On the uniqueness of the Kerr-(A)dS metric spacetime as a type II(D) solution in six dimensions[1]

Marcello Ortaggio

Institute of Mathematics
Czech Academy of Sciences

XVIII Black Holes Workshop, Lisbon – December 18th, 2025

[1] Joint work with D. Kokoška, Phys. Rev. D 112 (2025) 044050

Contents

Background: 4D Golden Age's results

Goldberg-Sachs theorem (1962)

1. vacuum: $R_{ab} = 0$
2. type II (or more special): $\ell_{[e} C_{a]bcd} \ell^b \ell^c = 0 \quad (\ell_a \ell^a = 0)$

$\Rightarrow$ $\boldsymbol{\ell}$ geodesic and shearfree

Background: 4D Golden Age's results

Goldberg-Sachs theorem (1962)

1. vacuum: $R_{ab} = 0$
2. type II (or more special): $\ell_{[e}C_{a]bcd}\ell^b\ell^c = 0 \quad (\ell_a\ell^a = 0)$

$\Rightarrow \boldsymbol{\ell}$ geodesic and shearfree

$$\Leftrightarrow \ell_{a;b}\ell^b = 0 \text{ and } \boldsymbol{\rho} = \begin{pmatrix} \theta & \omega \\ -\omega & \theta \end{pmatrix},$$

where $\rho_{ij} \equiv \ell_{a;b}m^{(i)a}m^{(j)b}$ (in a spatial o.n. frame)

Background: 4D Golden Age's results

Goldberg-Sachs theorem (1962)

1. vacuum: $R_{ab} = 0$
2. type II (or more special): $\ell_{[e}C_{a]bcd}\ell^b\ell^c = 0 \quad (\ell_a\ell^a = 0)$

$\Rightarrow \boldsymbol{\ell}$ geodesic and shearfree

$$\Leftrightarrow \ell_{a;b}\ell^b = 0 \text{ and } \boldsymbol{\rho} = \begin{pmatrix} \theta & \omega \\ -\omega & \theta \end{pmatrix},$$

where $\rho_{ij} \equiv \ell_{a;b}m^{(i)a}m^{(j)b}$ (in a spatial o.n. frame)

$\longrightarrow$ 1st step in the Newman-Penrose integration.

Background: 4D Golden Age's results

Goldberg-Sachs theorem (1962)

1. vacuum: $R_{ab} = 0$
2. type II (or more special): $\ell_{[e}C_{a]bcd}\ell^b\ell^c = 0 \quad (\ell_a\ell^a = 0)$

$\Rightarrow \boldsymbol{\ell}$ geodesic and shearfree

$$\Leftrightarrow \ell_{a;b}\ell^b = 0 \text{ and } \boldsymbol{\rho} = \begin{pmatrix} \theta & \omega \\ -\omega & \theta \end{pmatrix},$$

where $\rho_{ij} \equiv \ell_{a;b}m^{(i)a}m^{(j)b}$ (in a spatial o.n. frame)

$\longrightarrow$ 1st step in the Newman-Penrose integration.

Notably:

- Kerr'63: "Gravitational Field of a Spinning Mass as an Example of Algebraically Special Metrics"
- Kinnersley'69: complete classification and integration for type D.

Some developments in HD

1. Weyl tensor classification [Coley-Milson-Pravda-Pravdová’04]

Some developments in HD

1. Weyl tensor classification [Coley-Milson-Pravda-Pravdová'04]
2. NP formalism

   [Pravda-Pravdová-Coley-Milson'04, Coley-Milson-Pravda-Pravdová'04, M.O.-Pravda-Pravdová'07]

Some developments in HD

1. Weyl tensor classification [Coley-Milson-Pravda-Pravdová'04]
2. NP formalism
   [Pravda-Pravdová-Coley-Milson'04, Coley-Milson-Pravda-Pravdová'04, M.O.-Pravda-Pravdová'07]
3. Goldberg-Sachs theorem:
   - "violations"
     [Myers-Perry'86, Frolov-Stojković'03, Pravda-Pravdová-M.O.'07]
   - partial extensions
     [Pravda-Pravdová-Coley-Milson'04, M.O.-Pravda-Pravdová'09, M.O.-Pravda-Pravdová-Reall'12, M.O.-Pravda-Pravdová'13,'18, Tintěra-Pravda'19]

