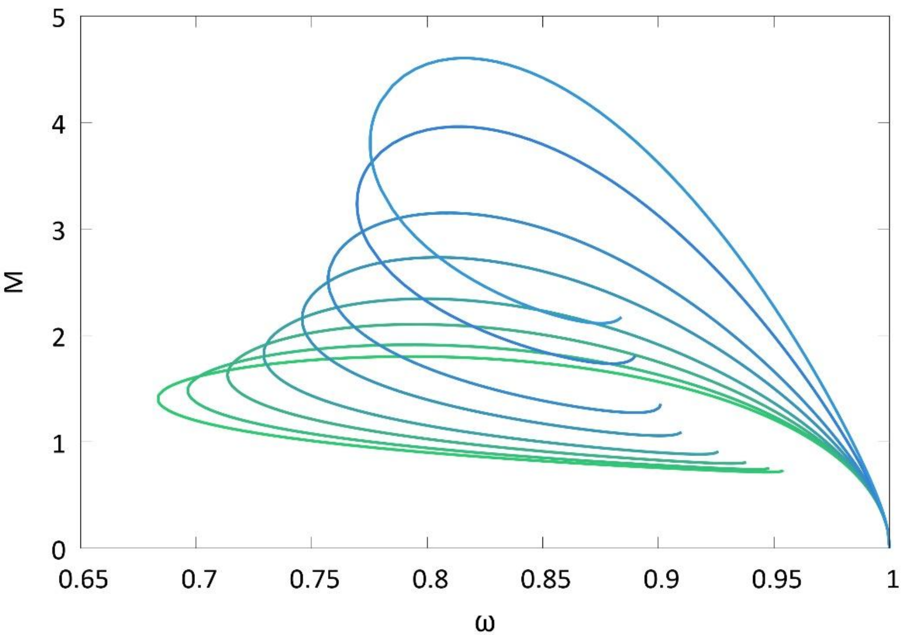

ADM mass vs. ω for λ = 0 to 300.

Vacuum Bach-Weyl metric

Canonical Weyl coordinates

$$ds^2 = -e^{2V(\rho,z)}dt^2 + e^{-2V(\rho,z)}\left[e^{2K(\rho,z)}(d\rho^2 + dz^2) + \rho^2 d\varphi^2\right]$$

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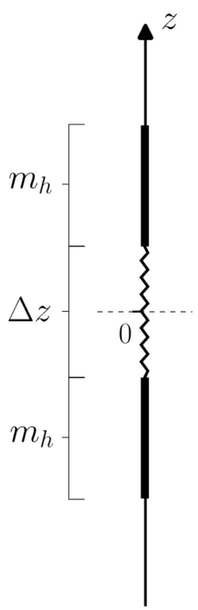

Rod structure

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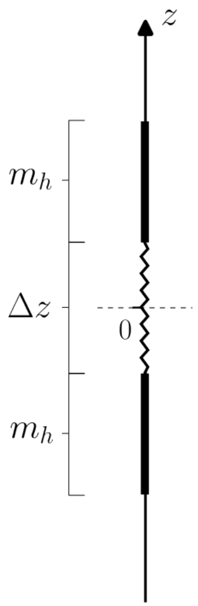
Conical excess

$$\delta = 2\pi\left(1 - \lim_{\rho\to 0}\sqrt{\frac{g_{\varphi\varphi}}{\rho^2 g_{\rho\rho}}}\right) = \underline{-2\pi m_h^2/(\Delta z^2 + 2m_h\Delta z) < 0}$$



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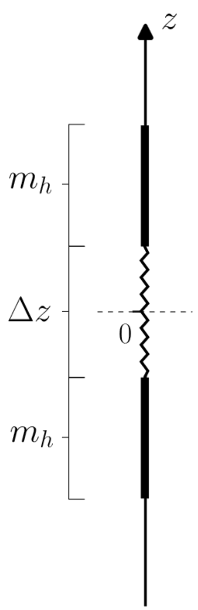

