Radio Orbiting a Black Hole

Masters Thesis in Applied Mathematics
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Overview

- Motivation
- Theoretical model
- Computational methodology
- Results
- Next steps

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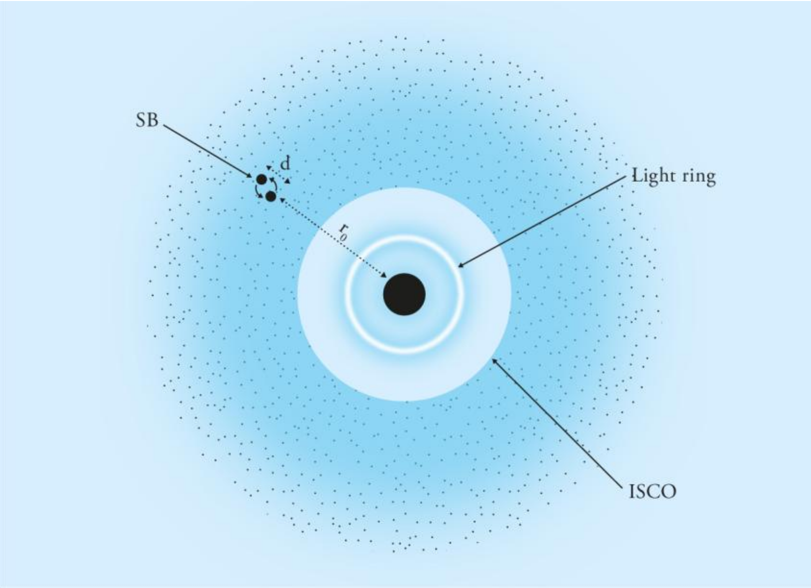

Binary EMRI orbiting a supermassive black hole

[this paper follows up on arXiv:2506.14868 by João Sieiro dos Santos et al, where they did the same procedure for a gravitational quadrupole]

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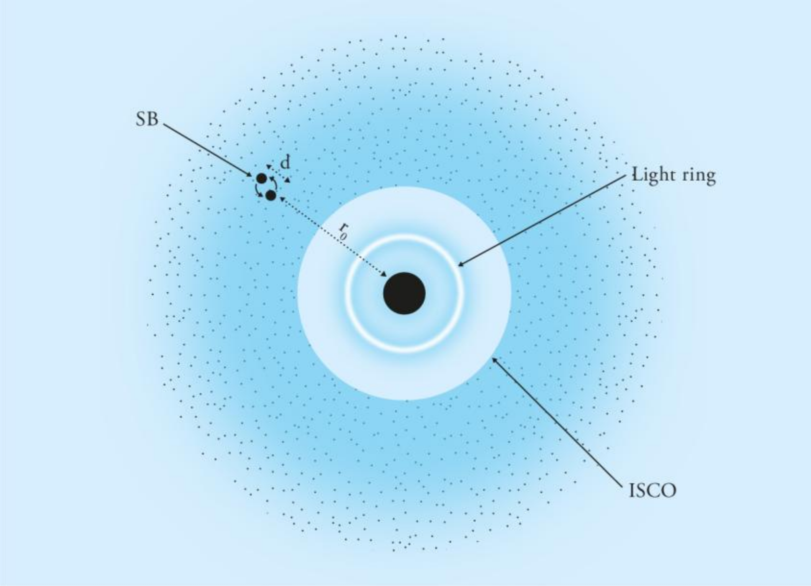

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We use Black Hole Perturbation Theory to study the radiation emitted by a dipole in this type of environment.

This is the **first ever** first principles model for a radiation source moving in Kerr spacetime.

