



Dyonic Reissner-Nordström black holes in Weyl conformal gravity

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XVIII Black Holes Workshop

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Agenda for the day

1. Introduction to Conformal Gravity
2. Reissner-Nordström black holes
 1. Null geodesics
 2. Horizon structures
 3. Exotic spacetimes

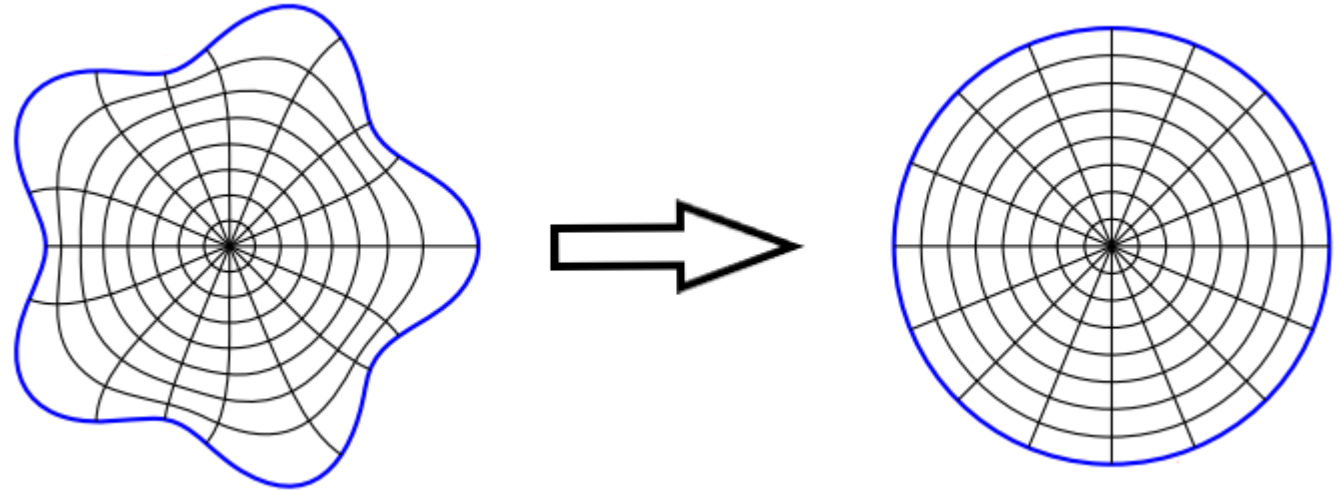
Weyl conformal gravity

Attempt to remedy GR

Invariances:

- Lorentz
- Coordinate
- Diffeomorphism
- Conformal

$$g_{\mu\nu}(x) \rightarrow \Omega^2(x)g_{\mu\nu}(x)$$



Cred.: R. Kusano from L. Trefethen (2025)

Weyl conformal gravity

Weyl tensor: $C_{\lambda\mu\nu\kappa}$

$$= R_{\lambda\mu\nu\kappa} - \frac{1}{2} (g_{\lambda\nu} R_{\mu\kappa} - g_{\lambda\kappa} R_{\mu\nu} - g_{\mu\nu} R_{\lambda\kappa} + g_{\mu\kappa} R_{\lambda\nu})$$
$$+ \frac{1}{6} R (g_{\lambda\nu} g_{\mu\kappa} - g_{\lambda\kappa} g_{\mu\nu})$$

Weyl action: $S_W = -\alpha_g \int d^4x \sqrt{-g} \underline{C_{\lambda\mu\nu\kappa}} C^{\lambda\mu\nu\kappa}$

Preserving
conformal
invariance

Weyl conformal gravity (CG)

Bach equations

$$W_{\mu\nu} = \frac{1}{4\alpha_g} T_{\mu\nu}$$

$$W^{\mu\nu} = 2C^{\mu\lambda\nu\kappa}_{;\lambda;\kappa} - C^{\mu\lambda\nu\kappa} R_{\lambda\kappa}$$

Bach tensor

Up to fourth
derivatives
of metric

CG Schwarzschild metric

$$g_{\mu\nu} = \text{diag} \left(-B(r), \frac{1}{B(r)}, r^2, r^2 \sin^2 \theta \right)$$

$$B(r) = 1 - 3\beta\gamma - \frac{\beta(2 - 3\beta\gamma)}{r} + \underbrace{\gamma r}_{\text{Galactic}} - \underbrace{kr^2}_{\text{Cosmological}}$$