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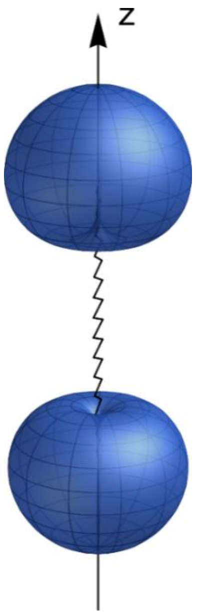

Surface Geometry

Vacuum Bach-Weyl metric

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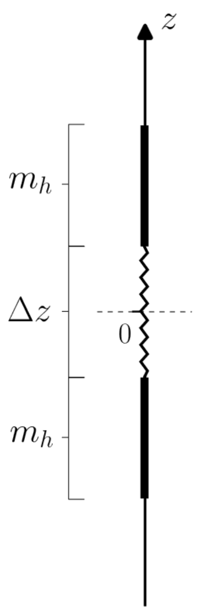

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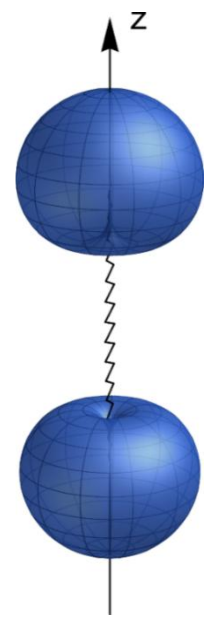


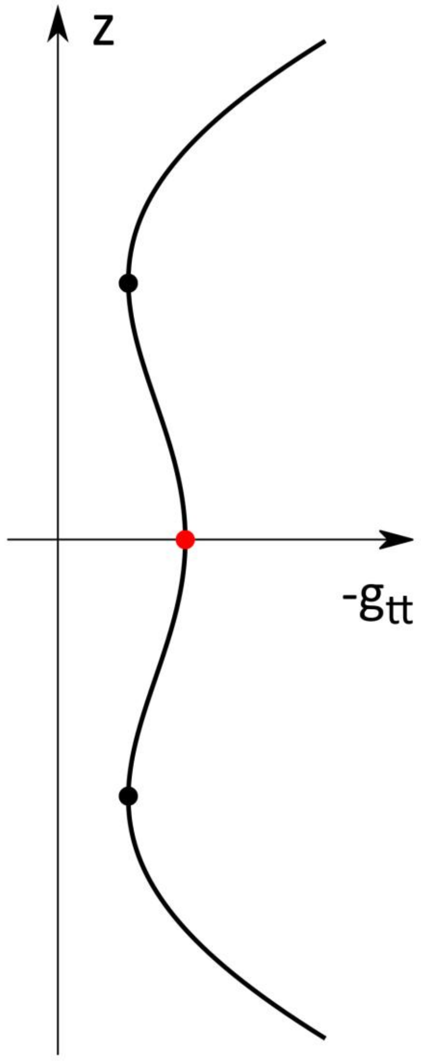
Surface Geometry

Ansatz

$$ds^2 = -e^{2(V+F_0)}dt^2 + e^{2(K-V+F_1)}(d\rho^2 + dz^2) + e^{2(-V+F_2)}\rho^2(d\varphi - W\,dt)^2$$

$$\Psi = \psi(\rho, z)e^{i(m\varphi-\omega t)} \qquad U(|\Psi|^2) = \mu^2|\Psi|^2 + \lambda|\Psi|^4$$