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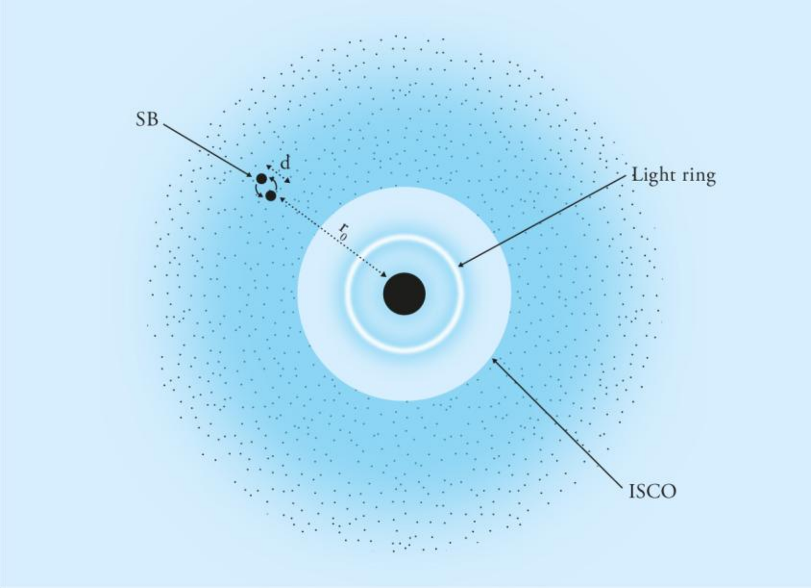

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Guiding question: how is the radiation of a dipole affected when it is orbiting a rotating black hole?

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$$\updownarrow$$

$$J^\mu = 2\int \nabla_\alpha \left(u^{[\alpha} p^{\mu]} \delta^{(4)}\left(x - z(\tau)\right)\right) d\tau \, .$$

Theoretical Model

We plug this 4-current into **Teukolsky's equation**:

$$\mathcal{D}\phi_2 = \mathcal{T}(J^\mu),$$

where $\mathcal{D}$ is a second order linear differential operator and $\mathcal{T}$ is the source term, which depends on J^μ.

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The frequencies excited by the system are

$$\omega_{mpq} = m\Omega_0 + p\Omega_p + q\Omega_d,$$

where Ω_0 is the **orbit** frequency, Ω_p is the **precession** frequency of the dipole, Ω_d is the **dipole** frequency, and p $\in \{0, \pm 1\}$, q $\in \{\pm 1\}$.

Computational Methodology

We implemented a code that computes a **semi-analytic expression** for the amplitudes Z^{∞} in the decomposition

$$\phi_2 \sim \sum_{l,m,p,q} Z^{\infty}_{lmpq} \, {}_{-1}S_{lm\omega_{mpq}} e^{im\varphi} \frac{e^{-i\omega_{mpq}\mathrm{u}}}{r}$$

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For now, we are using **BHPToolkit** to compute numerical values. However, since **high frequencies** excite higher harmonic modes, the toolkit becomes **inefficient** in this regime – we may need to upgrade.

BHPTookit logo

Results

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As a benchmark, we compute the **radiated power at infinity** for a standing dipole at fixed $\mathrm{r} = \mathrm{R}$ in Kerr spacetime. We compare the result with the classical **Larmor's formula**

