GRAN SASSO SCIENCE INSTITUTE
SCHOOL OF ADVANCED STUDIES
Scuola Universitaria Superiore

JOHNS HOPKINS UNIVERSITY

INFN

XVIII BLACK HOLES WORKSHOP

QUASINORMAL RINGING OF KERR BLACK HOLES FROM AN EQUATORIAL PLUNGE

BASED ON ARXIV:2512.07959

LAURA PEZZELLA

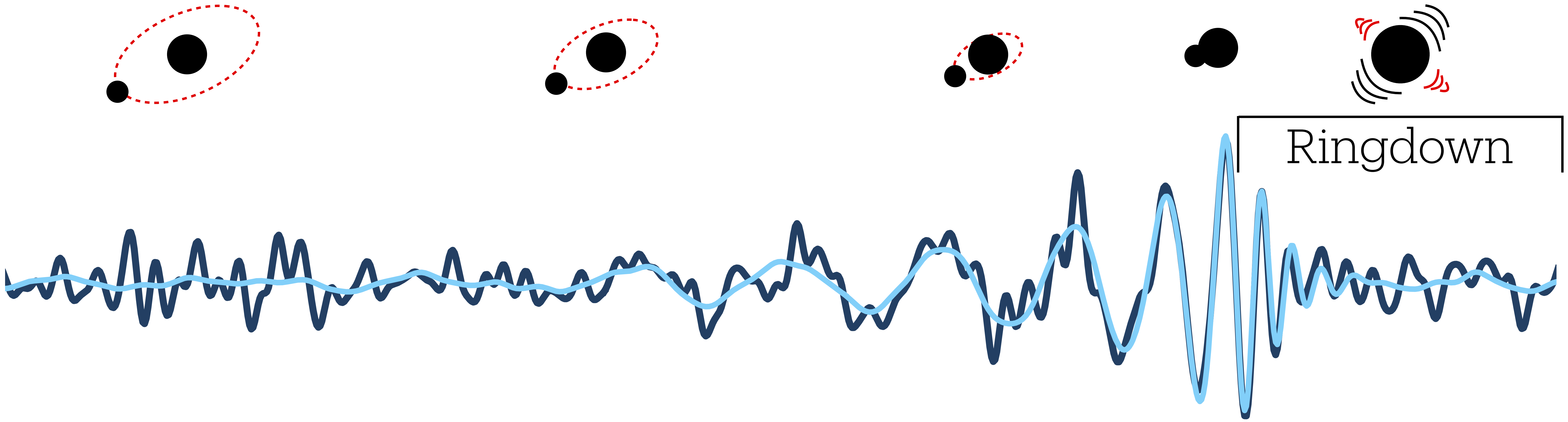
WITH M. DELLA ROCCA, E. BERTI, L.GUALTIERI, A.MASELLI

December 19th 2025

Illustration by Lia Halloran

RINGDOWN

QUASI NORMAL MODES EXCITATION



- The ringdown waveform originates from the **distorted final product** of the merger.
- The **gravitational signal** emitted during the ringdown is well modeled by a **superposition** of damped sinusoids.
- The characteristic complex frequencies are called quasi-normal modes (**QNMs**)

RINGDOWN

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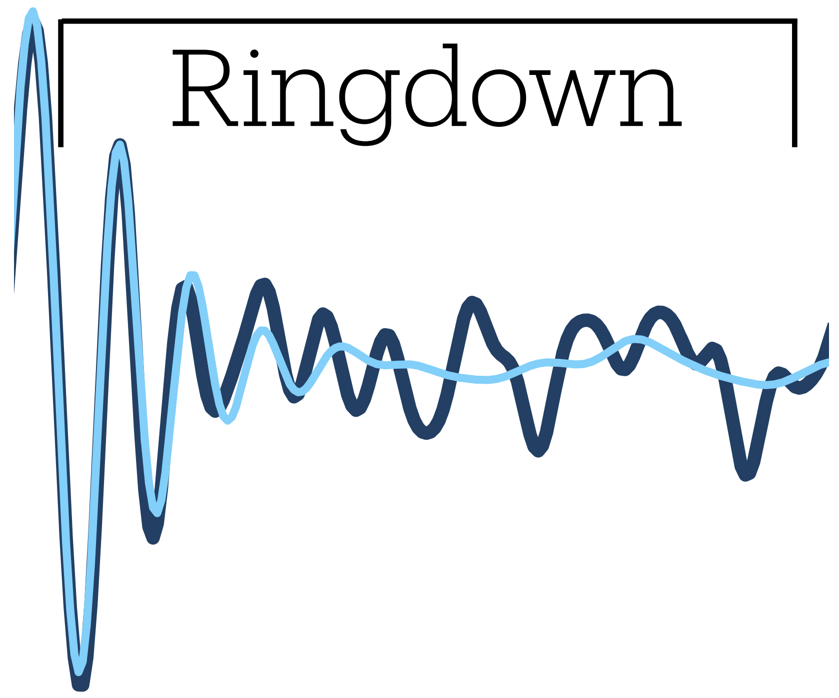