CG Reissner-Nordström metric

Starting from $A_\mu = \left(\frac{Q}{r}, 0, 0, -P \cos \theta\right)$

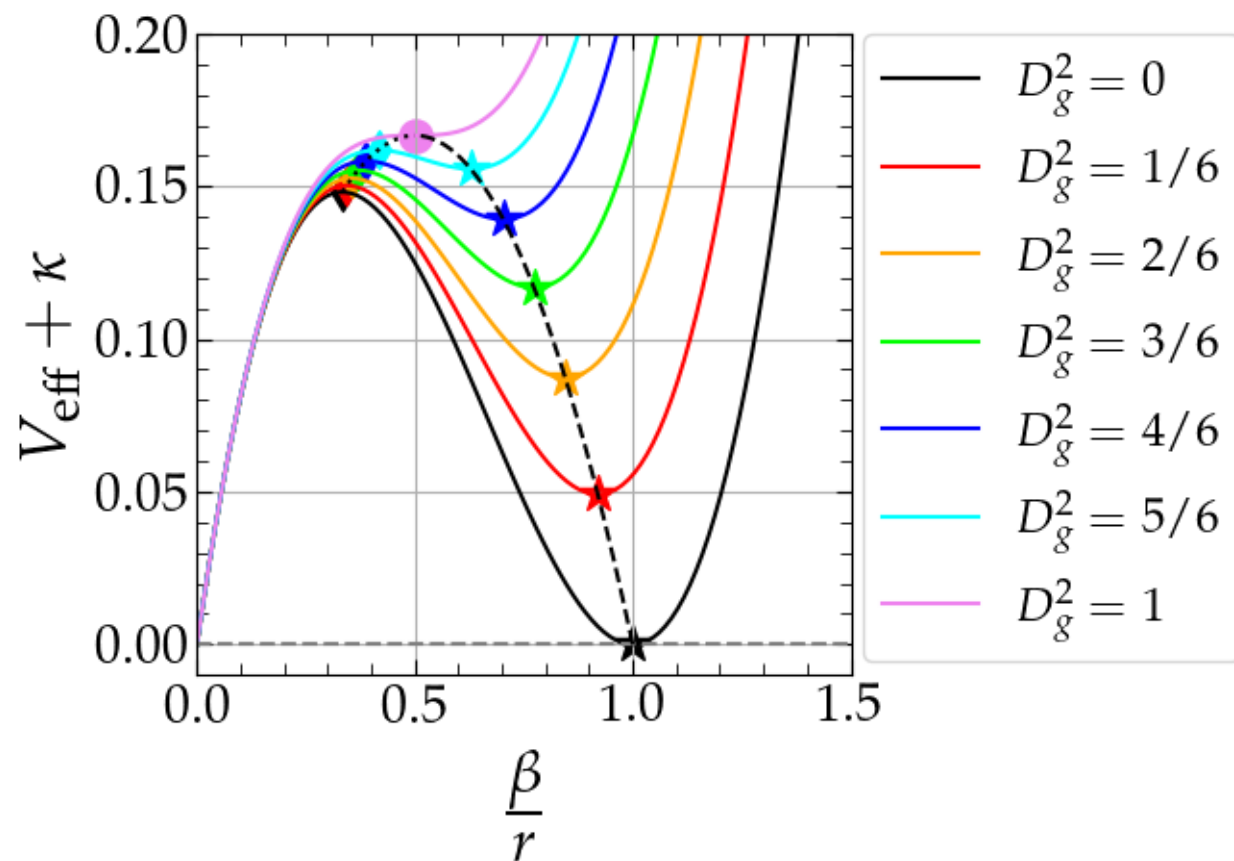
$$B(r) = 1 - 3\beta\gamma - \frac{\beta(2 - 3\beta\gamma) - \frac{D_g^2}{3\gamma}}{r} + \gamma r - \kappa r^2$$