$$P_L = \frac{p^2 \omega^4}{12 \pi \epsilon_0}.$$

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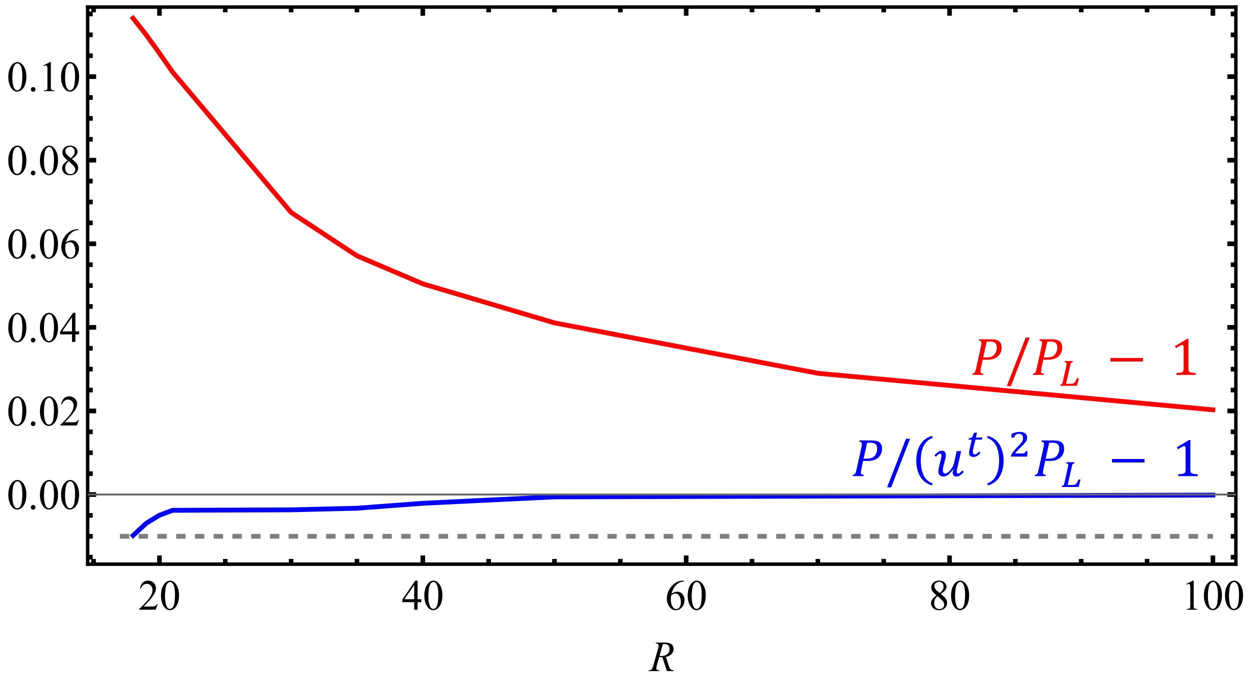

Remarkable agreement, even up to $R \sim 20M$!

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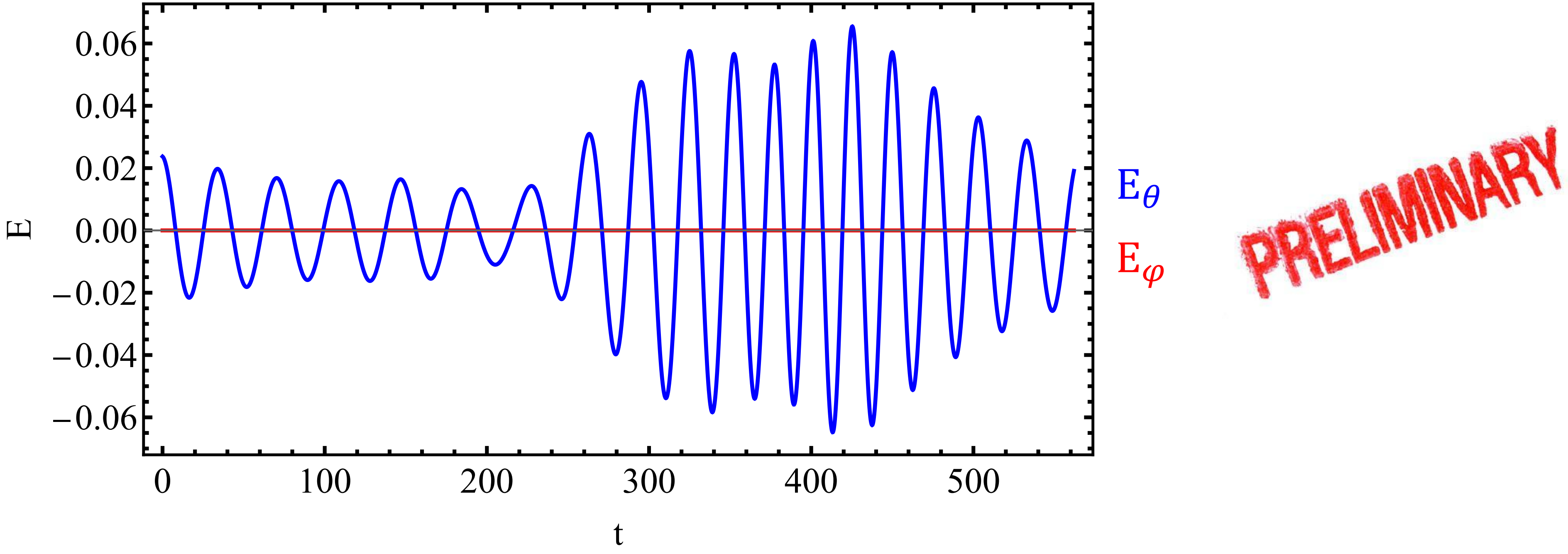
$M = 1,$
$a = 0,$
$\Omega_d = 0.2,$
$\mathrm{R} = 20$