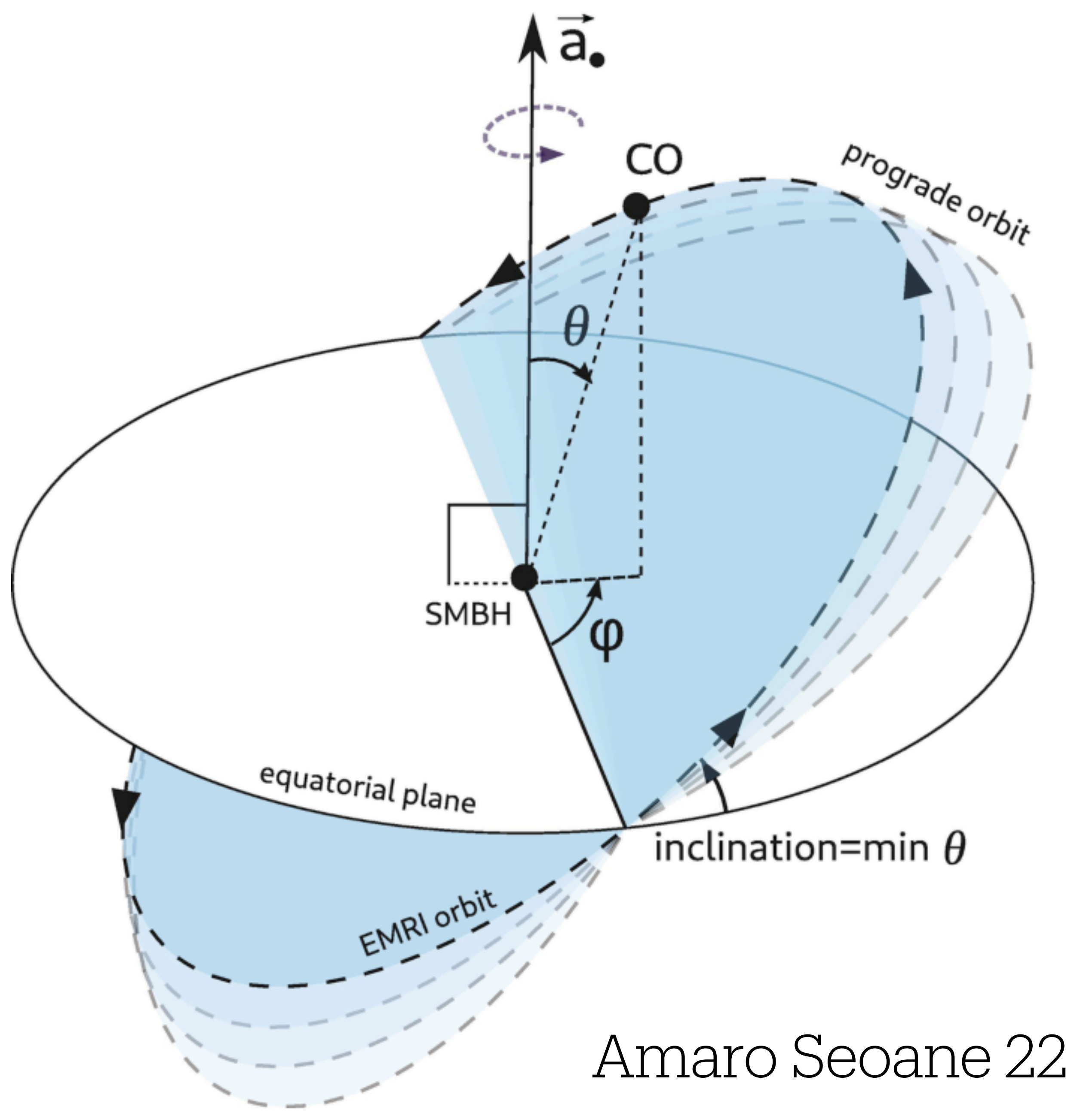
$$h(t, r, \theta, \phi) = \sum_{\ell mn} A_{\ell mn} e^{-t/\tau_{\ell mn}} \cos(\omega_{\ell mn} t + \phi_{\ell mn})$$

- QNMs **frequencies** depend entirely on the **final BH's parameters** (**mass M** and **spin J**)
- The **amplitudes** and **phases** of the signal depend on the **dynamics** of the **specific process** that formed the BH
- The **amplitudes** of the signal can be rewritten in terms of **excitation coefficients**

FRAMEWORK

WHAT DO WE DO AND HOW?

- **Kerr** BHs
- **Equatorial** orbits ($\theta = \pi/2$)
- **Circular orbit** near the ISCO
- Edge-on binary ($\iota = \pi/2$)



Amaro Seoane 22

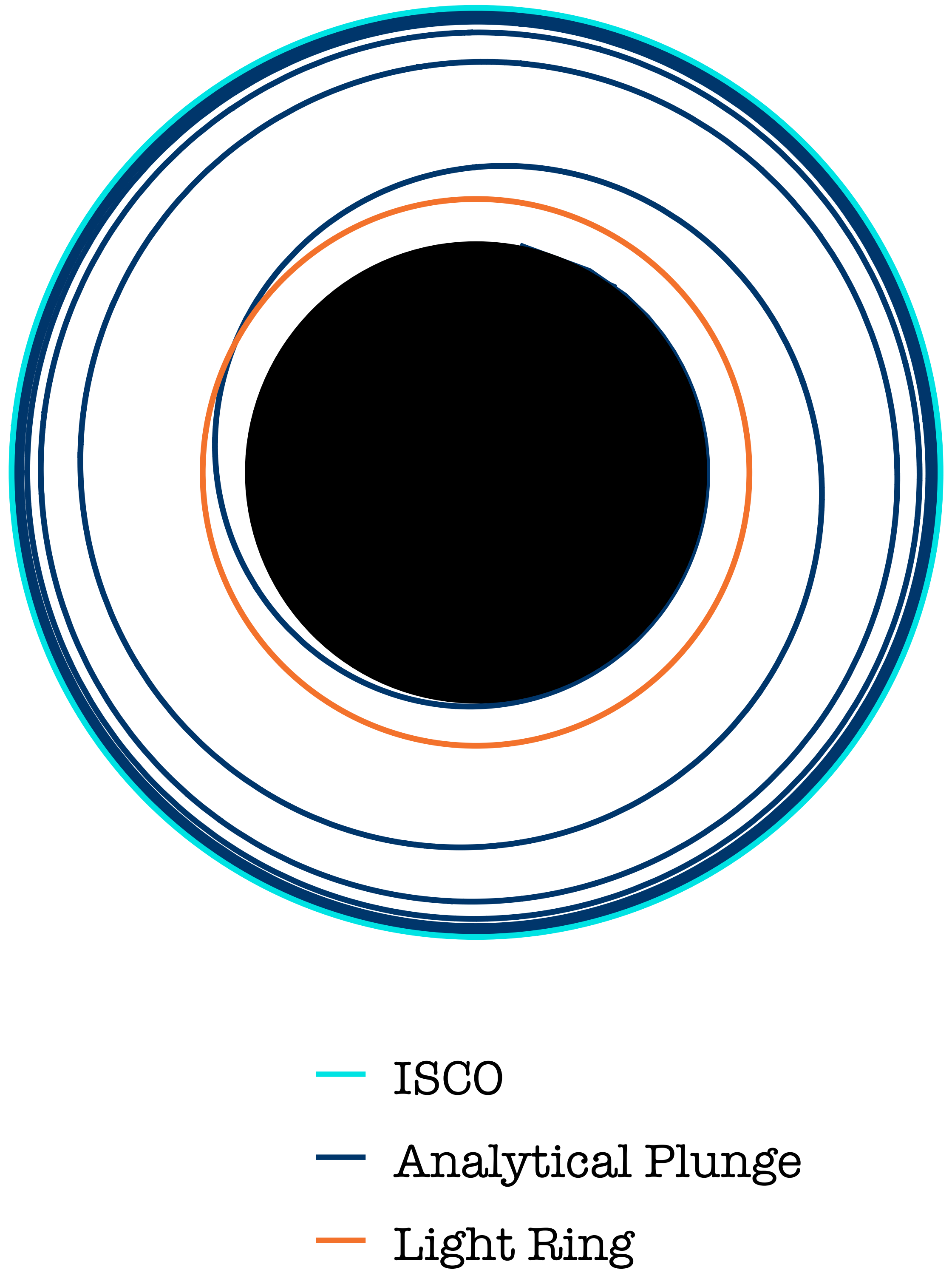
☑ **Goal:** Compute the excitation coefficients for particles plunging from the innermost stable circular orbit into a Kerr BH

