Thermodynamics of hot curved space in the canonical ensemble and black hole nucleation

Tiago V. Fernandes

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centra

Cavity surrounded by a heat reservoir

Euclidean path integral approach

Gibbons and Hawking[1]: $Z = \int Dg\, \mathrm{e}^{-I[g]}$, $Z_{0\mathrm{loop}} = \mathrm{e}^{-I[g_{\mathrm{cl}}]} = \mathrm{e}^{-\beta F}$;
Gross, Perry and Yaffe[2]: Black hole nucleation from hot flat space;
York's realization[3]: Finite cavity.

Entropy of self-gravitating radiation fluid

Sorkin, Wald and Zhang[4]: radiation as perfect fluid inside a spherical cavity.

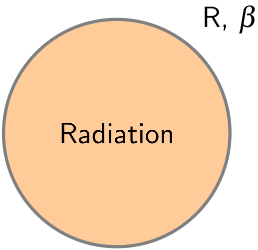
[1] G. W. Gibbons and S. W. Hawking, Phys. Rev. D **15**, 2752 (1977).
[2] D. J. Gross, M. J. Perry and L. G. Yaffe, Phys. Rev. D **25**, 330 (1982).
[3] J. W. York, Phys. Rev. D **33**, 2092 (1986).
[4] R. D. Sorkin, R. M. Wald and Z. J. Zhang, Gen. Rel. Grav. **13**, 1127 (1981).

Self-gravitating fluid inside a cavity

The setup

The action

$$I = -\frac{1}{16\pi l_{\mathrm{p}2}} \int \sqrt{g} R d^4x + \int \mathscr{F}_{\mathrm{m}} d^4x - \frac{1}{8\pi l_{\mathrm{p}2}} \int (K - K_0)\sqrt{\gamma} d^3x$$

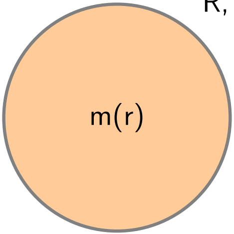


Spherical symmetry
$ds^2 = b(y)^2 d\tau^2 + a(y)^2 dy^2 + r^2 d\Omega^2$
Boundary conditions
Regularity: $b(0)$ finite, $\frac{b'}{a}|_{y=0} = 0$, $r(0) = 0$;
Heat reservoir: $\beta = 2\pi b(1)$, $r(1) = R$;
Perfect fluid: $\varepsilon = aT_{\mathrm{loc}}^4$;
Local temperature: $T_{\mathrm{loc}} = \frac{1}{2\pi b(y)}$ (ensures the conservation of the stress-tensor).

Imposing the Hamiltonian constraints, the action becomes $\left(f = 1 - \frac{2m(r)}{r}\right)$

$$I(R,\beta;m(r)) = \frac{R\beta}{l_{\mathrm{p}}^2}\left(1-\sqrt{f}\right) - 4\pi\int_0^R s\, r^2 f^{-1/2} dr \ ; \ m(r) = 4\pi\int_0^r \varepsilon\, r^2 dr$$

R, β



Equilibrium equation $(\delta I = 0)$

$$\frac{1}{\varepsilon+p}\frac{dp}{dr} + \frac{l_{\mathrm{p}}^2}{r^2 f}\left(4\pi r^3 p + m\right) = 0 \ \text{(TOV equation)}$$

Stability conditions $(\delta^2 I > 0)$

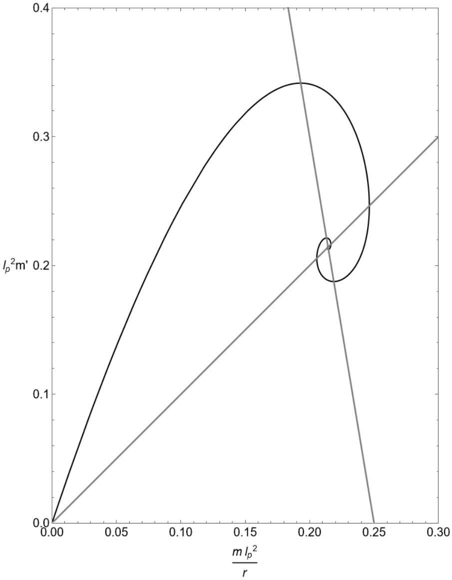
$$\hat{L}\delta m = -\chi\frac{\delta m \pi T_{\mathrm{loc}}}{r^2 f^{\frac{3}{2}}(\varepsilon+p)} \ ; \ \delta m'(R) = 0$$

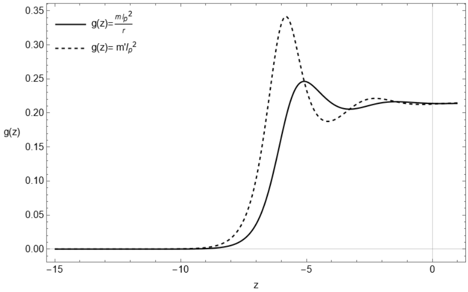
Stable if $\chi > 0$ for all modes δm

$$\hat{L}\delta m = \left(\frac{\delta m'}{4\pi r^2\sqrt{f}\,T_{\mathrm{loc}}^2}\frac{dT_{\mathrm{loc}}}{d\varepsilon}\right)' + \frac{2 l_{\mathrm{p}}^2}{r T_{\mathrm{loc}}^2 f^{\frac{3}{2}}}\left(\frac{T_{\mathrm{loc}}}{2r} - \frac{dT_{\mathrm{loc}}}{dr}\right)$$