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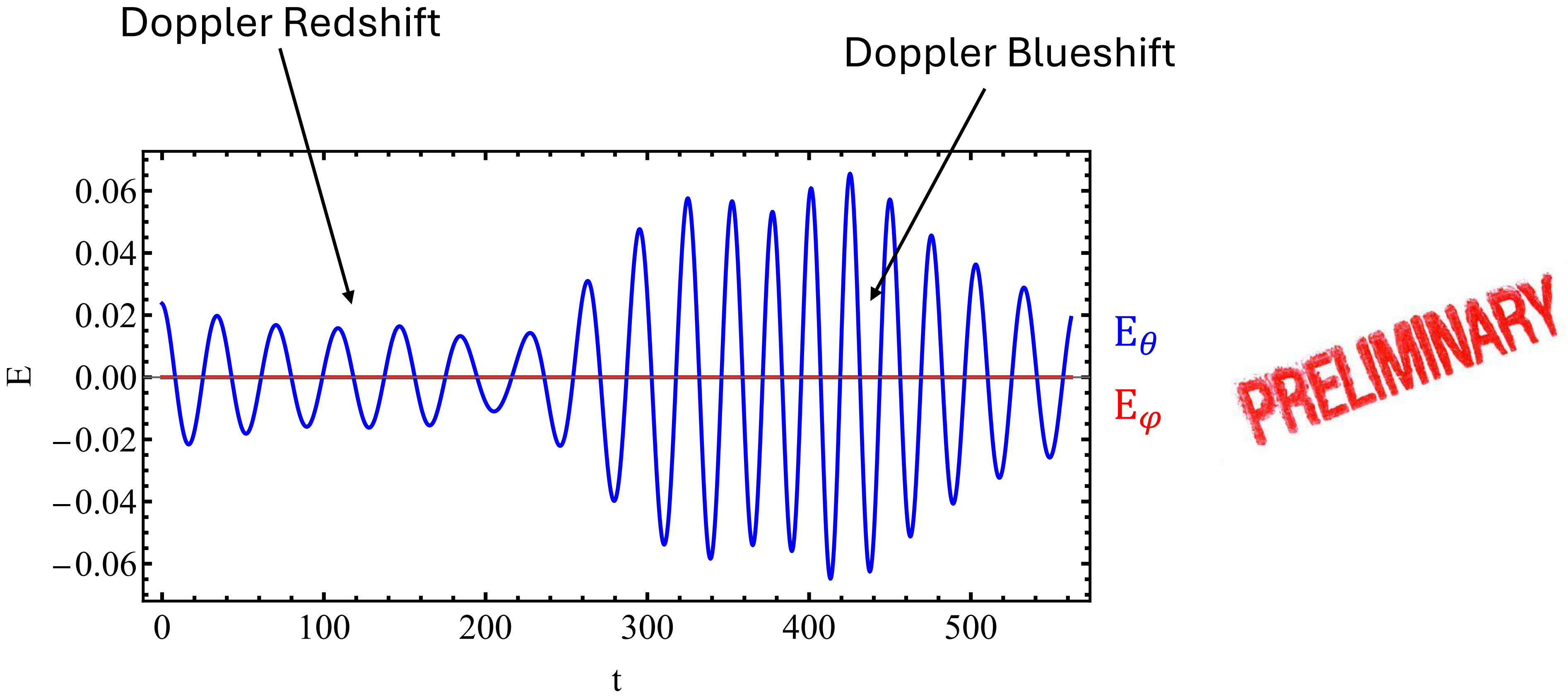