☑ **How:** Compute these quantities for extreme mass-ratio binaries using **BH perturbation theory**

PLUNGING GEODESICS

DYSON-VAN DE MEERT

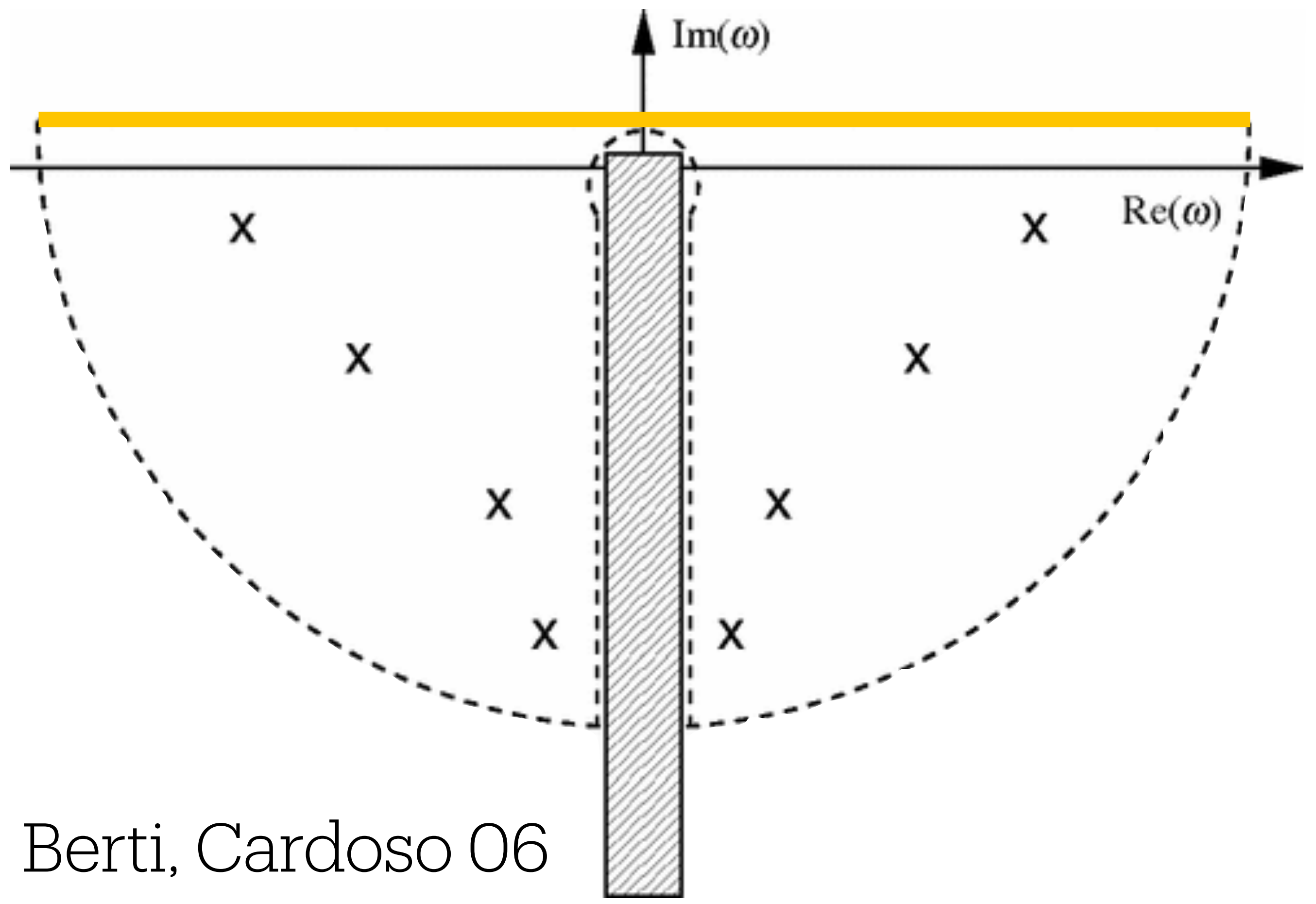
- **Analytical expression** for geodesics plunging found by Dyson and van de Meent in terms of elliptic functions.
- In this model, the plunge is **analytically extended up to the ISCO**: the transition is not modeled.
- The body is in **free fall**, with energy and angular momentum fixed at the ISCO values



RESIDUAL THEOREM

QNMS AS POLES

- QNMs are the **poles** of the Green Function
- Integration along the real axis is replaced by the sum of residuals



