A greybody-factor approach to modeling black hole ringdown

XVIII BLACK HOLES WORKSHOP

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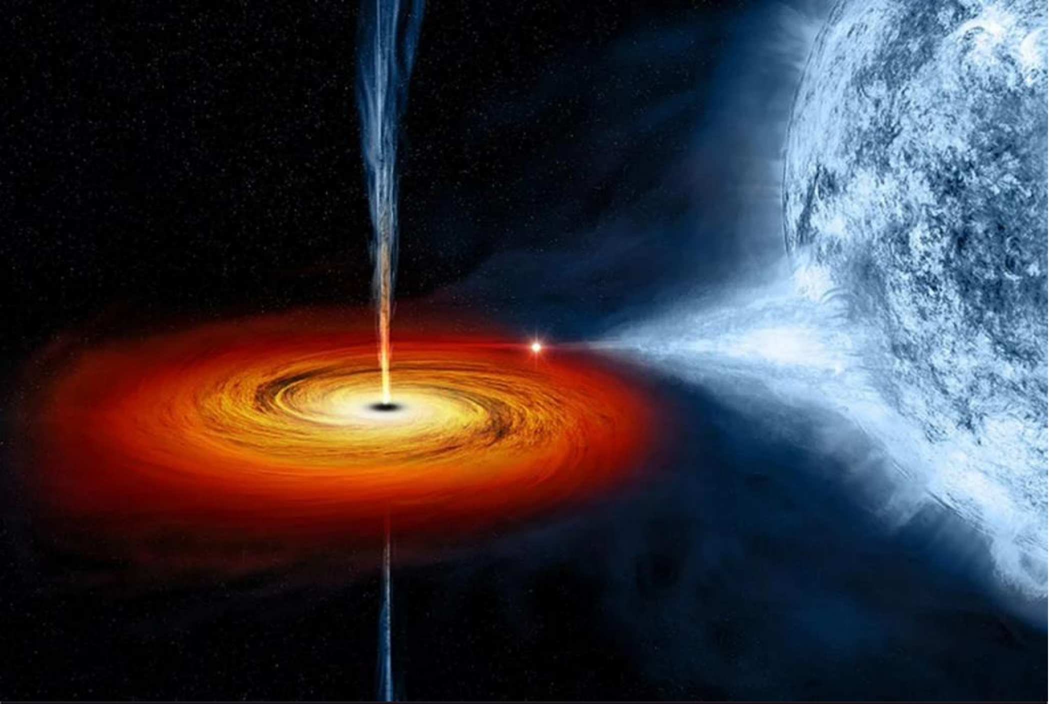
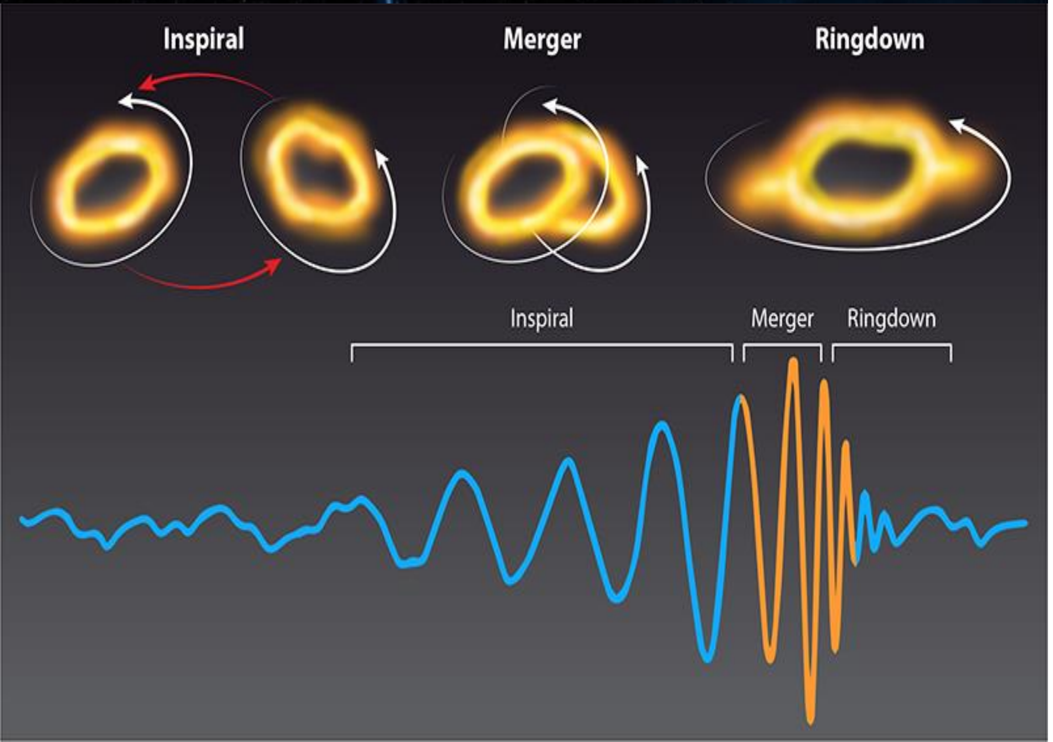
Lisbon 19/12/2025

https://arxiv.org/pdf/2512.15877

In collaboration with Sophia Yi, Emanuele Berti and Paolo Pani





OVERVIEW

1. **Ringdown signals**: greybody factors vs quasinormal modes.