- No $1/r^2$ term like in GR
- Mass and charge follow $1/r$

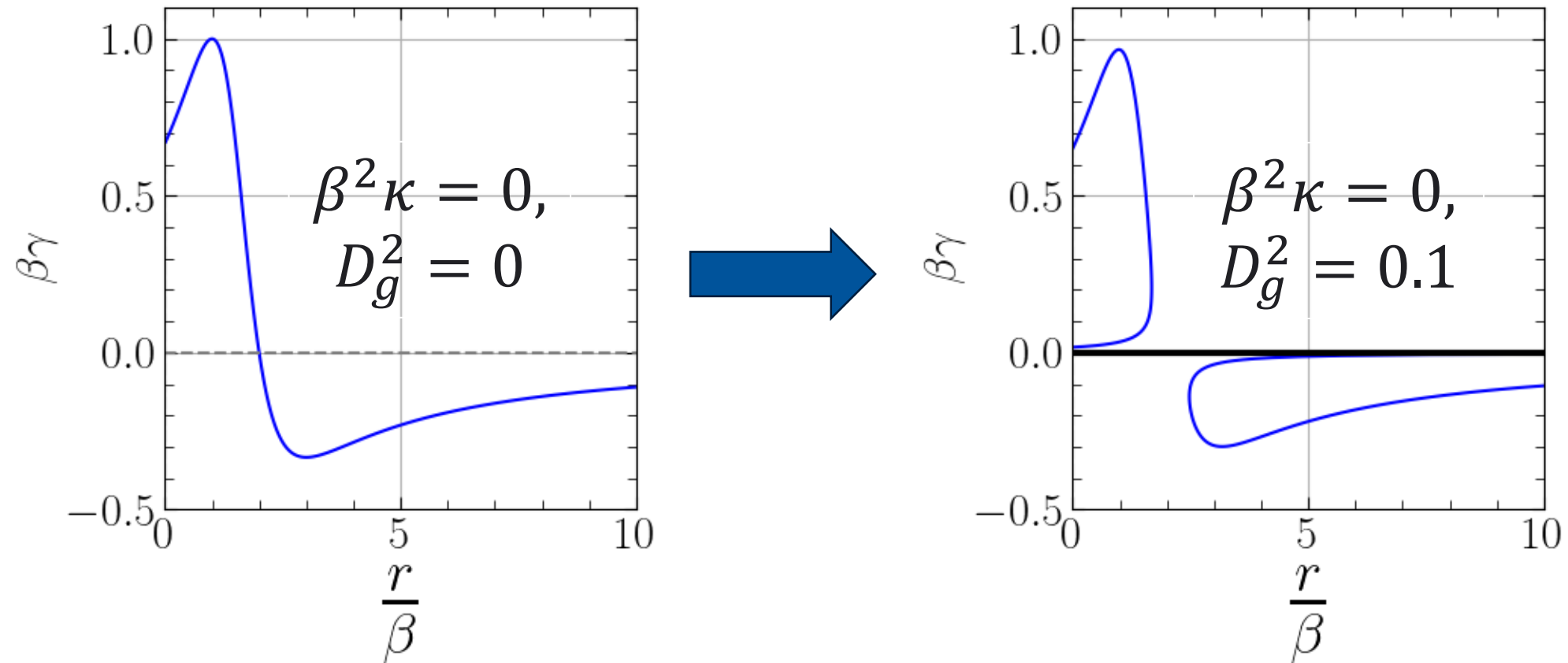
$$D_g^2 \equiv \frac{-3(Q^2 + P^2)}{8\alpha_g}$$