Berti, Cardoso 06

RESIDUAL THEOREM

QNMS AS POLES

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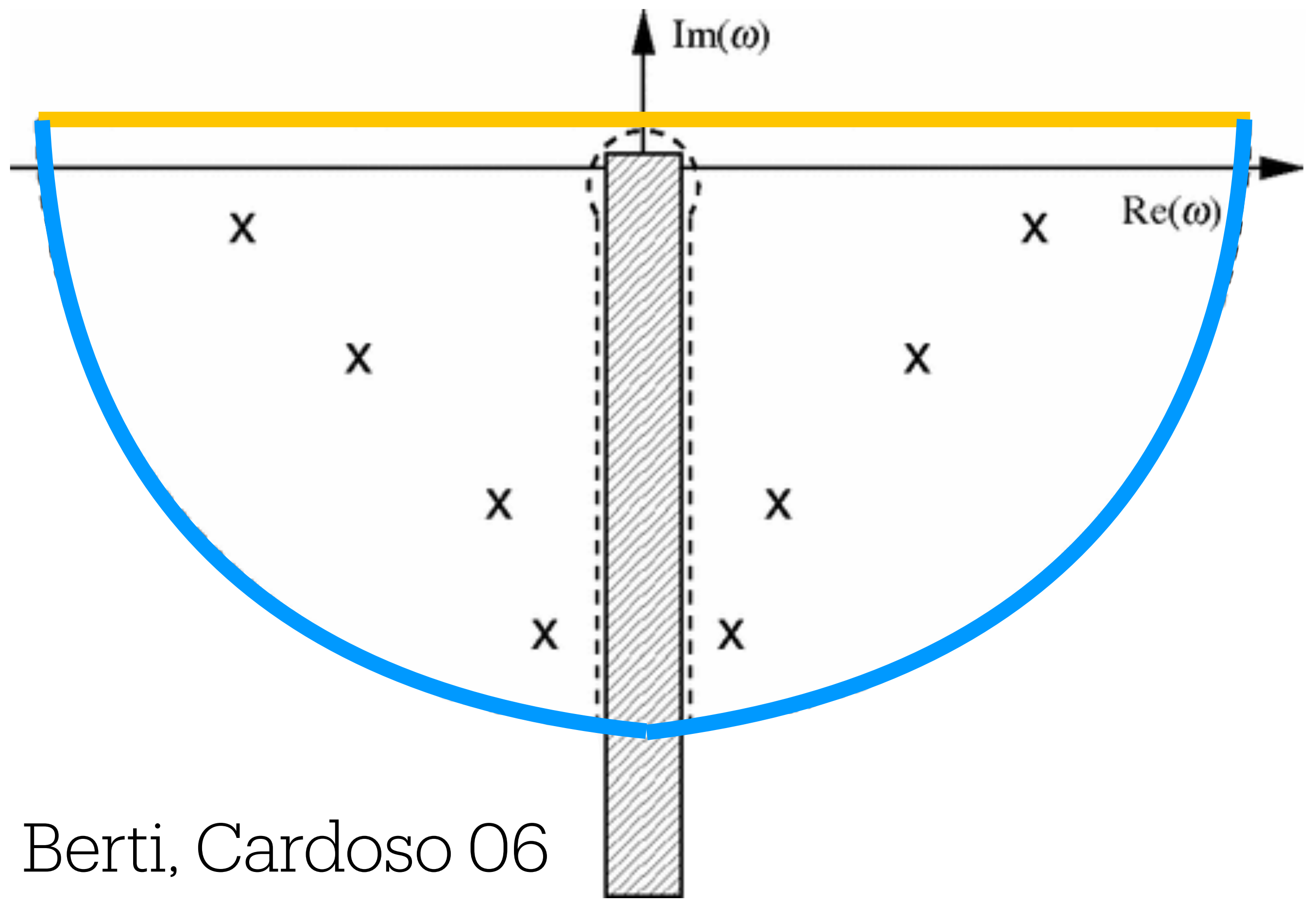
- Integration along the real axis is replaced by the sum of residuals
- Applying the residual theorem, the **time domain** solution reads



Berti, Cardoso 06

$$
\begin{aligned}
\hat{\tilde{X}}_p^{\ell m}(u, v \to \infty) &\approx -\sum_n 2\pi i \underset{\omega \to \omega_{\ell mn}}{\mathrm{Res}} \int_{\mathbb{R}} \mathrm{d}\omega \; e^{-i\omega t} \tilde{X}_p^{\ell m\omega}(r \to +\infty) \\
&= -2\pi \sum_n \tilde{C}^{\mathrm{SN}}_{\ell mn} e^{-i\omega_{\ell mn} u}
\end{aligned}
$$

EXCITATION COEFFICIENTS

QUASI NORMAL MODES EXCITATION

$$\hat{\tilde{X}}_p^{\ell m}(u, v \to \infty) = -\sum_n \tilde{C}^{\mathrm{SN}}_{\ell mn} e^{-i\omega_{\ell mn} u}$$

- The quasinormal **excitation coefficients** are a concrete measure of the QNM content of a waveform
- Excitation coefficients can be written as the product of two contributions: the quasinormal **excitation factors**, which are **initial-data independent**, and the **source-dependent integral** $I_{\ell mn}$

$$\tilde{C}^{\mathrm{SN}}_{\ell mn} = B_{\ell mn} I_{\ell mn} \qquad B_{\ell mn} = \frac{\tilde{A}_{\ell mn}}{2\omega_{\ell mn}\alpha_{\ell mn}}$$

EXCITATION COEFFICIENTS

FROM SASAKI -NAKAMURA TO GRAVITATIONAL STRAIN

$$\hat{\tilde{X}}_p^{\ell m}(u, v \to \infty) = -\sum_n \tilde{C}^{\mathrm{SN}}_{\ell mn} e^{-i\omega_{\ell mn} u}$$

- By Fourier-transforming and expanding in harmonics the gravitational strain, we have:

$$h_{\ell m} = 8\frac{1}{r}\sum_n C_{\ell mn} e^{-i\omega_{\ell mn} u} \, {}_{-2}S^{a\omega}_{\ell m}(\theta, \phi)$$