Two Kerr black hole with self-interacting scalar hair



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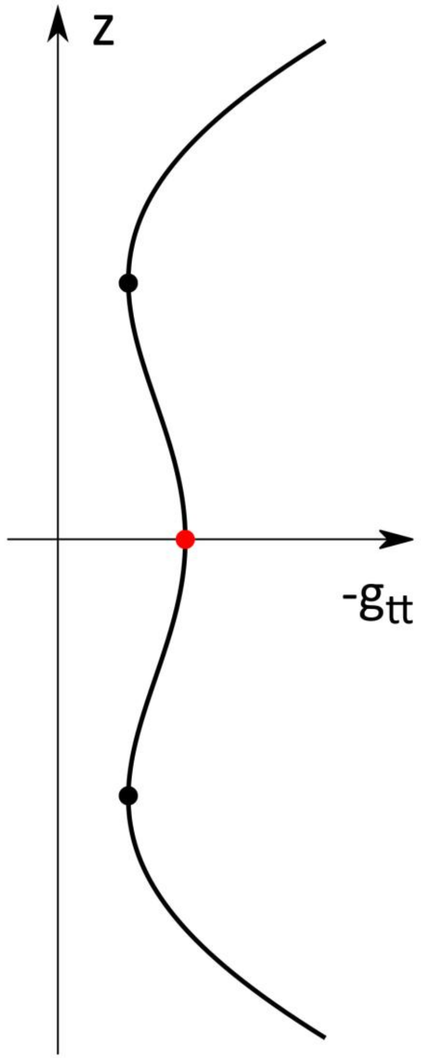

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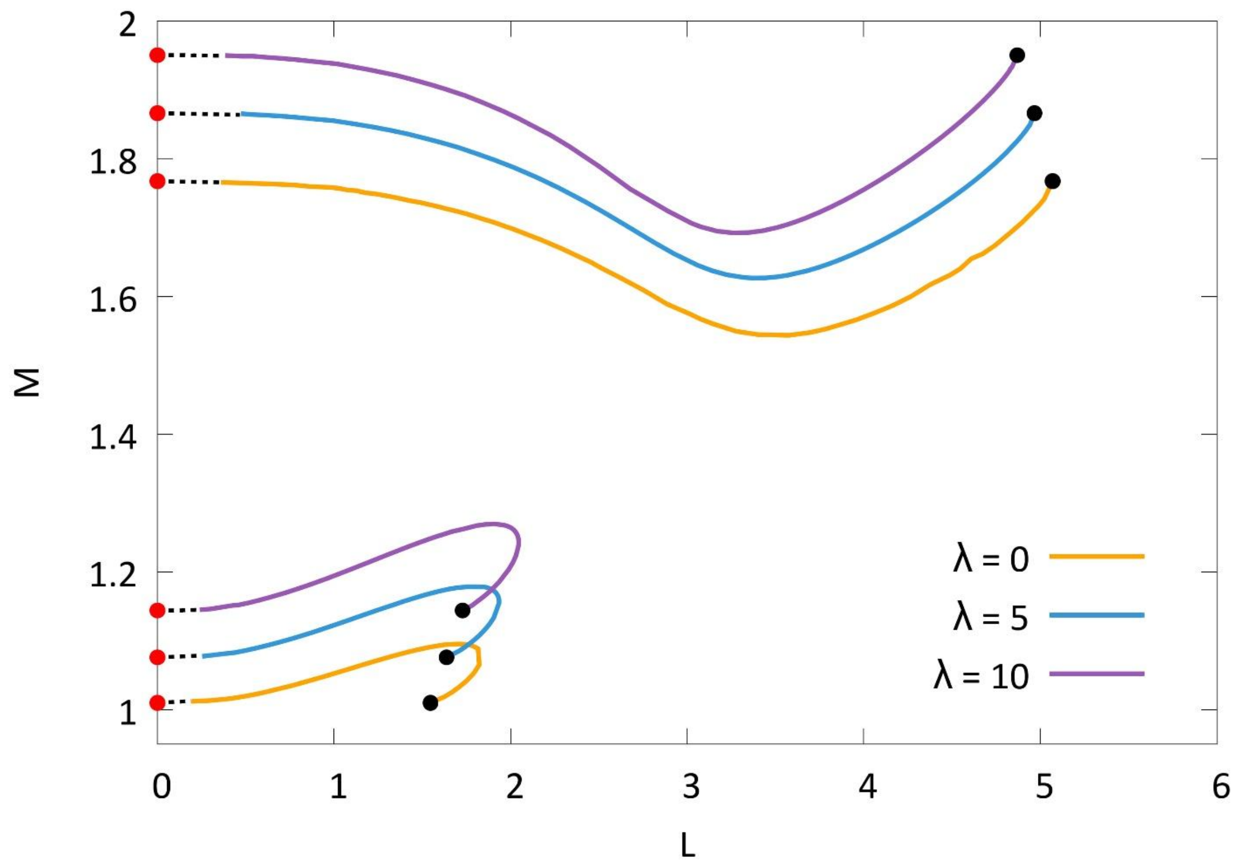

