Spectroscopy of Accelerating Black Holes

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Contents

Motivation

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- This was done numerically by (K. Destounis, R. D. B. Fontana and F. C. Mena). Thus we are interested on getting **analytical expressions** for the quantities above.
- In this work, the focus will be on the **eikonal limit** and its relation with **circular null geodesics** given that this black hole **lacks spherical symmetry**.

Accelerated black holes: $C-$ Metric

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where

$$f(r) = \left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right)\left(1 - a^2r^2\right); \quad P(\theta) = 1 - 2aM\cos\theta + a^2Q^2\cos^2\theta, \tag{2}$$

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where a the acceleration parameter, Q charge and M mass.

- Our observer is somewhere in between $M + \sqrt{M^2 - Q^2} < r < 1/a$.

Equation of Motion for a Scalar Field

The Klein Gordon equation for a massless test scalar field Ψ gives

$$\Box\Psi = 0 \Leftrightarrow \frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu\Psi) = 0 \tag{3}$$

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Under some transformations and the ansatz:

$$\tilde{\Psi} = \Lambda^{-1}\Psi = e^{-i\omega t}e^{im\varphi}\frac{\phi(r)}{r}\chi(\theta), \tag{4}$$

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where

$$dx = \frac{dr}{f(r)}, \quad dz = \frac{d\theta}{P(\theta)\sin\theta} \tag{6}$$

Form of Potentials

- The potentials are:

$$V_r = f(r)\left(\frac{\lambda}{r^2} - \frac{f(r)}{3r^2} + \frac{f'(r)}{3r} - \frac{f''(r)}{6}\right), \tag{7}$$

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If the metric was spherically symmetric, then:

- $\chi(\theta)e^{im\varphi} \to Y_l^m(\theta,\varphi)$;
- $\lambda \to l(l+1) + 1/3$, where l is the angular momentum number.

Boundary Conditions (QNMs)

1. Nothing comes out of the horizon:

$$\Psi \sim e^{-i\omega(t+x)}, \quad x \to -\infty \quad (r \to r_+), \tag{9}$$

where $r_+ = M + \sqrt{M^2 - Q^2}$ is the radius of the black hole horizon.

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where $r_+ = M + \sqrt{M^2 - Q^2}$ is the radius of the black hole horizon.

2. Just outgoing waves near the acceleration horizon:

$$\Psi \sim e^{-i\omega(t-x)}, \quad x \to +\infty \quad (r \to 1/a). \tag{10}$$

Eikonal Limit and WKB QNM (Uncharged)

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- In this limit, the equations of motion are equivalent for perturbations of other spins, differing only at the next-to-leading order correction in the QNM frequency.
- By applying the WKB method we have:

$$\omega = \frac{\sqrt{\lambda f(r_c)}}{r_c} - i\left(n + \frac{1}{2}\right)\sqrt{-\frac{r_c^2}{2f(r_c)}\left(\frac{d^2}{dx^2}\frac{f(r)}{r^2}\right)_{r=r_c}}, \tag{12}$$

where r_c is the critical point of V_r and n the overtone number.

Quasinormal Modes Solutions

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The real part of ω becomes:

$$\frac{\Re(\omega)}{\sqrt{\lambda}} = \frac{\sqrt{1 + \sqrt{1 + 12a^2 M^2} + 12a^2 M^2(-3 + \sqrt{1 + 12a^2 M^2})}}{3\sqrt{6}M}, \tag{15}$$

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And the imaginary part:

$$\frac{\Im(\omega)}{(n+\frac{1}{2})} = -\frac{\sqrt{1+\sqrt{1+12a^2M^2}+12a^2M^2(2+12a^2M^2-3\sqrt{1+12a^2M^2})}}{3\sqrt{6}M} \quad (16)$$

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$$\lambda = l(l+1) + \frac{1}{3} + \\ + \Big[\frac{1 - l(l+1)(3l^2+3l-1) - m^2(15l^2+15l-11)}{2(2l-1)(2l+3)} r_+^2 \left(1 + \frac{Q^4}{r_+^4}\right) - \\ \frac{l(l+1)(3l^2+3l-2) + 3m^2(9l^2+9l-7)}{3(2l-1)(2l+3)} Q^2\Big] a^2 + O(a^3). \quad (17)$$

Relation with Circular Null Geodesics

- For spherically symmetric black holes we have (V. Cardoso, A. S. Miranda, E. Berti, H. Witek and V. T. Zanchi) in the eikonal limit

$$\omega = \left(l + \frac{1}{2}\right)\Omega_c - i\left(n + \frac{1}{2}\right)\Lambda, \tag{18}$$

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- In this black hole, it is not straightforward that this still holds. Though, with the geodesic equations of motion we can prove that

$$\Omega_c = \frac{\sqrt{(\frac{d\theta}{d\eta})^2/P(\theta) + (\frac{d\varphi}{d\eta})^2 P(\theta)\sin^2\theta}}{\frac{dt}{d\eta}} = \frac{\sqrt{f(r_c)}}{r_c} \tag{19}$$

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Thus $\Re(\omega) = \sqrt{\lambda}\Omega_c$, and identically for the Lyaponouv exponent.