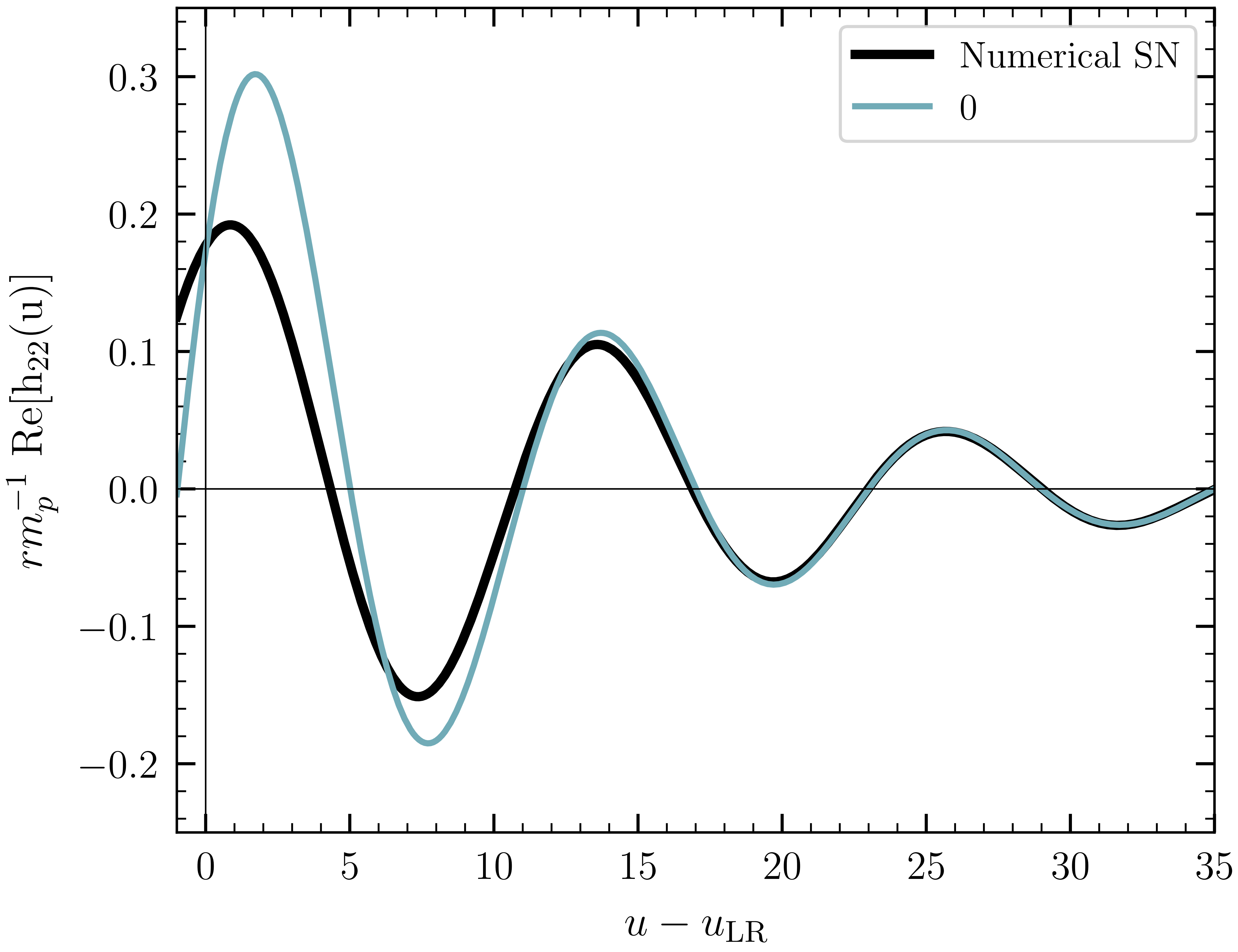
where $C_{\ell mn} = \dfrac{8}{\sqrt{2\pi c_0}} \tilde{C}^{\mathrm{SN}}_{\ell mn}$

GRAVITATIONAL WAVE EMISSION

COMPARISON WITH NUMERICAL SOLUTIONS

M. Della Rocca, LP + (2025)

- By adding higher overtones improves the agreement with the numerical waveforms.
- The more the modes, the **smaller the discrepancy**



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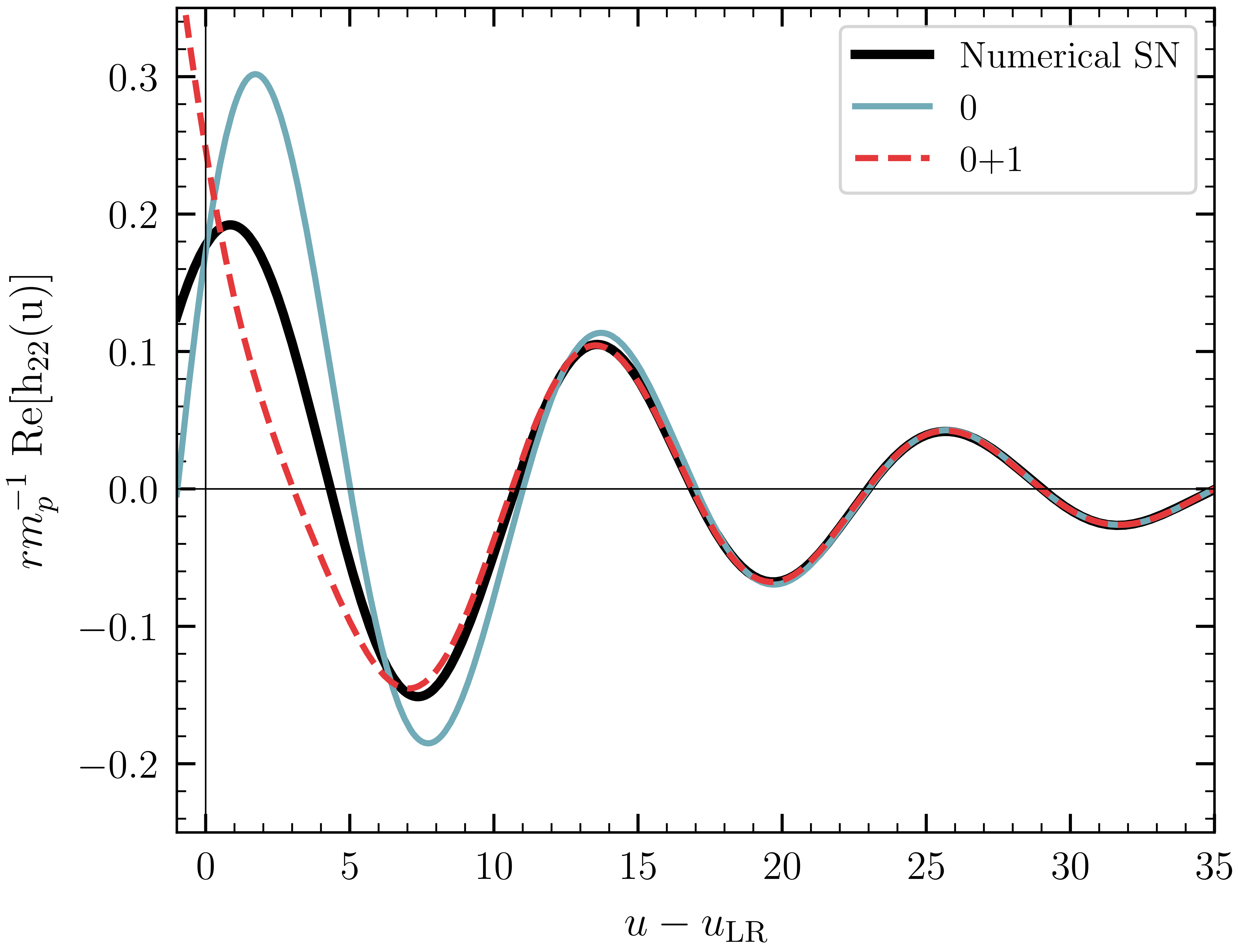