Self-gravitating fluid inside a cavity

Equilibrium solutions





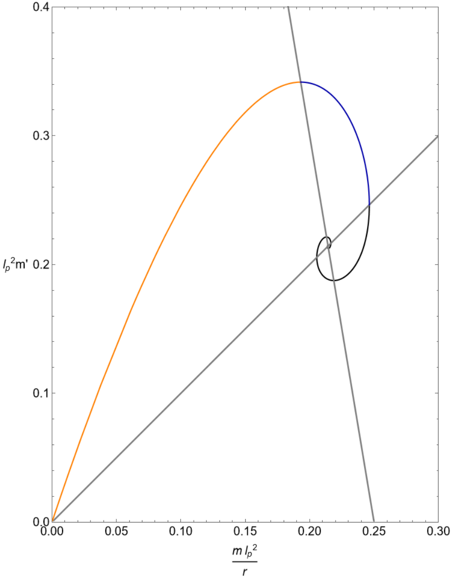
$$r\frac{d(m')}{dr} = \frac{2m'(1-4\frac{l_\mathrm{p}^2 m}{r} - \frac{2}{3}l_\mathrm{p}^2 m')}{(1-2\frac{l_\mathrm{p}^2 m}{r})}$$

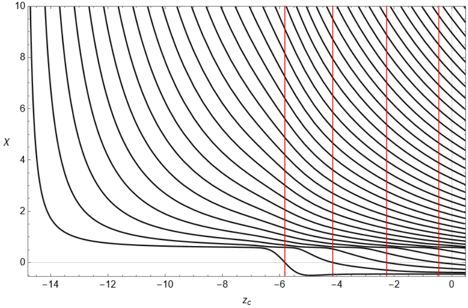
$$r\frac{d(m/r)}{dr} = m' - \left(\frac{m}{r}\right)$$

$$z \propto \log(r)$$

Self-gravitating fluid inside a cavity

Thermodynamic and mechanical stability





Thermodynamic and mechanical stability (orange)

$$\hat{L}\delta m = -\chi \frac{\delta m \pi T_{\rm loc}}{r^2 f^{\frac{3}{2}}(\varepsilon + p)}\ ;\ \delta m'(R) = 0$$

Mechanical stability (blue)

$$\delta m(R) = 0$$

Self-gravitating fluid inside a cavity
Thermodynamics

Entropy

$$S = \frac{2(4\pi a)^{\frac{1}{4}}}{9}\left(\frac{R}{l_{\mathrm{p}}}\right)^{\frac{3}{2}} \frac{l_{\mathrm{p}}^2(m' + \frac{m}{R})}{(l_{\mathrm{p}}^2 m')^{\frac{1}{4}}\sqrt{1 - \frac{2l_{\mathrm{p}}^2 m}{R}}}$$

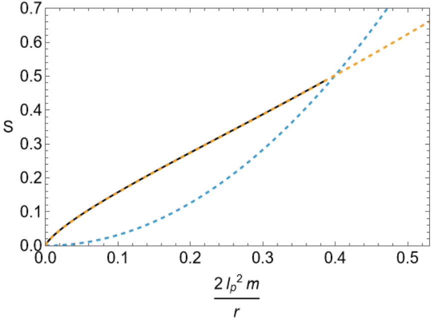
$R = l_{\mathrm{p}}$:

$$S \propto (\frac{2l_{\mathrm{p}}^2 m}{R})^{0.755} + 0.5(\frac{2l_{\mathrm{p}}^2 m}{R})^{2.24}$$

$$S_{\mathrm{bh}} = \pi\left(\frac{r_+}{R}\right)^2$$

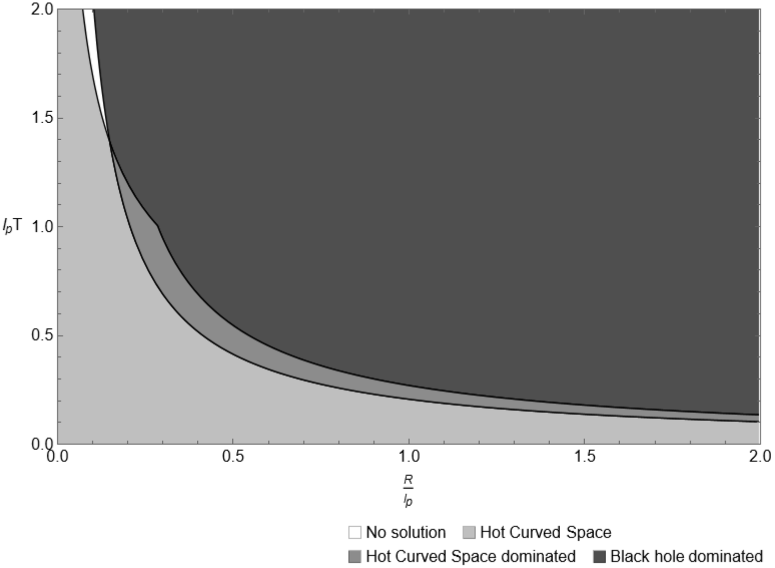
Energy

$$E = \frac{R}{l_{\mathrm{p}}^2}\left(1 - \sqrt{1 - \frac{2l_{\mathrm{p}}^2 m}{R}}\right)$$



Black - radiation entropy;
Orange - fit to radiation entropy;
Blue - black hole entropy;

Phase diagram Self-gravitating radiation vs Black hole



Summary

Construction of the canonical ensemble of a radiation fluid inside a cavity:

- There is a stable solution that exists for a maximum compactness ($\frac{l_{\mathrm{p}}^2 m}{R} \approx 0.19$) and temperature;
- Thermodynamic and mechanic stability is represented by one single condition;
- Entropy goes as $\left(\frac{l_{\mathrm{p}}^2 m}{R}\right)^{0.755}$ at first order;
- There is black hole nucleation;
- Picture becomes intricate beyond the Planck regime;