ADM mass vs. distance

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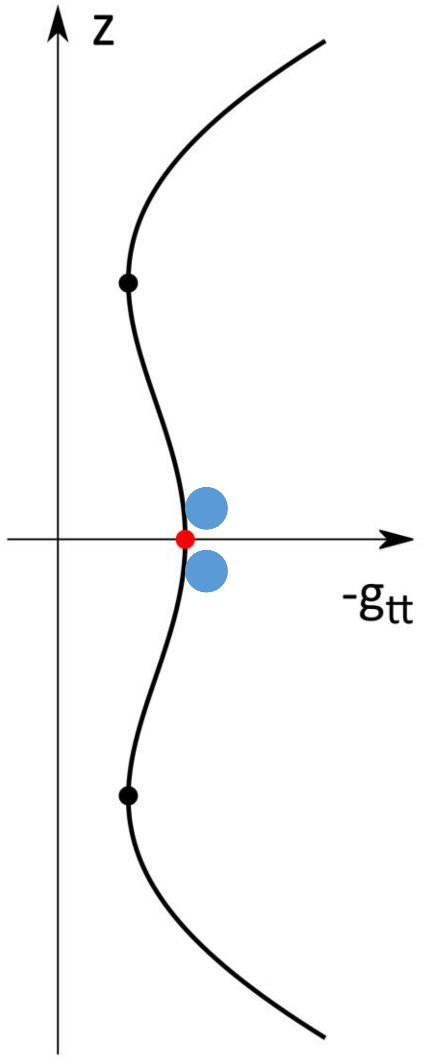

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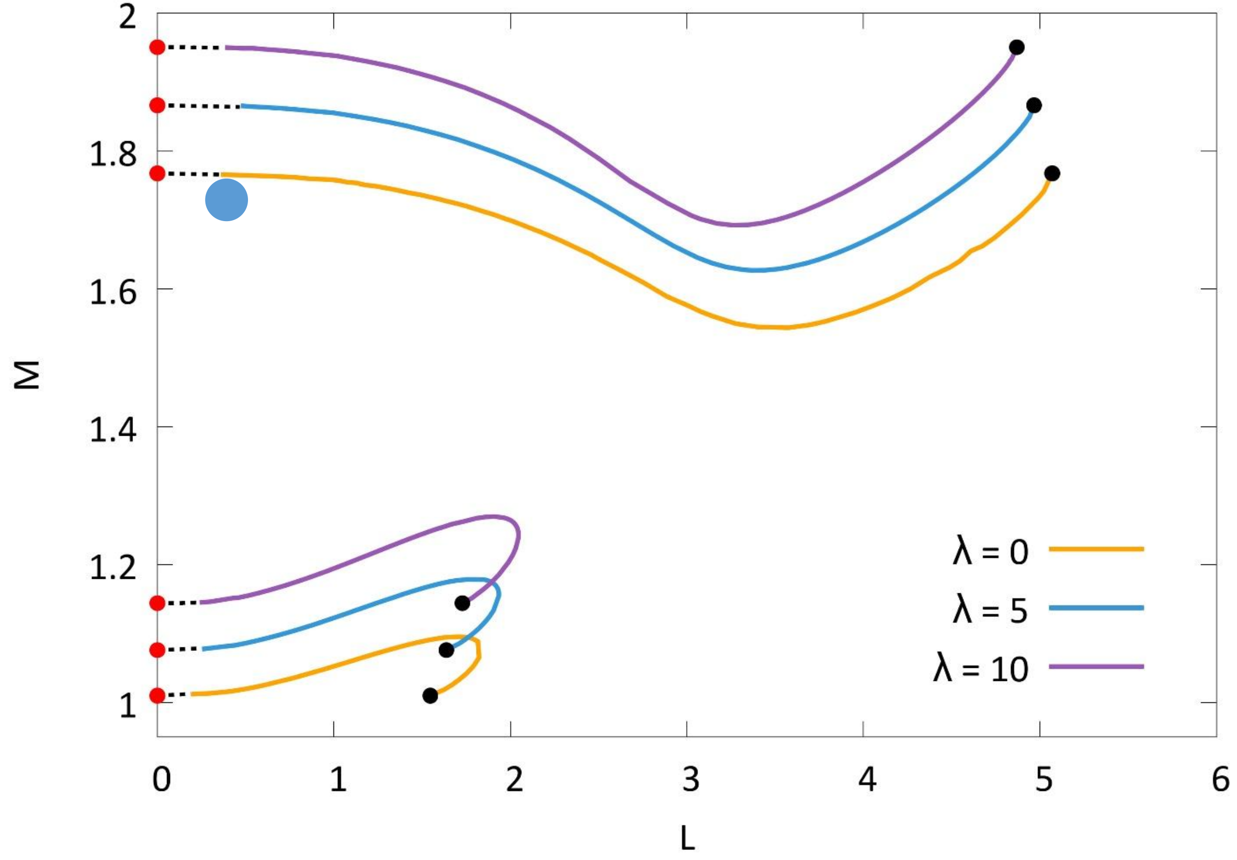