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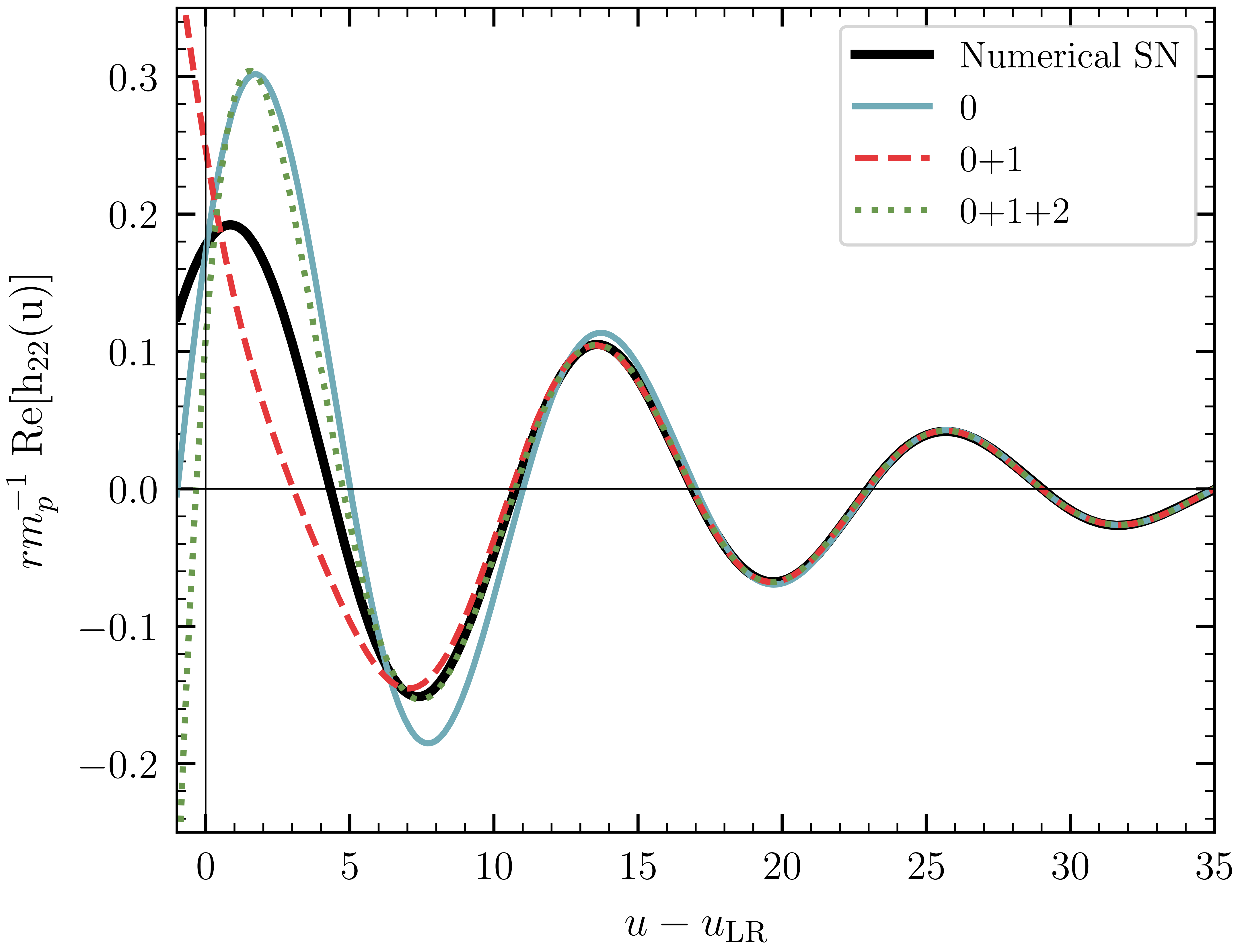
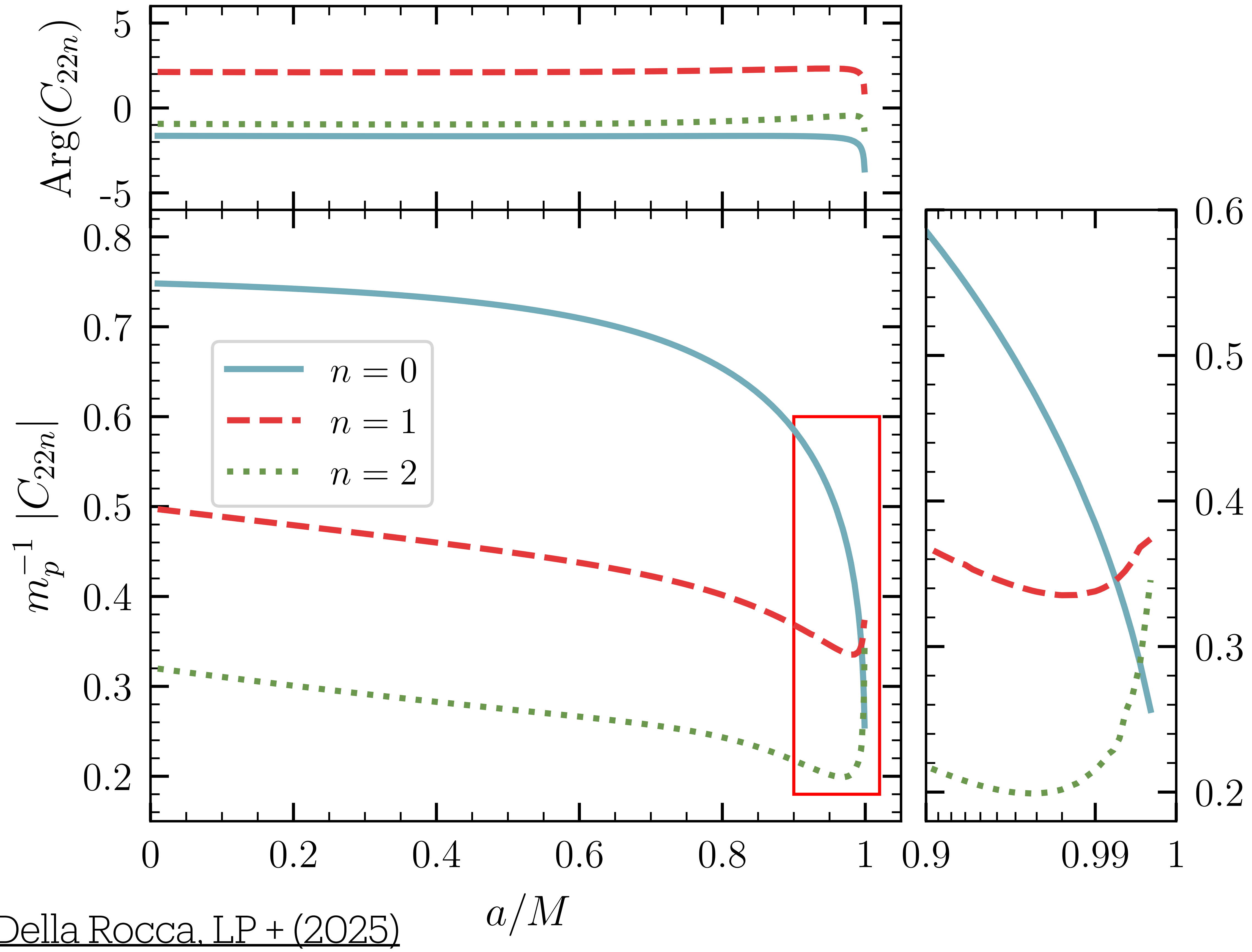


