Rotating Thin Shells in Einstein-Gauss-Bonnet Gravity

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XVIII Black Holes Workshop, 18 & 19 December 2025

Supported by **2024.04456.CERN**

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    - Coupled to non-linear electrodynamics
    - etc.

2404.04691: Rotating BH in 5D Einstein-Gauss-Bonnet gravity

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where

$$A(r) = r^2 - M - \frac{b}{r} + \frac{j^2}{4r^2} \,, \quad N^\psi = -\frac{j}{2r^2} \,.$$

But there's a catch

Reviewing Lovelock Gravity

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5D Lovelock = Einstein-Gauss-Bonnet

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Except for a specific point

Chern-Simons Point

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$$ds^2 = -V^2(r)dt^2 + V^{-2}(r)dr^2 + r^2 d\Omega^2 \,,$$

with

$$V^2(r) = 1 + \frac{r^2}{4\alpha} - \frac{r^2}{4\alpha}\sqrt{1 + \frac{16\alpha M}{r^4} + \frac{4\Lambda\alpha}{3}} \,.$$

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Chern-Simons:

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2404.04691: Rotating BH in 5D Einstein-Gauss-Bonnet gravity

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2404.04691: Rotating BH in 5D Einstein-Gauss-Bonnet gravity **in the Chern-Simons point**

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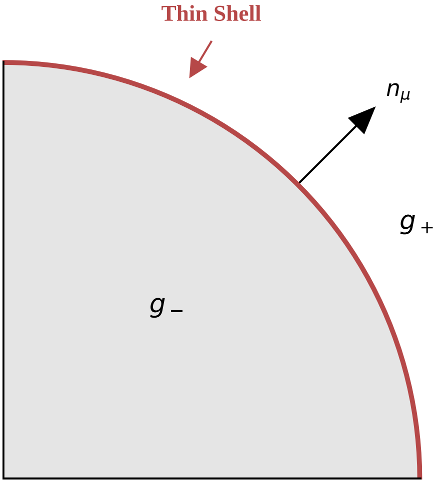
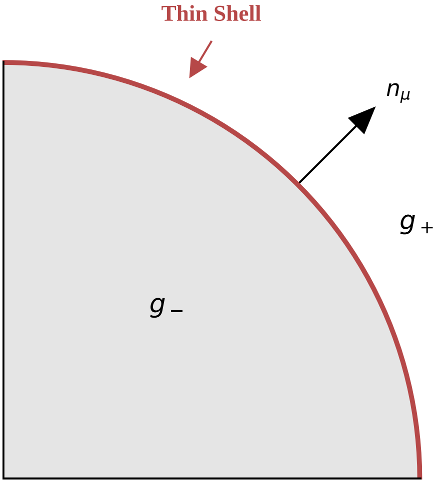
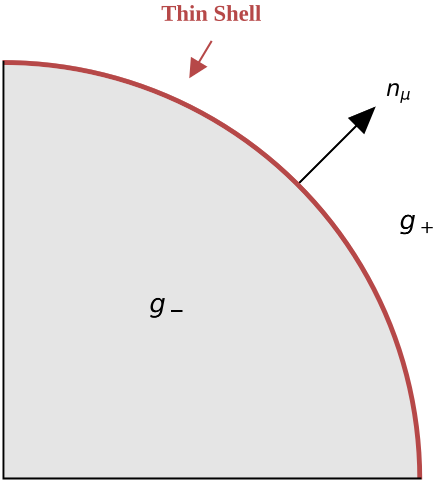

Thin Shell
n_μ
g_+
g_-



Connecting Spacetimes

- Match induced metric on thin shell

$$h^-_{ij} = h^+_{ij}$$



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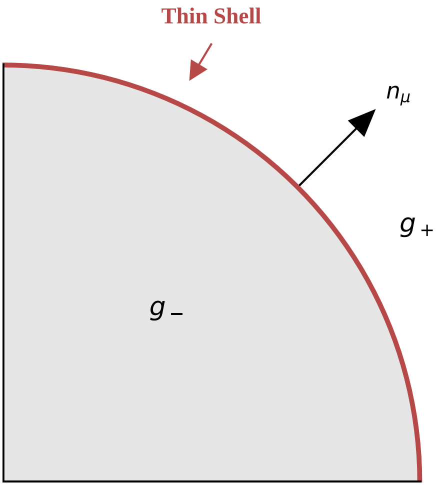
$$[K_{ij} - K h_{ij}] = -8\pi S_{ij}$$



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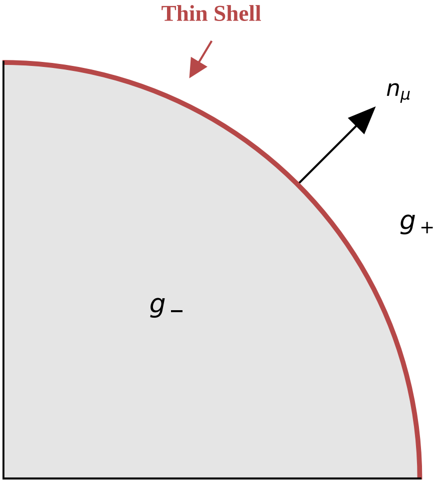

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- Lovelock: +1 term (Y_{ij})

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Rotating Openheimer-Snyder?