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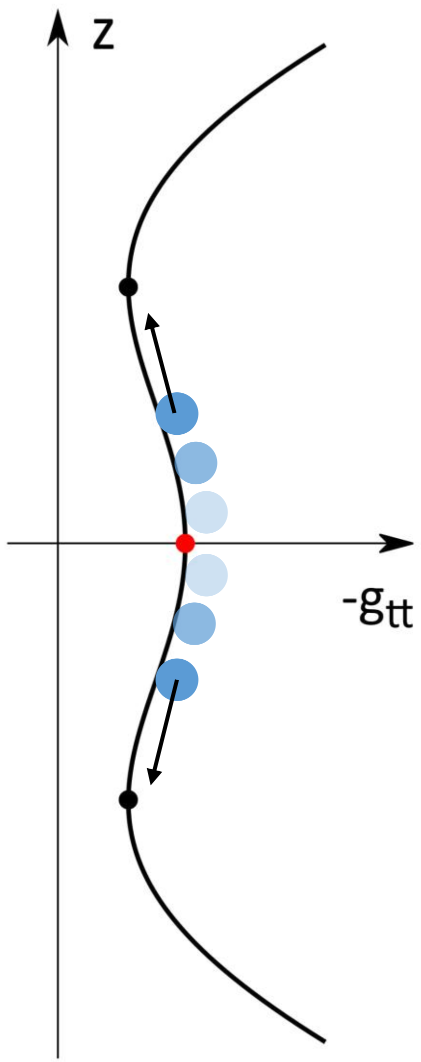