EXCITATION COEFFICIENTS

AT DIFFERENT SPINS (2,2)

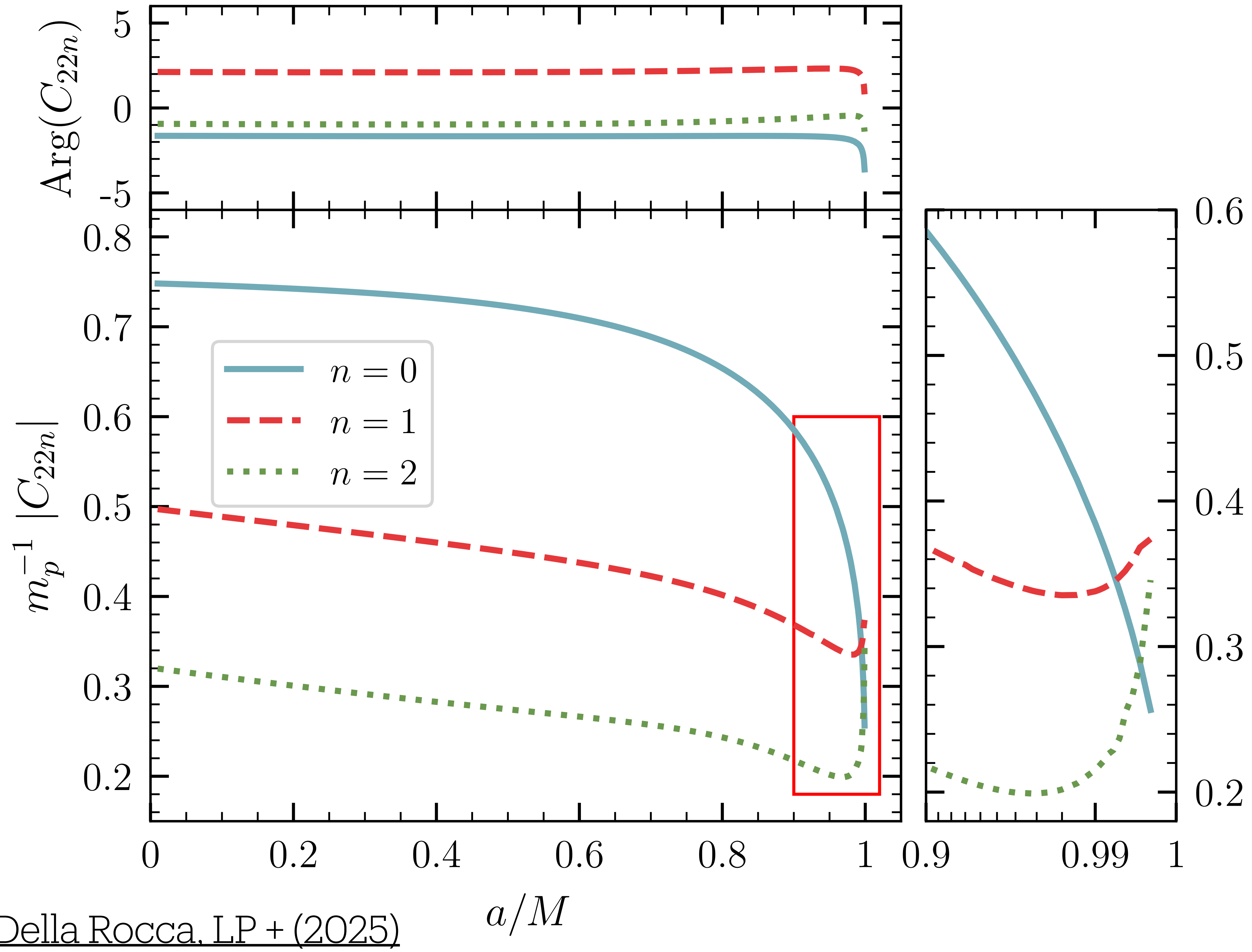


M. Della Rocca, LP + (2025)

- Computation of the excitation coefficient C_{22n} for different spins
- Different trends increasing the overtone number
- Overtones may play a dominant role for near-extremal Kerr BHs