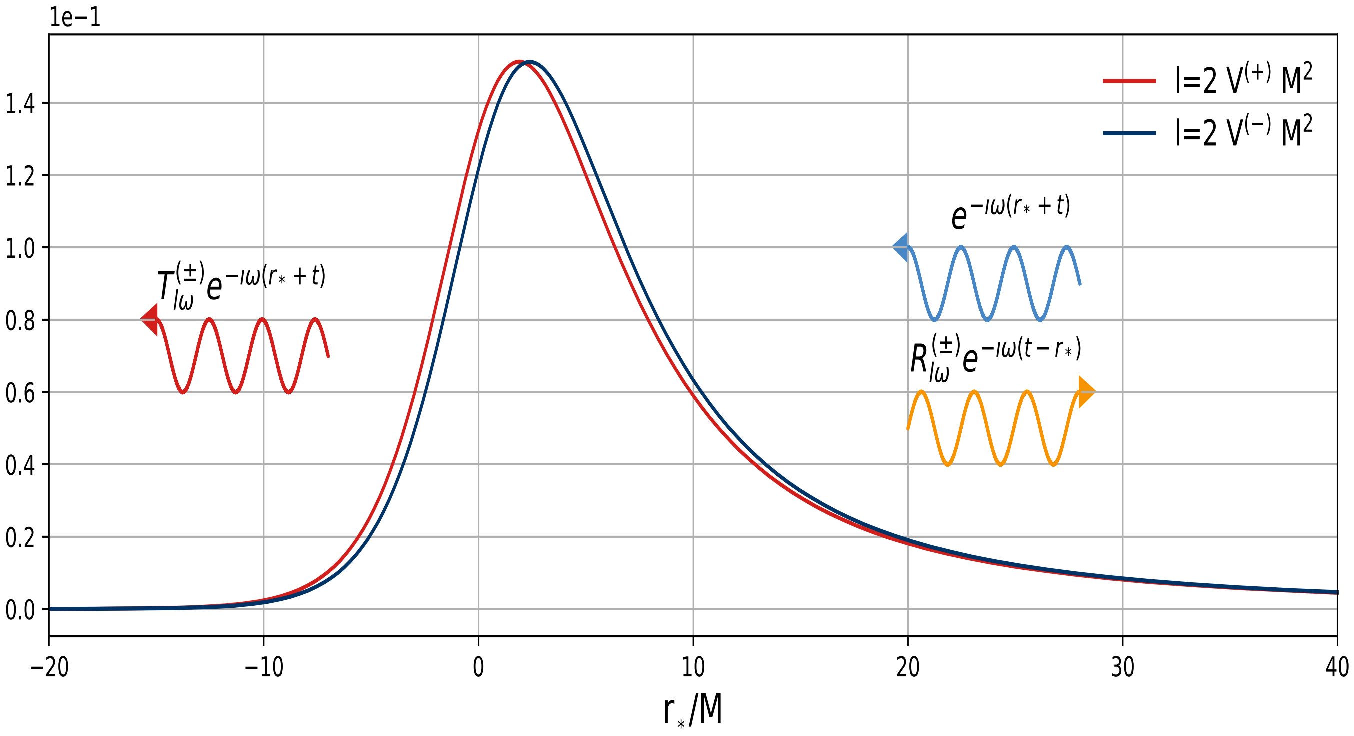
2. **Comparable mass mergers:** fitting frequency domain ringdown amplitude with greybody factors.

QUASINORMAL MODES vs GREYBODY FACTORS

Perturbations of a Schwarzschild black hole can be described through **Regge-Wheeler** and **Zerilli** equations.

$$\left(\frac{d^2}{dr_*^2} - V_l^{(\pm)}(r) + \omega^2\right) Z_l^{(\pm)}(r_*) = 0$$

Greybody factors (GFs) describe how the black hole reacts to a scattering process.



$$Z_l^{(\pm)}(r_*) \begin{cases} A_{l\omega}^{(\pm),out} e^{\imath\omega r_*} + A_{l\omega}^{(\pm),in} e^{-\imath\omega r_*} & r_* \to \infty \\ e^{-\imath\omega r_*} & r_* \to -\infty \end{cases}$$