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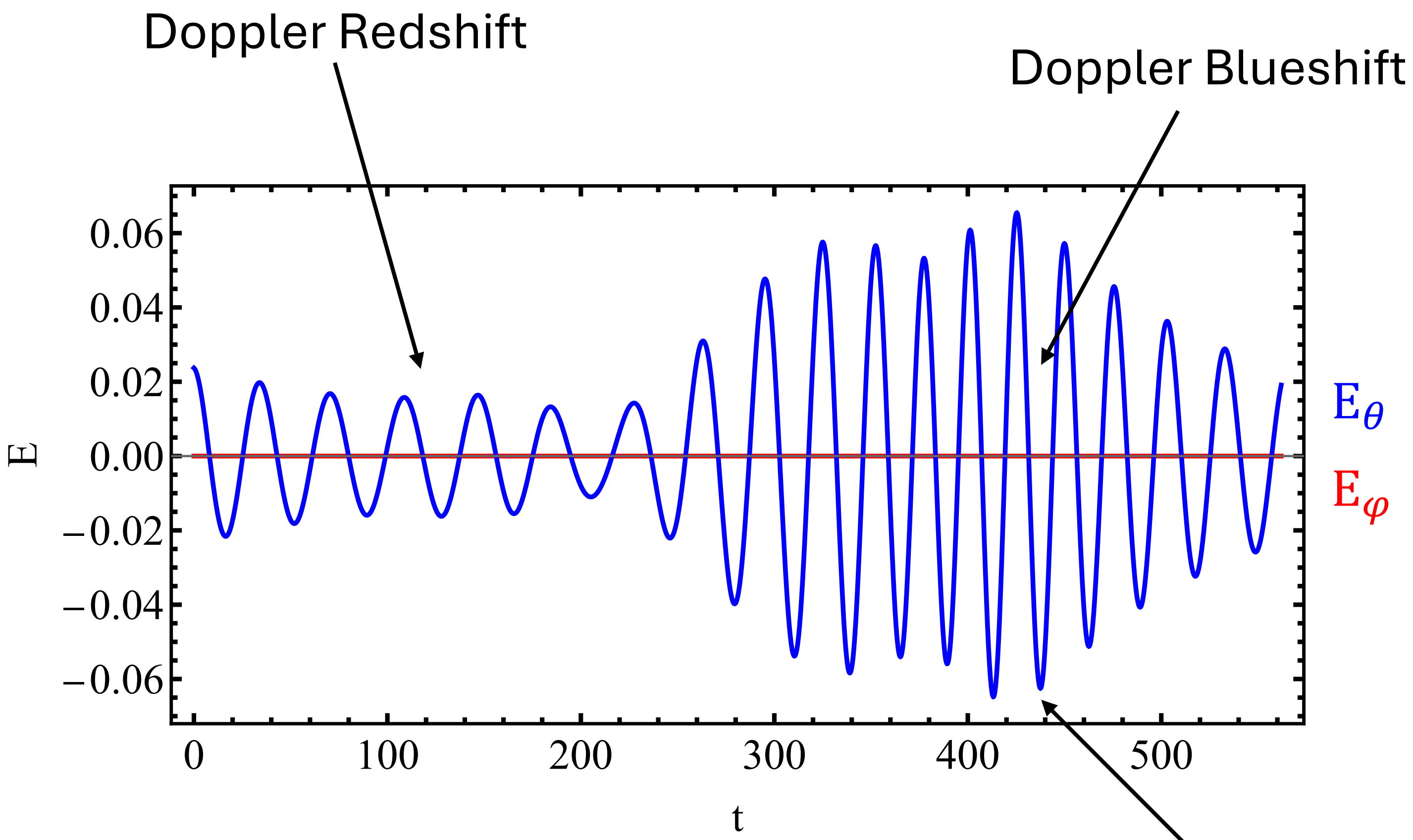