EXCITATION COEFFICIENTS

AT DIFFERENT SPINS (2,2)

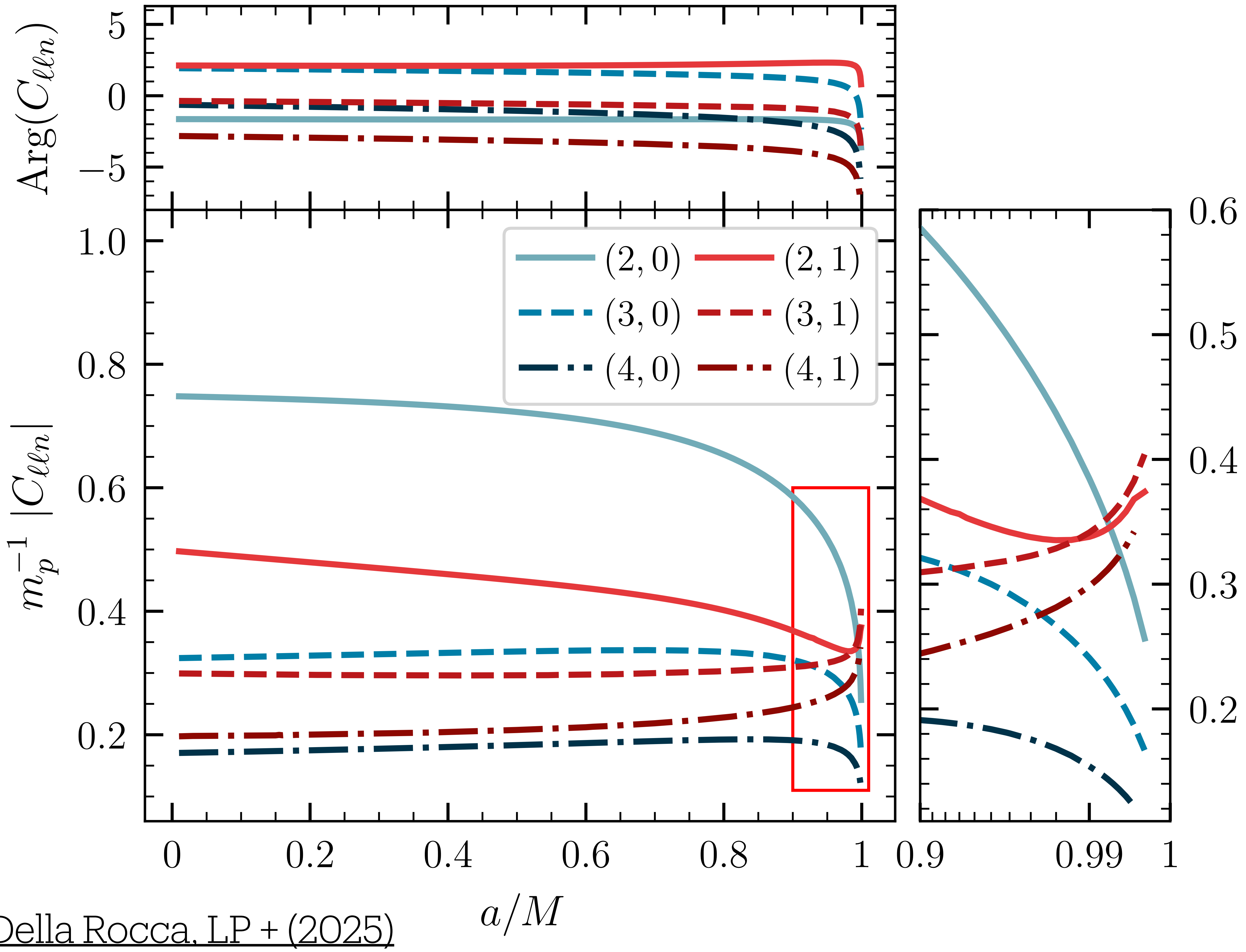


M. Della Rocca, LP + (2025)

- For nearly extremal BH $a/M \gtrsim 0.994$, the **overtone contribution** becomes more **relevant**
- $|C_{221}| > |C_{222}|$ at any spin

EXCITATION COEFFICIENTS

AT DIFFERENT SPINS (ℓ, ℓ, n)



M. Della Rocca, LP + (2025)

- Computation of the excitation coefficient $C_{\ell\ell 0}$ and $C_{\ell\ell 1}$ for different spins
- For nearly extremal BH, the **overtone contribution** becomes more **relevant** at any ℓ
- The amplitude of the coefficients are **comparable** for spins $a/M \lesssim 0.9$

CONCLUSIONS

RESULTS AND PROSPECTS

- ☑ Developed a code to **treat the plunge** of the secondary in **frequency domain**
- ☑ The **complete waveform** can be reconstructed as the **superposition of excitation coefficient**, provided that more modes are included
- ☑ New catalogue of **excitation coefficients** C_q for plunging events
- ⭐ **Overtones** and **higher modes** might play a **dominant role** for **near-extremal** Kerr BHs

What can be done:

- ☐ Characterize the plunge as a function of **eccentricity** and **inclination**
- ☐ Check the **validity** of the method for **comparable mass binaries**

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THANKS FOR THE ATTENTION!

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LAURA.PEZZELLA@GSSI.IT

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