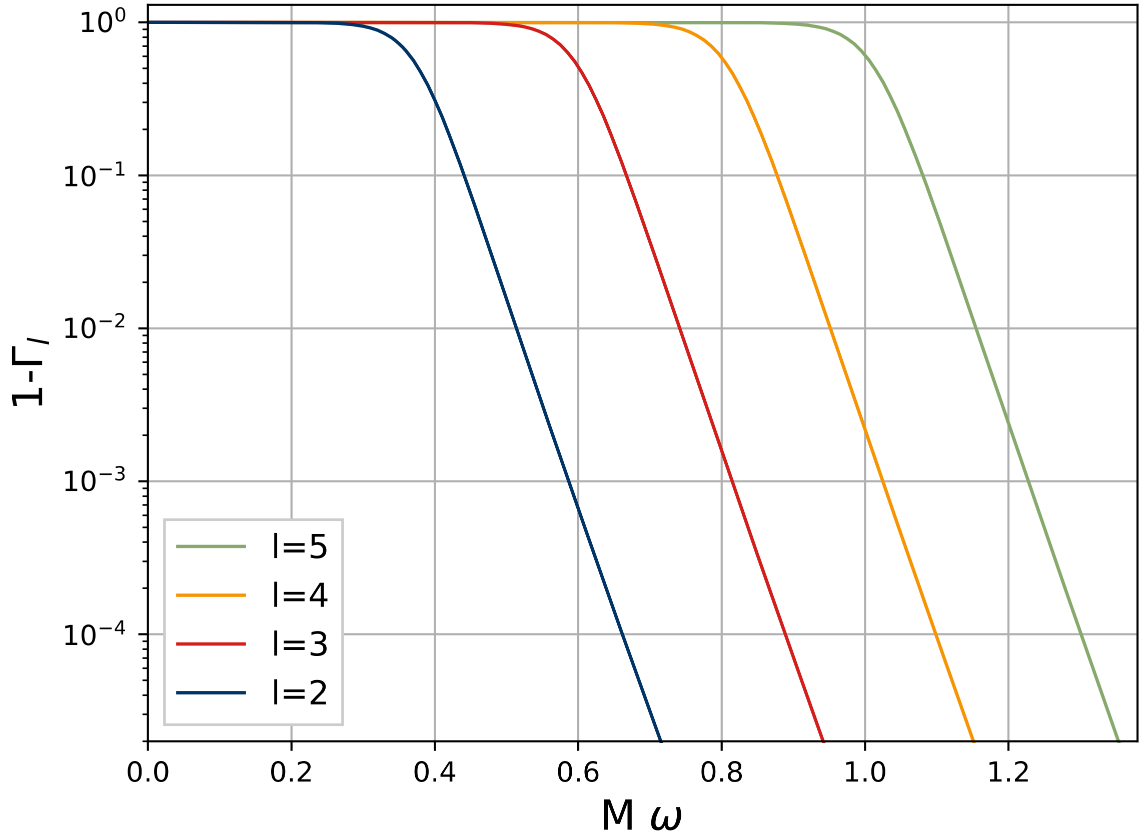
GREYBODY FACTORS

From this, one can define black hole reflectivity and black hole transmission amplitude (i.e. the **greybody factor** $\Gamma_l(\omega)$).

GFS AND REFLECTIVITY

$$\Gamma_l(\omega) = \left|\mathrm{T}^{(\pm)}_{\mathrm{l}\omega}\right|^2 = \left|\frac{1}{A^{(\pm),in}_{l\omega}}\right|^2$$

$$\mathcal{R}_l(\omega) = \left|\mathrm{R}^{(\pm)}_{\mathrm{l}\omega}\right|^2 = \left|\frac{A^{(\pm),out}_{l\omega}}{A^{(\pm),in}_{l\omega}}\right|^2 = 1 - \Gamma_l(\omega)$$



SOME RESULTS

- GFs have been showed to be imprinted the frequency domain ringdown amplitude in the point particle infall scenario for BHs.
  [see N.Oshita https://arxiv.org/pdf/2309.05725, **RFR**, K. Destounis and P. Pani https://arxiv.org/abs/2406.01692]

- GFs have been showed to be stable under small perturbations of the wave equation (differently from QNMs that are highly unstable)
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HOW TO MODEL GRAVITATIONAL WAVES WITH GFS

We all know that the time domain ringdown can be described as a superposition of quasinormal oscillations.

In the greybody factor framework however we focus on **frequency domain.**

$$H_{\ell m}(\omega) = |h_{\ell m}(\omega)| \quad \text{with} \quad h_{\ell m}(\omega) = \frac{1}{2\pi}\int dt\, h_{\ell m}(t) e^{-i\omega t}$$

The idea is really simple. The riflectivity is imprinted in the Fourier transform in the sense:

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Information about the **initial binary.**

