On Folding Calabi-Yau Diagrams in M-theory and Black Holes

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ESMaR-FSR, Physics Department, Mohammed V University in Rabat

JXVIII BHs Workshop

12 décembre 2025

Talk information

Talk information

- AB, Abderrahim Bouhouch, On Folding Calabi-Yau Diagrams in M-theory Black Brane Scenarios, arXiv :2505.12948, hep-th math-ph

- Based on works and discussions with
  - Saad Eddine Badis (Rabat)
  - Hajar Belmahi (Rabat)
  - Abderrahim Bouhouch (Rabat)
  - Salah Eddine Ennadifi (Rabat)
  - Maryem Jemri (Rabat)
  - Moulay Brahim Sedra (Kénitra/Meknès).

Plan

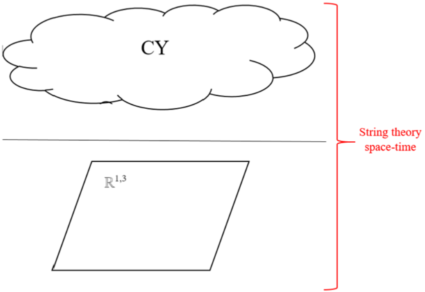
- Black hole physics
- Black objects in non-trivial theories
  - String theory in $D = 10$
  - M-theory in $D = 11$.
- Approaching and understanding new features of Calabi-Yau spaces.
- Connecting mathematics with physics via black holes

C. Long, A. Sheshmani, C. Vafa and S. T. Yau, *Non-Holomorphic Cycles and Non-BPS Black Branes*, Commun. Math. Phys. **399** (2023)1991.

- M-Theory scenarios in 5D

$$\text{Black brane objects} = \begin{cases} \text{0-branes}: & \text{Black holes} \\ \text{1-branes}: & \text{Black strings} \end{cases}$$

- String theory space-time



- Calabi-Yau space 3-folds (CY^3)
  - Comapct (finite size)
  - Complex
  - Kahler
  - $SU(3)$ holonomy group
  - Vanishing first Chern class.

Calabi-Yau constructions

- Orbifold constructions

$$\mathrm{CY}^3 = \frac{\mathbb{T}^{2x3}}{G}, \qquad \mathbb{T}^2 = \mathbb{S}^1 \times \mathbb{S}^1$$

  - G= Discrete subgroup of $SU(3)$ holonomy group.

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Complete intersection Calabi-Yau scenarios

- Embedding space

$$\mathcal{A} := \mathbb{CP}^{n_1} \times \ldots \times \mathbb{CP}^{n_m}$$

- Matrix configuration

$$\mathrm{CY}^3 \equiv \left[\begin{array}{c||ccc} \mathbb{CP}^{n_1} & d_1^1 & \ldots & d_K^1 \\ \vdots & \vdots & \ddots & \vdots \\ \mathbb{CP}^{n_m} & d_1^m & \ldots & d_K^m \end{array} \right]^{h^{1,1},h^{1,2}}_{\chi(\mathcal{CY}^3)}$$

- CY^3 and dimension conditions

$$\sum_{i=1}^{h^{1,1}} n_i - K = 3, \qquad n_i + 1 = \sum_{r=1}^{K} d_r^i$$

Graph theory representation of CICY

- Graph theory

$$\text{Graph=Diagram} = (V(\mathcal{D}), E(\mathcal{D}))$$

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- Example : bi-cubic CY^3

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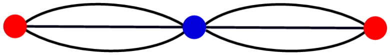

Figure – Diagram of bi-cubic in $\mathbb{CP}^2 \times \mathbb{CP}^2$.

Folding CICY diagrams

- Outer-automorphism group Γ

$$\Gamma : \mathcal{D} \rightarrow \mathcal{D}$$

- Folded CY diagram

$$\mathcal{D} \rightarrow \frac{\mathcal{D}}{\Gamma}$$

Tetra-quadric CY^3 scenarios

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$$\mathcal{A} := \left[\begin{array}{c||c} \mathbb{CP}^1 & 2 \\ \mathbb{CP}^1 & 2 \\ \mathbb{CP}^1 & 2 \\ \mathbb{CP}^1 & 2 \end{array}\right]^{4,68}_{-128} .$$

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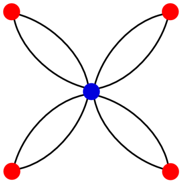

Figure – Tetra-quadric CY^3 diagram.