Some developments in HD

1. Weyl tensor classification [Coley-Milson-Pravda-Pravdová'04]
2. NP formalism
   [Pravda-Pravdová-Coley-Milson'04, Coley-Milson-Pravda-Pravdová'04, M.O.-Pravda-Pravdová'07]
3. Goldberg-Sachs theorem:
   - "violations"
     [Myers-Perry'86, Frolov-Stojković'03, Pravda-Pravdová-M.O.'07]
   - partial extensions
     [Pravda-Pravdová-Coley-Milson'04, M.O.-Pravda-Pravdová'09, M.O.-Pravda-Pravdová-Reall'12, M.O.-Pravda-Pravdová'13,'18, Tintěra-Pravda'19]
4. Complete classification of all 5D vacua of type II(D)
   [Bernardi de Freitas-Godazgar-Reall'15,'16, Wylleman'15, García-Parrado&Wylleman'11, Reall-Graham-Turner'13]

Assumptions in this work:

1. $R_{ab} = (n-1)\lambda, \quad n = 6$

Assumptions in this work:

1. $R_{ab} = (n-1)\lambda, \quad n = 6$
2. $\ell_{[e} C_{a]b[cd} \ell_{f]} \ell^b = 0$ (type II), $\quad \boldsymbol{\ell} = \partial_r$ geodesic

Assumptions in this work:

1. $R_{ab} = (n-1)\lambda, \quad n = 6$
2. $\ell_{[e}C_{a]b[cd}\ell_{f]}\ell^b = 0$ (type II), $\quad \boldsymbol{\ell} = \partial_r$ geodesic
3. rank($\boldsymbol{\rho}$)=4

Assumptions in this work:

1. $R_{ab} = (n-1)\lambda, \quad n = 6$
2. $\ell_{[e}C_{a]b[cd}\ell_{f]}\ell^{b} = 0$ (type II), $\quad \boldsymbol{\ell} = \partial_r$ geodesic
3. rank($\boldsymbol{\rho}$)=4
4. $C_{ijkl} = o(r^{-2})$ for $r \to \infty$ (asymptotic (A)dS)

Assumptions in this work:

1. $R_{ab} = (n-1)\lambda, \quad n = 6$
2. $\ell_{[e} C_{a]b[cd} \ell_{f]} \ell^b = 0$ (type II), $\quad \boldsymbol{\ell} = \partial_r$ geodesic
3. rank($\boldsymbol{\rho}$)=4
4. $C_{ijkl} = o(r^{-2})$ for $r \to \infty$ (asymptotic (A)dS)

$\Rightarrow$ partial extension of Goldberg-Sachs [M.O.-Pravda-Pravdová'09]:

$$\boldsymbol{\rho}^{-1} = \mathrm{diag}\left(\begin{bmatrix} r & -y_1 \\ y_1 & r \end{bmatrix}, \begin{bmatrix} r & -y_2 \\ y_2 & r \end{bmatrix} \right).$$

Assumptions in this work:

1. $R_{ab} = (n-1)\lambda, \quad n = 6$
2. $\ell_{[e}C_{a]b[cd}\ell_{f]}\ell^b = 0$ (type II), $\quad \boldsymbol{\ell} = \partial_r$ geodesic
3. rank($\boldsymbol{\rho}$)=4
4. $C_{ijkl} = o(r^{-2})$ for $r \to \infty$ (asymptotic (A)dS)

   $\Rightarrow$ partial extension of Goldberg-Sachs [M.O.-Pravda-Pravdová'09]:

$$\boldsymbol{\rho}^{-1} = \mathrm{diag}\left(\begin{bmatrix} r & -y_1 \\ y_1 & r \end{bmatrix}, \begin{bmatrix} r & -y_2 \\ y_2 & r \end{bmatrix}\right).$$

5. $\boldsymbol{\rho}$ is "generic": $|y_1| \neq |y_2|$, $\mathrm{d}y_1 \neq 0 \neq \mathrm{d}y_2$

Result:

explicit form of all metrics in terms of 1+3 parameters $\hat{\mathcal{U}}^0, c_0, d_0, \mu$