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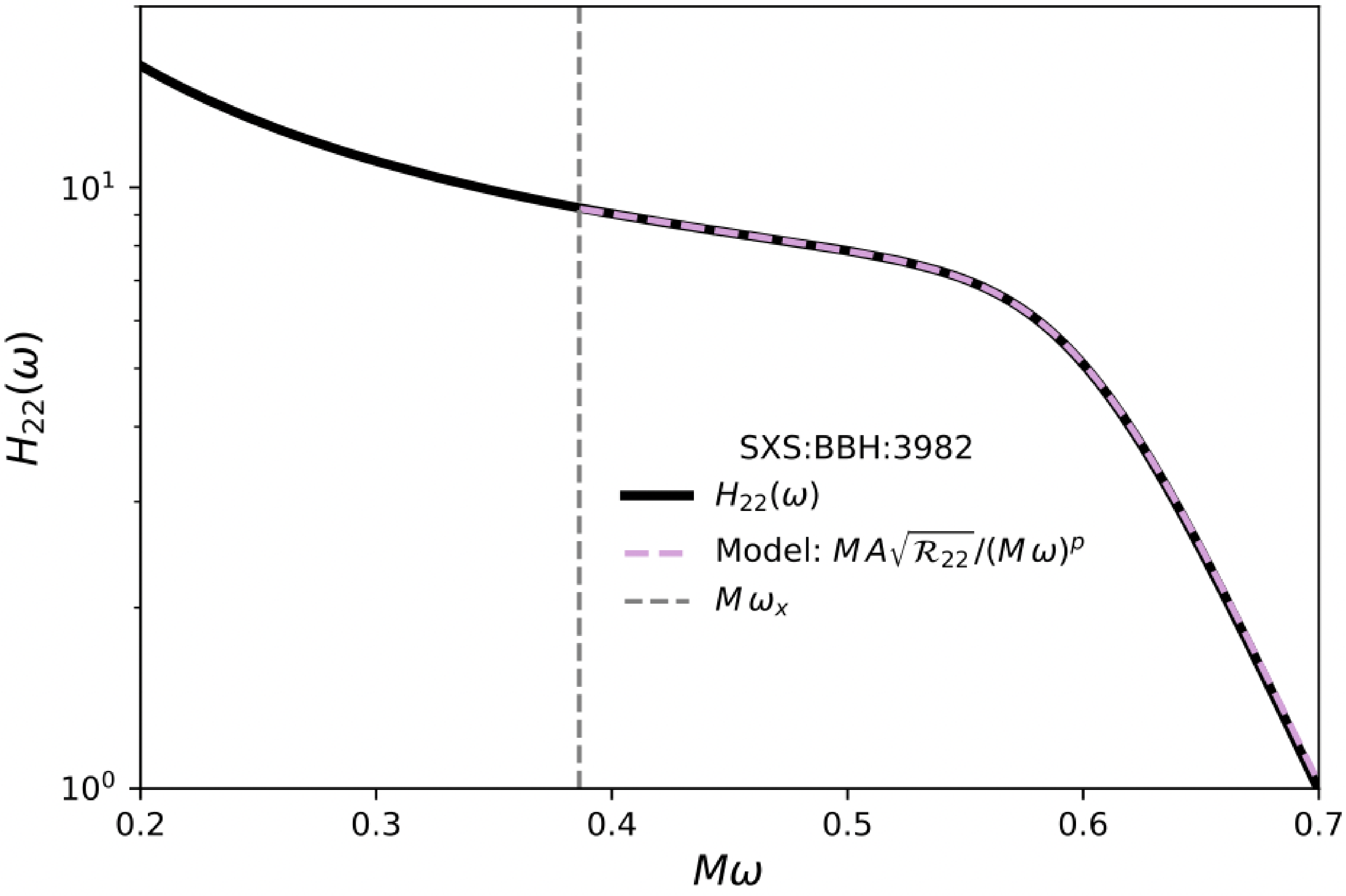