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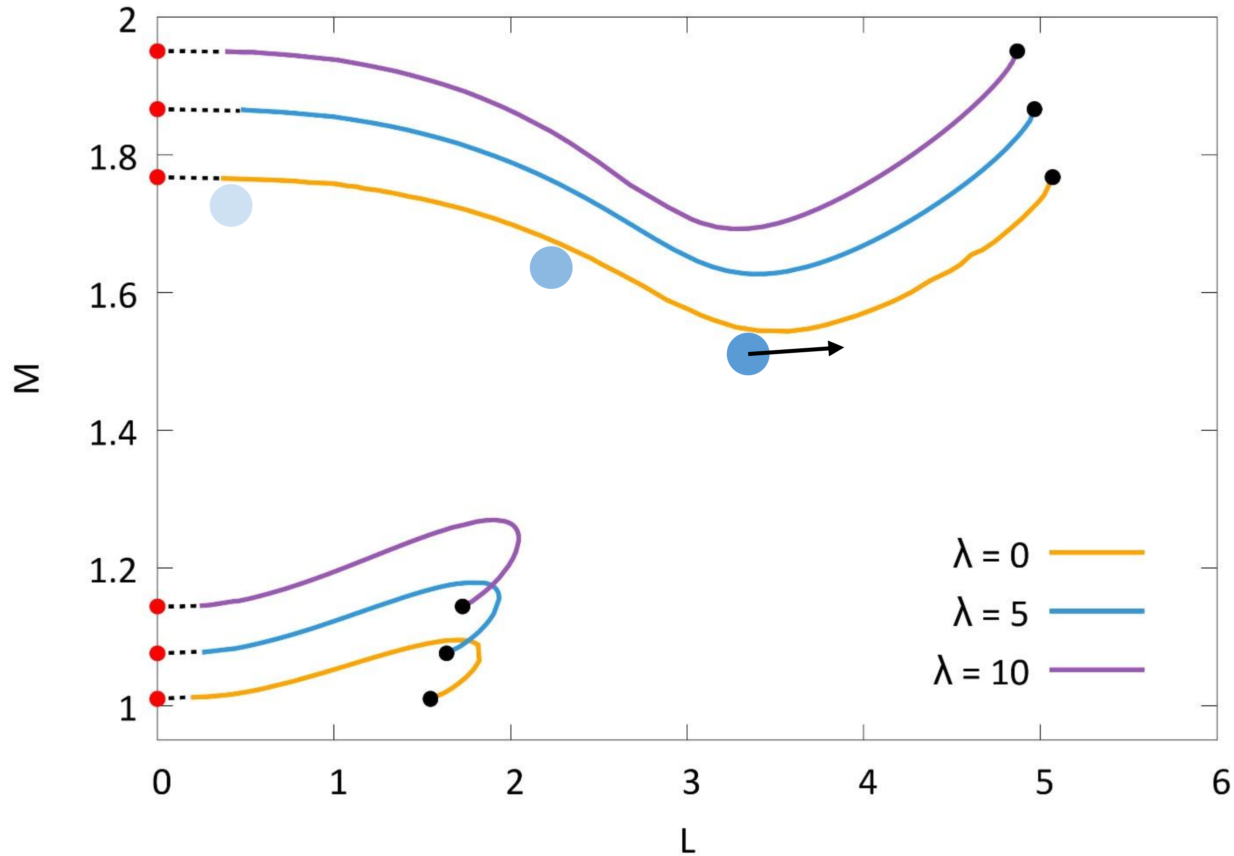

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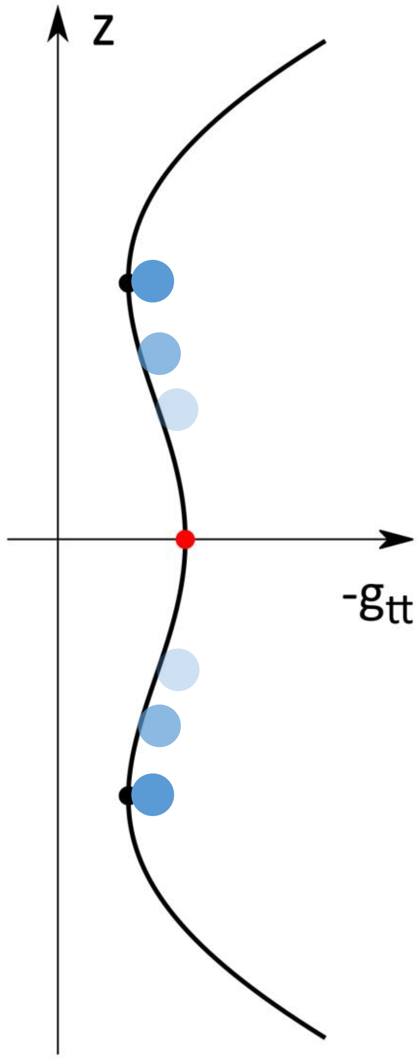