Shadow Radius

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Black holes and Black Bodies

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- This is due to the gravitational attraction that the black hole exerts on its radiation, leading to the absorption of the latter.
- A **greybody factor** $\Gamma(\omega)$ is introduced: it is interpreted as the probability of radiation escaping the black hole.
- The number of particles emitted by a BH per unit time per unit frequency becomes:

$$\frac{dN_i}{dtd\omega_i} = \frac{1}{2\pi} \sum_{\lambda,m} \frac{g_i \Gamma^s_{\lambda,m}(\omega)}{e^{\omega/T} \pm 1}, \tag{21}$$

where g is the degeneracy of the particle species.

Greybody Factors

These are obtained by solving the scattering problem of fields in the background geometry. The boundary conditions are:

1. Near the horizon some radiation is **absorbed**:

$$\phi = Te^{-i\omega x}, \quad x \to -\infty \quad (r \to r_+), \tag{22}$$

   where T is the **transmission coefficient** and r_+ is the radius of the black hole horizon.

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Conservation of energy holds:

$$|T|^2 + |R|^2 = 1 \tag{24}$$

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The solution is:

$$\Gamma(\omega) = \left(\exp\left[\left(\sqrt{\lambda} - \frac{54M^2\omega^2}{\sqrt{\lambda}(2-\sqrt{1+12a^2M^2})(1-12a^2M^2+\sqrt{1+12a^2M^2})}\right)\frac{\pi}{(1+12a^2M^2)^{1/4}}\right] + 1\right)^{-1}. \tag{27}$$

Generalization to Charged Accelerating Black Hole

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$$r_c = \frac{-1+a^2Q^2}{3a^2M} + \frac{2\sqrt{1+a^4Q^4+a^2(9M^2-2Q^2)}}{3a^2M} \cos\left(\frac{1}{3}\arccos\left[\frac{-2+a^2(-27M^2(1+a^2Q^2)+2Q^2(3-3a^2Q^2+a^4Q^4))}{2(1+a^4Q^4+a^2(9M^2-2Q^2))^{3/2}}\right]\right) \qquad (29)$$

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$$\frac{\Im(\omega)}{n+\frac{1}{2}} = -\frac{\sqrt{(Q^2-2Mr_c+r_c^2)(1-a^2 r_c^2)(-10Q^2+12Mr_c+3(-1+a^2 Q^2)r_c^2-2a^2 Mr_c^3)}}{r_c^3} \tag{31}$$

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- Given the WKB solution the QNMs are:

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$$\frac{\Im(\omega)}{n+\frac{1}{2}} = -\frac{\sqrt{(Q^2-2Mr_c+r_c^2)(1-a^2r_c^2)(-10Q^2+12Mr_c+3(-1+a^2Q^2)r_c^2-2a^2Mr_c^3)}}{r_c^3} \tag{31}$$

- The greybody factor is

$$\Gamma(\omega) = \left(\exp\left[\left(\sqrt{\lambda} - \frac{\omega^2 r_c^4}{\sqrt{\lambda}(Q^2-2Mr_c+r_c^2)(1-a^2r_c^2)}\right)\frac{\pi r_c}{\sqrt{-10Q^2+12Mr_c+3(-1+a^2Q^2)r_c^2-2a^2Mr_c^3}}\right]+1\right)^{-1} \tag{32}$$

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Appendix A - Teukolsky equation (Uncharged Black Hole)

- Using the Newman Penrose formalism gives us the master equation

$$(r^2 f(r))^{-s} \frac{d}{dr}\left((r^2 f(r))^{s+1} \frac{dR(r)}{dr} \right) + V(r)R(r) = 0, \tag{33}$$

with

$$V(r) = -2ra^2(r-M)(1+s)(1+2s) + \frac{r^2\omega^2}{f(r)} - \\ - 2is\omega r \left[\frac{M}{r-2M} - \frac{1}{1-a^2r^2} \right] - B \tag{34}$$

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- Briefly, we'll see that B is related to λ.

Change of Variables

- By performing the following transformation

$$\Phi(r) = r(r^2 f(r))^{s/2} R(r) \quad \text{and} \quad x = \int \frac{dr}{f(r)}, \tag{35}$$

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- The potential is given by

$$V_r = \frac{(r-2M)(1-a^2r^2)(2M+rs) + (M-r+a^2Mr^2)^2 s^2}{r^4} + \\ + \frac{(r-2M)(1-a^2r^2)}{r^3} B - \frac{2i(r+M(-3+a^2r^2))s\omega}{r^2}. \tag{37}$$

Take the Eikonal Limit $B \to \infty$

- By now taking the eikonal limit $B \to \infty$, which means saving terms proportional in B and ω^2, we get:

$$V_r(r) = \left(1 - \frac{2M}{r}\right)(1 - a^2r^2)\frac{B}{r^2} = B\frac{f(r)}{r^2}, \tag{38}$$

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- Thus the limit is the same for different spin perturbations and $B = \lambda$ in this approximation.