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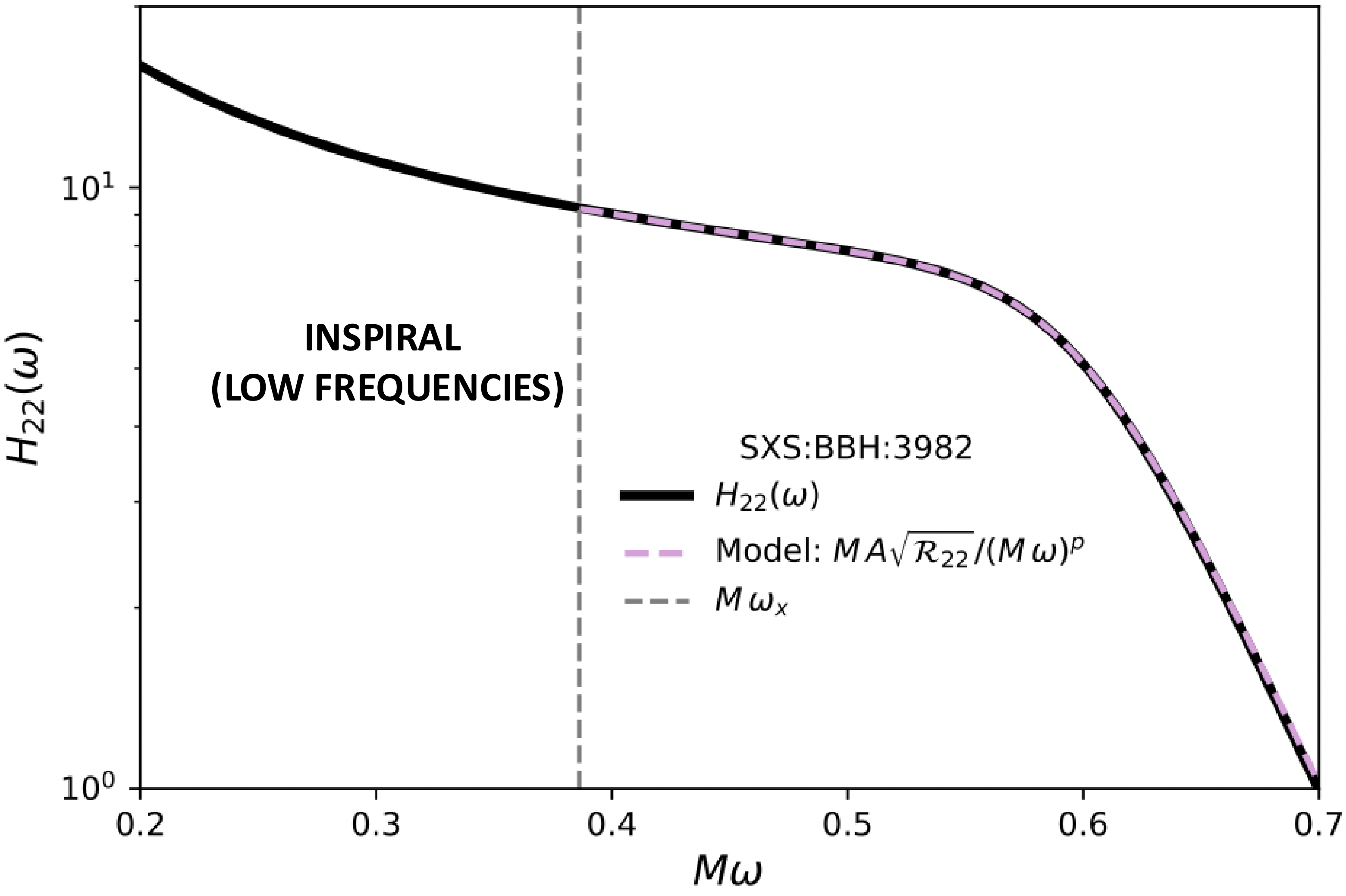