Result:

explicit form of all metrics in terms of 1+3 parameters $\hat{\mathcal{U}}^0, c_0, d_0, \mu$

$$
\begin{aligned}
\mathrm{d}s^2 = {} & 2\mathrm{d}r\left[\mathrm{d}u + \left(y_1^2 + y_2^2\right)\mathrm{d}\phi_1 + y_1^2 y_2^2 \mathrm{d}\phi_2\right] + \left(r^2 + y_1^2\right)\frac{y_2^2 - y_1^2}{\boldsymbol{\mathcal{P}(y_1)}}\mathrm{d}y_1^2 \\
& + \frac{\boldsymbol{\mathcal{P}(y_1)}}{(r^2 + y_1^2)(y_2^2 - y_1^2)}\left[\mathrm{d}u + \left(y_2^2 - r^2\right)\mathrm{d}\phi_1 - r^2 y_2^2 \mathrm{d}\phi_2\right]^2 + (y_1 \leftrightarrow y_2) \\
& + \frac{\boldsymbol{\mathcal{Q}(r)}}{(r^2 + y_1^2)(r^2 + y_2^2)}\left[\mathrm{d}u + \left(y_1^2 + y_2^2\right)\mathrm{d}\phi_1 + y_1^2 y_2^2 \mathrm{d}\phi_2\right]^2,
\end{aligned}
$$

$$
\begin{aligned}
\boldsymbol{\mathcal{P}(y_1)} &\equiv \lambda y_1^6 + 2\hat{\mathcal{U}}^0 y_1^4 - c_0 y_1^2 - d_0, \\
\boldsymbol{\mathcal{Q}(r)} &\equiv \lambda r^6 - 2\hat{\mathcal{U}}^0 r^4 - c_0 r^2 + d_0 + \mu r.
\end{aligned}
$$

Comments:

- all of type D and Kerr-Schild

Comments:

- all of type D and Kerr-Schild

  $\rightarrow$ double copy [Monteiro-O'Connel-White'14, Bahjat-Abbas&Luna&White'17, Carrillo-González&Penco&Trodden'18]

$$\boldsymbol{A} = \frac{er}{(r^2+y_1^2)(r^2+y_2^2)}\left[\mathrm{d}u + \left(y_1^2+y_2^2\right)\mathrm{d}\phi_1 + y_1^2 y_2^2 \mathrm{d}\phi_2\right]$$

cf. [Aliev'07, Krtouš'07, Chen-Lü'08]

Comments:

- all of type D and Kerr-Schild

  $\rightarrow$ double copy [Monteiro-O'Connel-White'14, Bahjat-Abbas&Luna&White'17, Carrillo-González&Penco&Trodden'18]

$$\boldsymbol{A} = \frac{er}{(r^2 + y_1^2)(r^2 + y_2^2)} \left[\mathrm{d}u + \left(y_1^2 + y_2^2\right) \mathrm{d}\phi_1 + y_1^2 y_2^2 \mathrm{d}\phi_2\right]$$

  cf. [Aliev'07, Krtouš'07, Chen-Lü'08]

- belong to the general Kerr-NUT-(A)dS class [Chen-Lü-Pope'06]

Comments:

- all of type D and Kerr-Schild

  $\rightarrow$ double copy [Monteiro-O'Connel-White'14, Bahjat-Abbas&Luna&White'17, Carrillo-González&Penco&Trodden'18]

$$\boldsymbol{A} = \frac{er}{(r^2+y_1^2)(r^2+y_2^2)}\left[\mathrm{d}u + \left(y_1^2+y_2^2\right)\mathrm{d}\phi_1 + y_1^2 y_2^2 \mathrm{d}\phi_2\right]$$

  cf. [Aliev'07, Krtouš'07, Chen-Lü'08]

- belong to the general Kerr-NUT-(A)dS class [Chen-Lü-Pope'06]
- special subfamily:

$$\mathcal{P}(s) = (\lambda s^2+\epsilon)(s^2-a_1^2)(s^2-a_2^2), \quad \epsilon = 0, \pm 1$$

  $\rightarrow$ (extended) Kerr-(A)dS metrics

  [Gibbons-Lü-Page-Pope'05, Mars&Peón-Nieto'22, Chruściel-Cong-Gray'25]

In progress: non-generic branches (single/equal spins, Taub-NUT and shearfree limits, . . .)

1. $\mathrm{d}y_1 = 0 \neq \mathrm{d}y_2$ ($y_1 = 0$ vs. $y_1 \neq 0$)
2. $\mathrm{d}y_1 = 0 = \mathrm{d}y_2$
3. $y_1 = y_2$.

Thank you!