Null geodesics

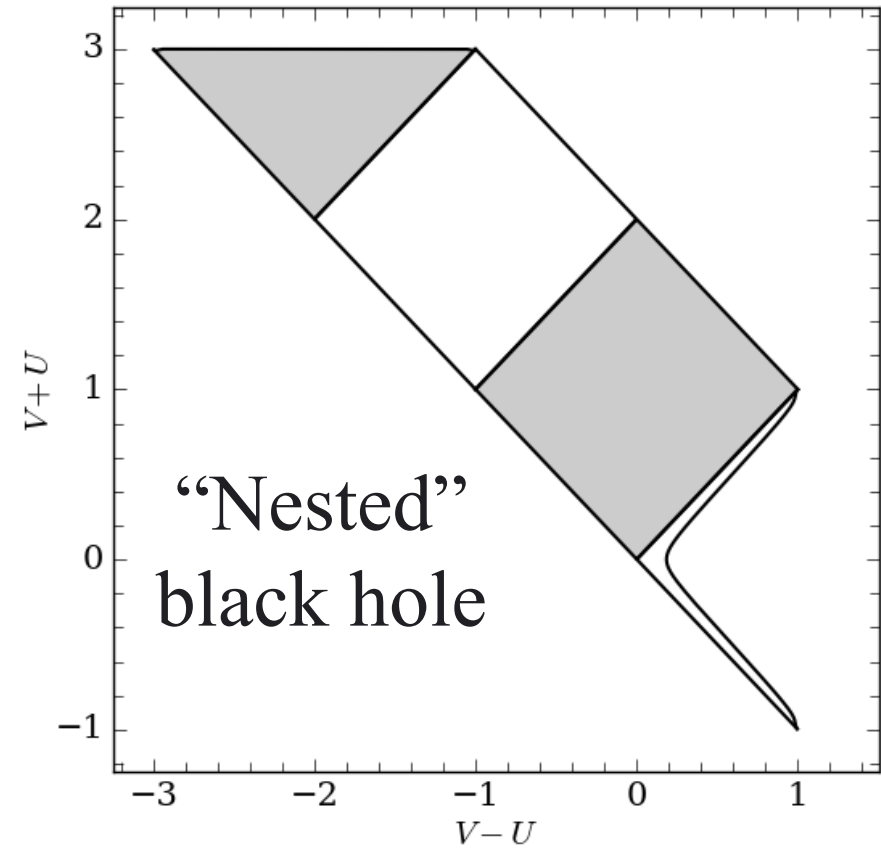
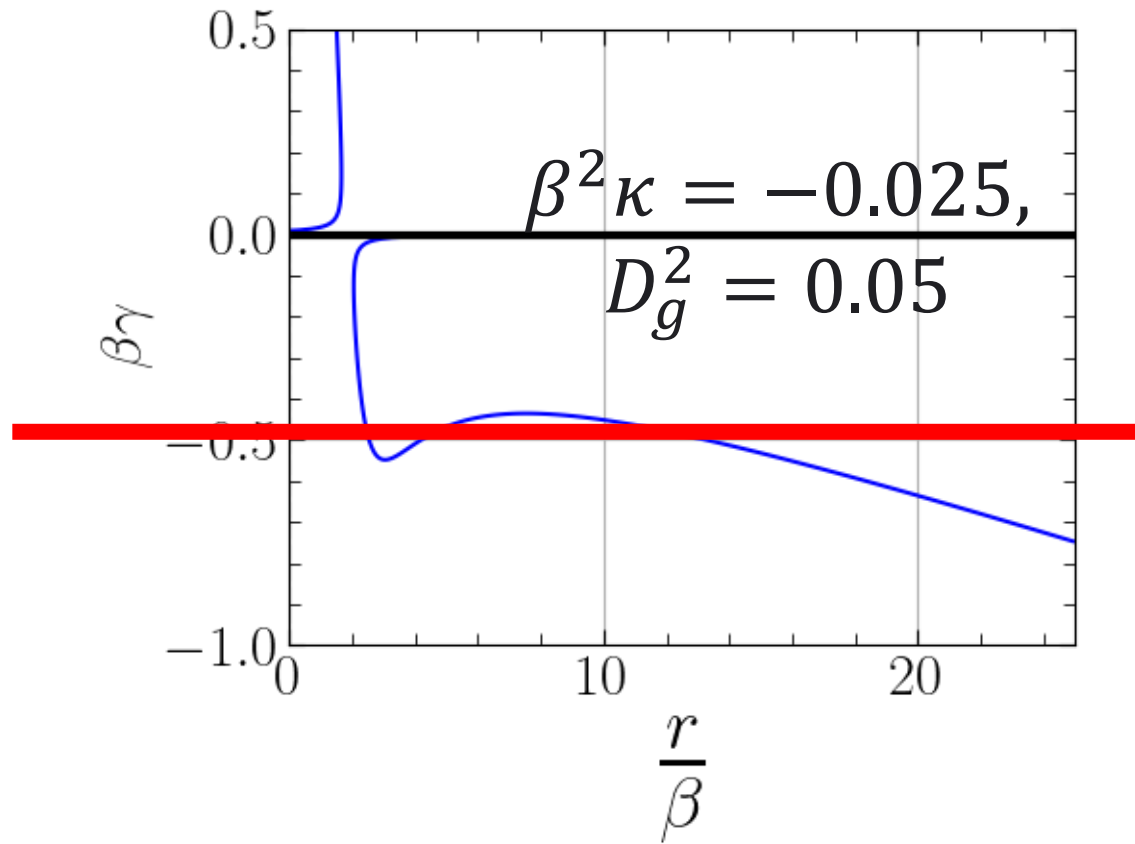
- Two photon spheres
 - Stable
 - Unstable
- At critical charge $D_g^2 = 1$:
 - *Saddle point* photon sphere



Horizon structures



Black hole spacetimes



Conclusions

1. Conformal gravity: alternative to GR
2. Reissner-Nordström solutions
3. Further findings – check out our paper!

Kusano, Yulo Asuncion, Horne: gr-qc/2510.05701 (2025)



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Horne, Yulo Asuncion & Kusano collab.

- “*Structure of Kerr black hole spacetimes in Weyl conformal gravity*”
Yulo Asuncion, Horne, Kusano, and Dominik. *CQG*. **42**(21) 215005 (2025)
- “*Dyonically charged black holes in Weyl conformal gravity*”
Kusano, Yulo Asuncion, and Horne. *arXiv* gr-qc/2510.05701 (2025)
- “*Loading the stable photon sphere in Weyl conformal gravity metrics*”
Kusano, Horne, König, and Yulo Asuncion. *arXiv* gr-qc/2510.18750 (2025)

Other things coming (hopefully) soon!!