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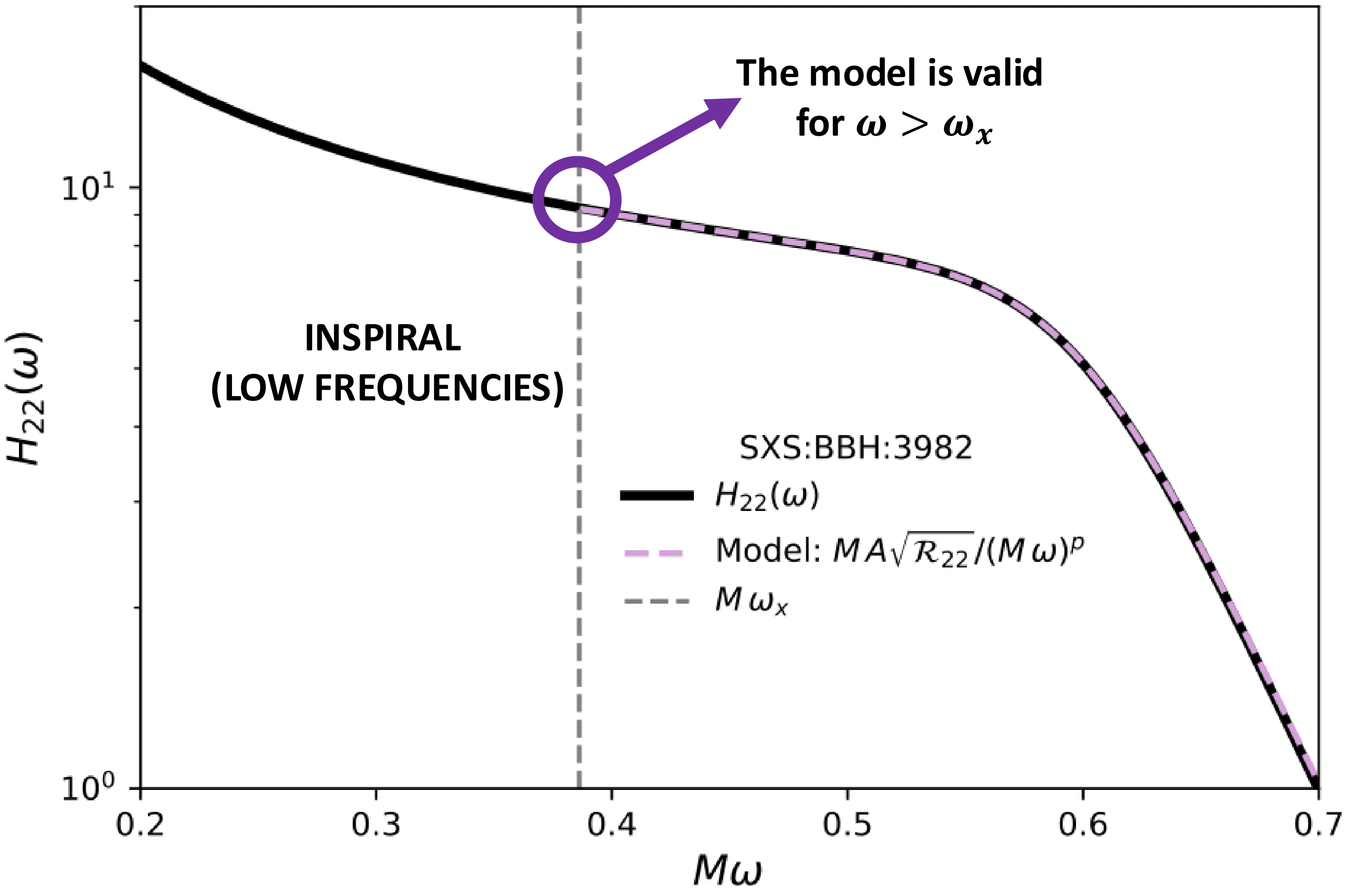
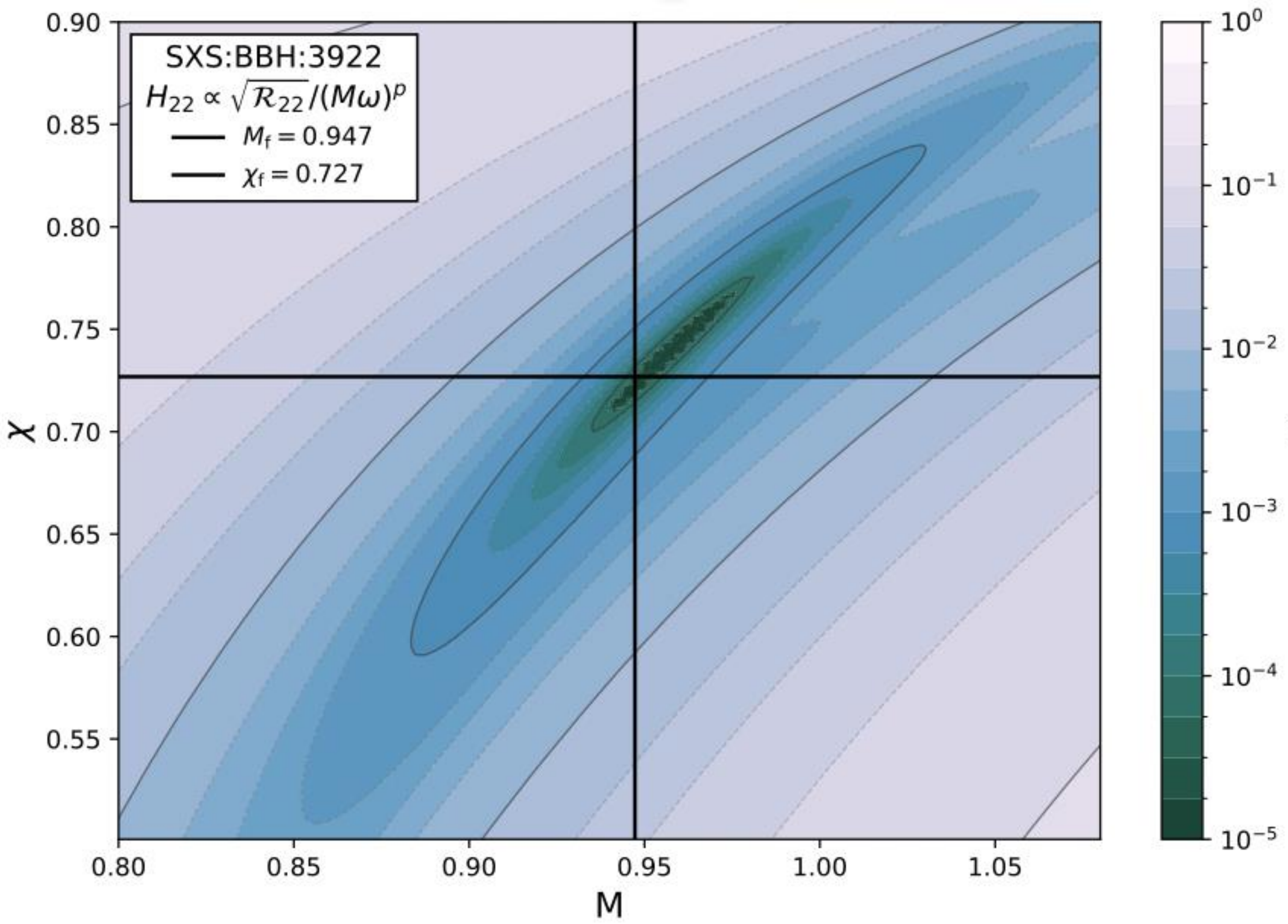