Tetra-quadric CY folding scenarios

- Outer-automorphism group Γ

$$\Gamma = \mathbb{Z}_2, \quad \mathbb{Z}_2 \times \mathbb{Z}_2, \quad \mathbb{Z}_3, \quad \mathbb{Z}_4.$$

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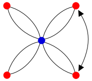
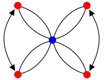
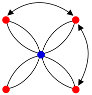
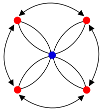
	$\Gamma = \mathbb{Z}_2$
	$\Gamma = \mathbb{Z}_2 \times \mathbb{Z}_2$
	$\Gamma = \mathbb{Z}_3$
	$\Gamma = \mathbb{Z}_4$

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- Non vanishing interesting numbers

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M-theory black holes from the tetra-quadric CY[3]

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$$\mathcal{V} = \frac{1}{3!} C_{ijk} t^i t^j t^k = 2t_1 t_2 t_3 + 2t_1 t_3 t_4 + 2t_2 t_3 t_4 + 2t_1 t_2 t_4$$

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$$G_{ij} = -\frac{1}{2} \partial_i \partial_j \log(\mathcal{V})$$

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- Black hole scalar potential

$$V_{eff}^{BH}(q_i, t_i) = G^{ij}(t_i, t_j) q_i q_j, \qquad i, j = 1, \ldots, 4.$$

Folding procedure in M-theory scenarios

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- $\mathbb{Z}_2$ folding procedure in M-theory scenarios

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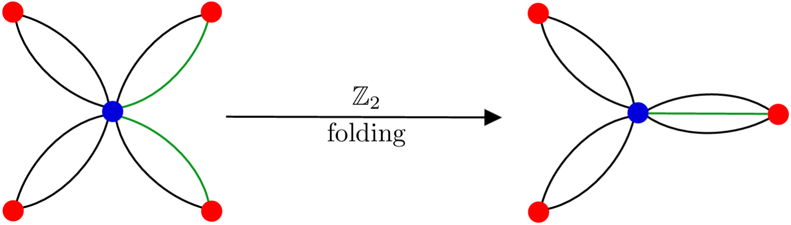

Figure – $\mathbb{Z}_2$ folding of tetra-quadric CY^3 diagram

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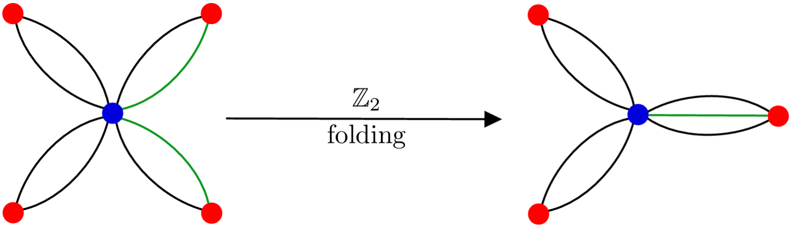
- $\mathbb{Z}_2$ folding procedure in M-theory scenarios



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- Potential computations

$V_{eff}^{BH}(\mathbb{CP}^1 \times \mathbb{CP}^1 \times \mathbb{CP}^1 \times \mathbb{CP}^1) - \mathbb{Z}_2$ Folding $- > V_{eff}^{BH}(\mathbb{CP}^1 \times \mathbb{CP}^1 \times \mathbb{CP}^2)$.

Conclusion

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  - M-theory compactification on the tetra-quadric CY^3
  - Folding of tetra-quadric CY diagram
  - Outer-automorphism groups

$$\mathbb{Z}_2, \quad \mathbb{Z}_2 \times \mathbb{Z}_2, \quad \mathbb{Z}_3, \quad \mathbb{Z}_4$$

  - Obtained geometries

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- Open questions
  - Extension to other CY^3 manifolds ?
  - Useful for stability and thermodynamics behaviors of stringy black holes ?
  - ....

Thank You For Your Attention.