- No analogy for **Kerr** yet
- In 5D, **1405.1433**: Collapsing thin shells with rotation (J. V. Rocha et. al)
    - Inner metric (g_-): Vacuum
    - Outer metric (g_+): Myers-Perry
    - Thin Shell (S_{ij}): Dust

What if inner and outer spacetimes are:

$$ds^2 = l^2 \cosh^2(\rho)\left[-A(r)dt^2 + \frac{dr^2}{A(r)} + r^2(d\psi + N_\psi dt)^2\right] + l^2 d\rho^2 + l^2 \cosh^2(\rho - \rho_0)dz^2\,,$$

with different M's, j's, and b's?

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Shell's Equation of Motion

$$\ddot{R} = -\frac{8\pi R}{m}(p_R - p)\sqrt{(A_- + \dot{R}^2)(A_+ + \dot{R}^2)}$$

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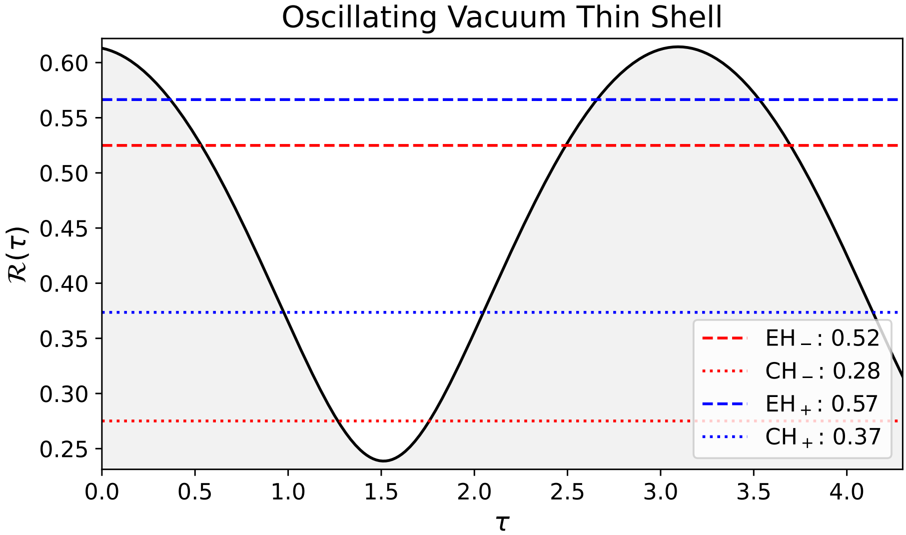
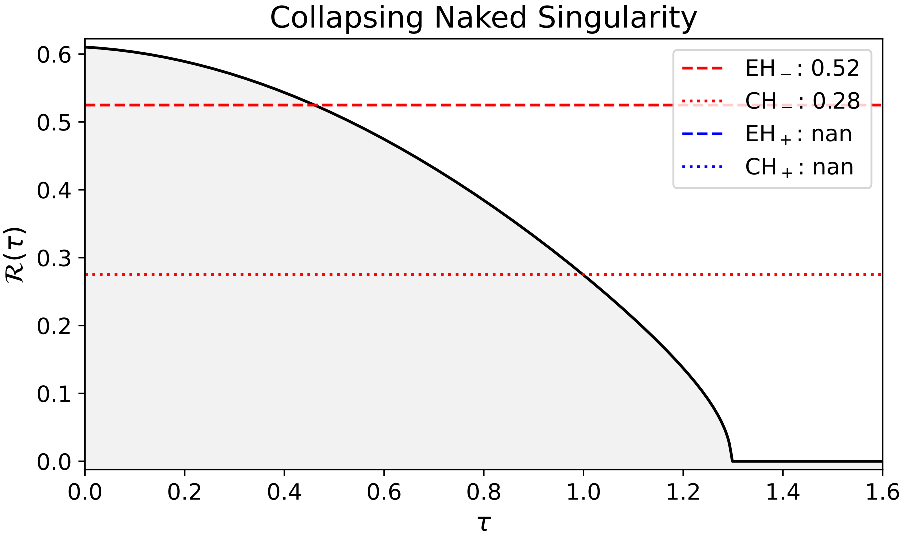
or

$$m = R\left(\sqrt{A_-(R) + \dot{R}^2} - \sqrt{A_+(R) + \dot{R}^2}\right) ,$$

with m constant



Parameters: $M_+ = 0.47,\ j_+ = 0.46,\ M_- = 0.4,\ j_- = 0.38,\ b_\pm = 0$



Parameters: $M_+ = 0.47,\ j_+ = 1.0,\ M_- = 0.4,\ j_- = 0.38,\ b_\pm = 0$

IV. Conclusions

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- Soon on **arXiv**

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A rotating solution in Einstein-Gauss-Bonnet gravity with a negative cosmological constant was recently found in the Chern-Simons point. Contrary to the attempts of applying a rotating thin shell in Kerr, here we show a clear and straightforward way to describe the way a rotating thin shell behaves. The inner and outer spacetimes are replicas of the same rotating metric, with different values of mass and angular momenta. We explore the parameter-space possibilities and discuss the mathematical correctness of the standard junction conditions used up until now in the Einstein-Gauss-Bonnet theory, which is still a matter of debate.