I. DETERMINATION OF THE REMNANT PARAMETERS



$$\mathcal{M} = 1 - \frac{\langle H^{\rm data} | H^{\rm model} \rangle}{\sqrt{\langle H^{\rm data} | H^{\rm data} \rangle \langle H^{\rm model} | H^{\rm model} \rangle}}$$

The true values of the remnant parameters lie in the minimal mismatch region, achieving mismatches of $\mathcal{O}(10^{-5})$.

II. FITTING $A_{\ell m}$ and $p_{\ell m}$ WITH BINARY PARAMETERS

Information about the **initial binary** is contained in $A_{\ell m}$ and $p_{\ell m}$.

$$A_{\ell m} = A_{\ell m}(\mathrm{m}_1, \mathrm{m}_2, \vec{\chi}_1, \vec{\chi}_2) \qquad p_{\ell m} = p_{\ell m}(\mathrm{m}_1, \mathrm{m}_2, \vec{\chi}_1, \vec{\chi}_2)$$

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We fitted a subset of the SXS catalog simulations as polynomials of the binary parameters, using the following parametrization

$$\chi_+ = \frac{q\chi_1 + \chi_2}{1+q}, \qquad \chi_- = \frac{q\chi_1 - \chi_2}{1+q}, \qquad \delta = \sqrt{1-4\eta} \qquad \text{with} \quad \eta = \frac{q}{(1+q)^2}$$

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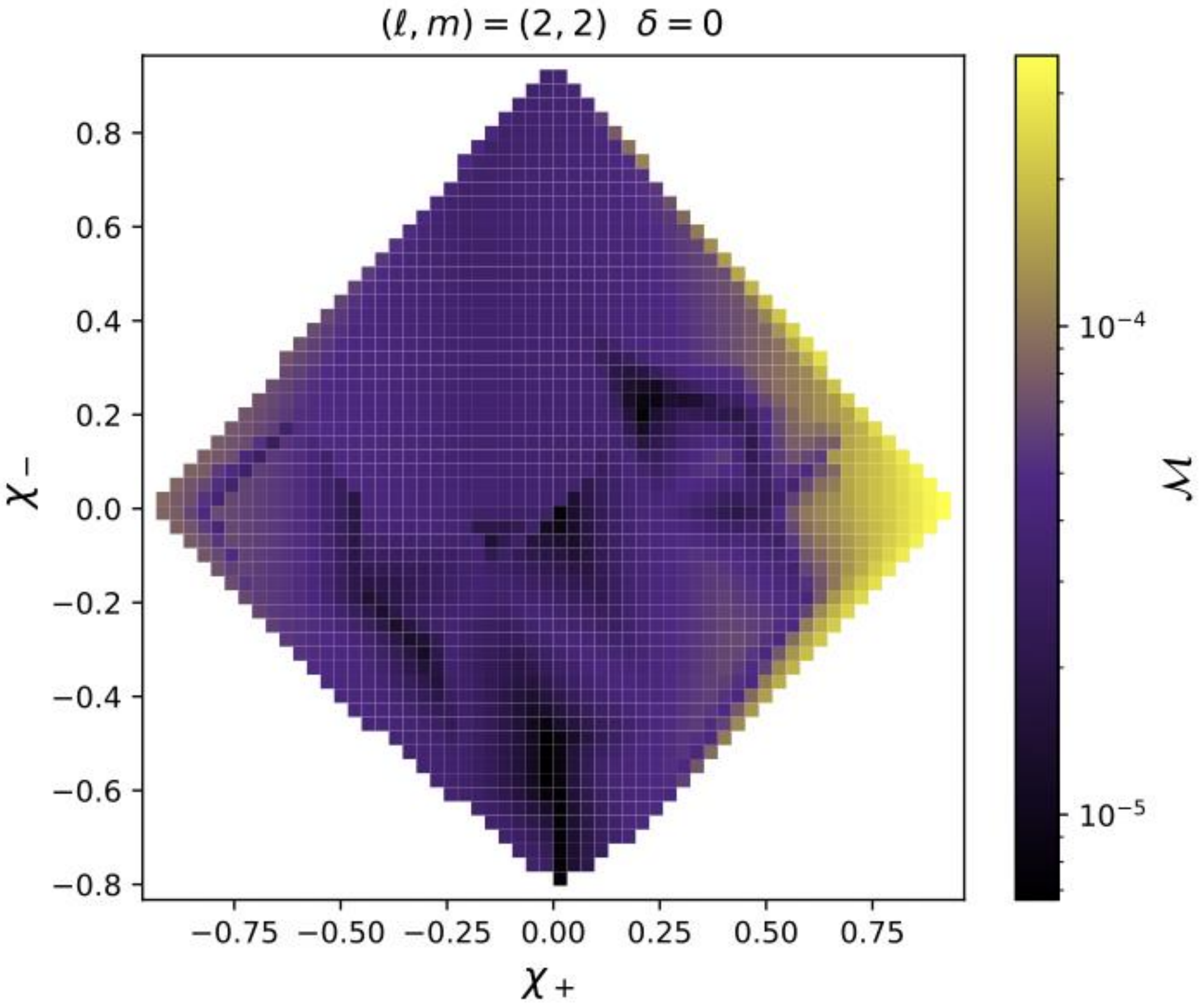
- **Spin aligned**
- **Non eccentric**
- **Comparable mass ($\eta \in [0.15, 0.25]$)**