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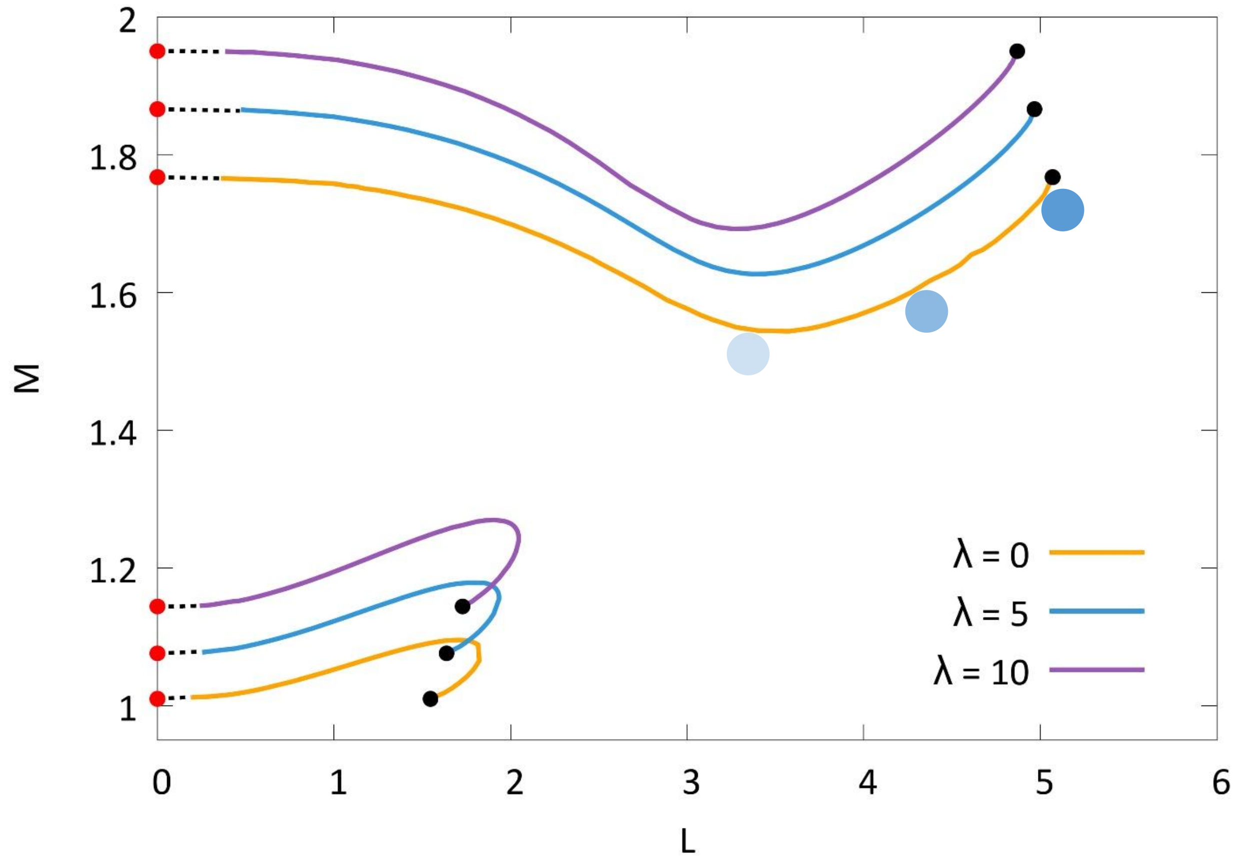