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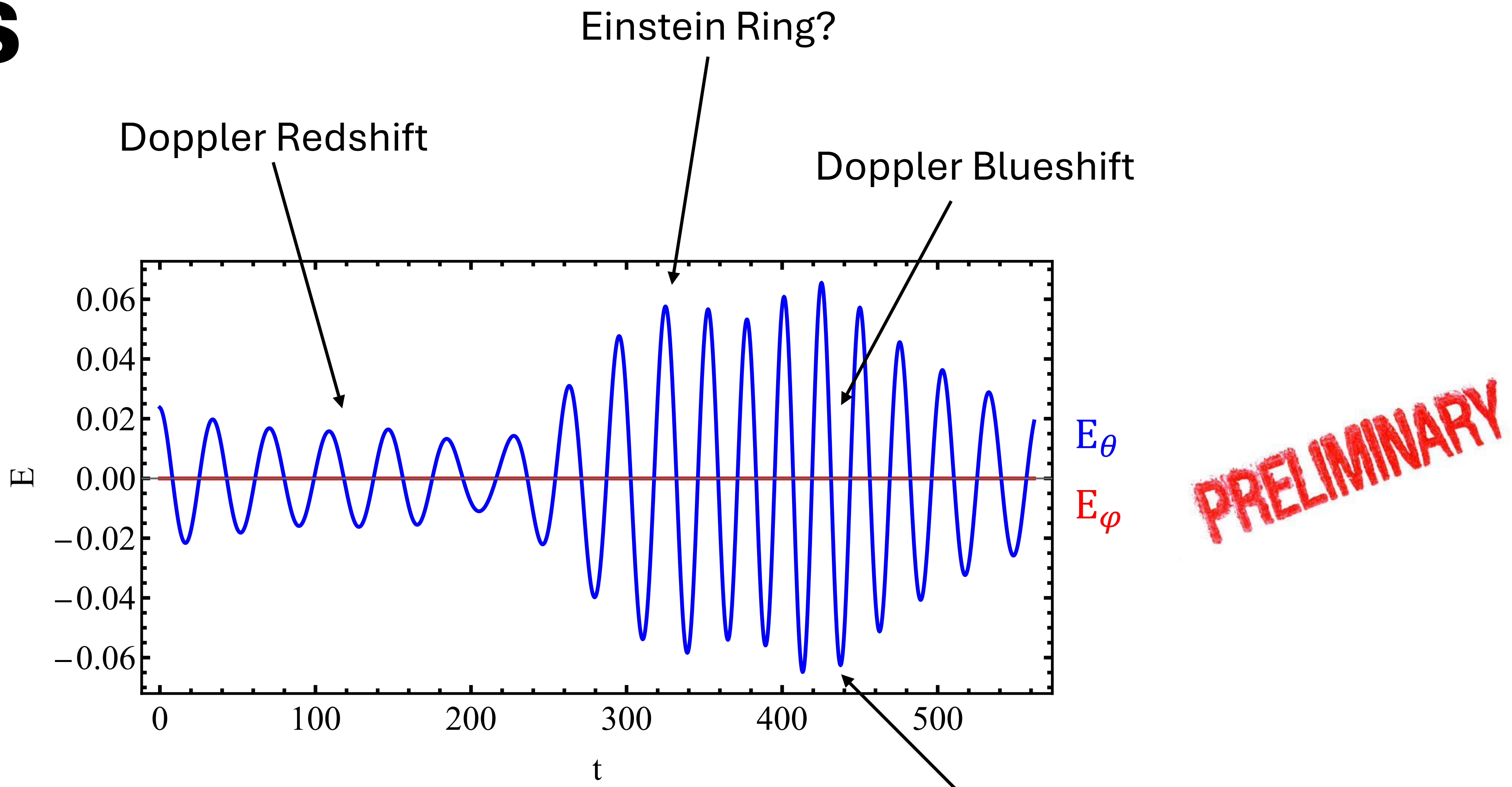

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Next Steps

Our next objectives include:

- Compute waveforms and amplitudes for the most general case
- Study other effects such as helicity-dependent scattering
- Implement incoherent radiation
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Thank You!