II. FITTING $A_{\ell m}$ and $p_{\ell m}$ WITH BINARY PARAMETERS

We performed the fit on a subset of ~250 simulations and then tested its performance on a completely independent subset of ~100 simulations.



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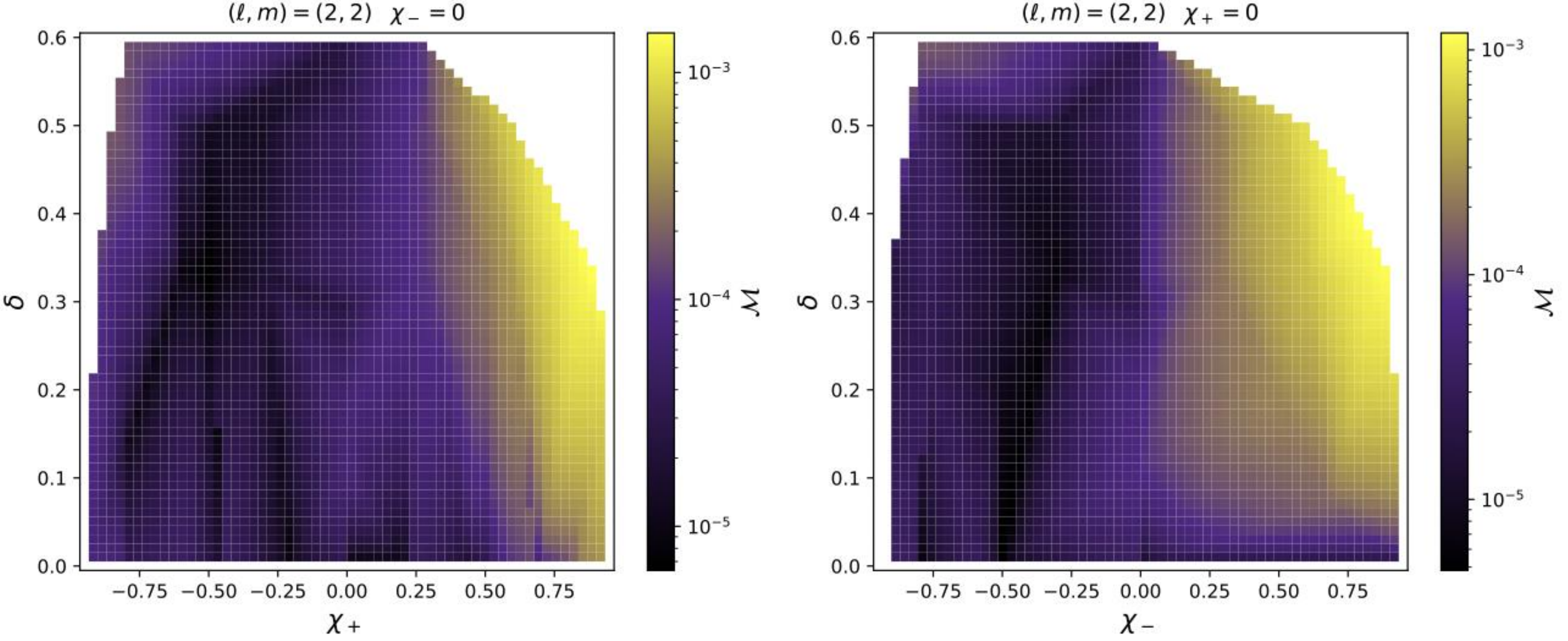

TAKE-ON MESSAGES

1. Greybody factors can represent a robust gravitational wave observable.
2. Comparable mass mergers can be fitted with GFs.

FUTURE PROSPECTS

1. Understanding the role of spin induced spherical-spheroidal mixing (in preparation)
2. Including precession.
3. Applying the model to real data (in preparation).

Thank you for listening ☺

You can find everything about comparable mass mergers here:
Modeling the frequency-domain ringdown amplitude of comparable-mass mergers with greybody factors, **Romeo Felice Rosato, Sophia Yi, Emanuele Berti and Paolo Pani.**
https://arxiv.org/pdf/2512.15877