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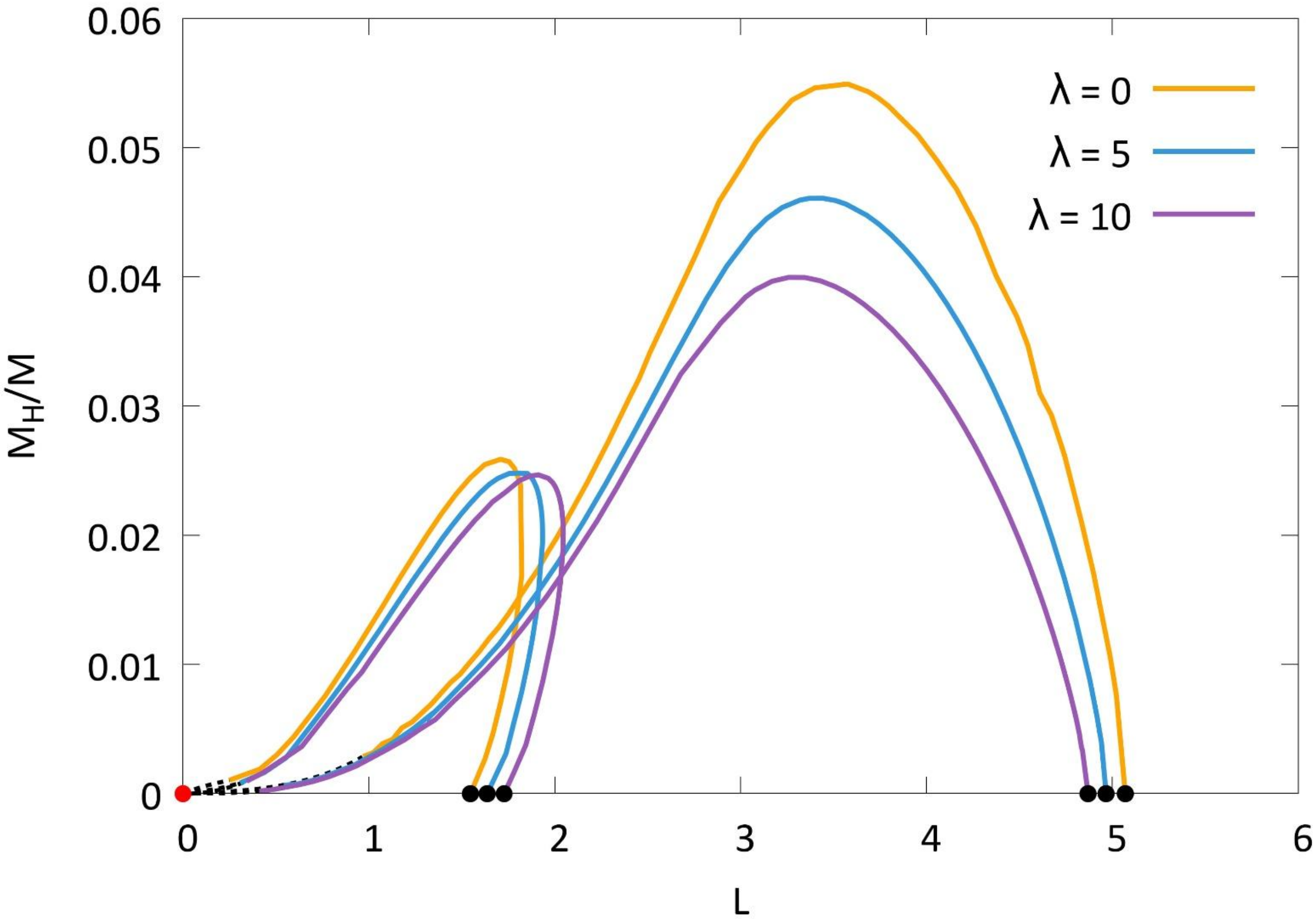

M_H/M vs. distance

Conclusions

- We constructed dipolar spinning boson stars with self-interactions and two spinning BHs in the boson star environment.

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- Electro-vacuum

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- Electro-vacuum
- Proca hair

Thank you!