Dynamics of Black Holes with Resonant Hair

José Ferreira, Carlos Herdeiro, Eugen Radu, Miguel Zilhão

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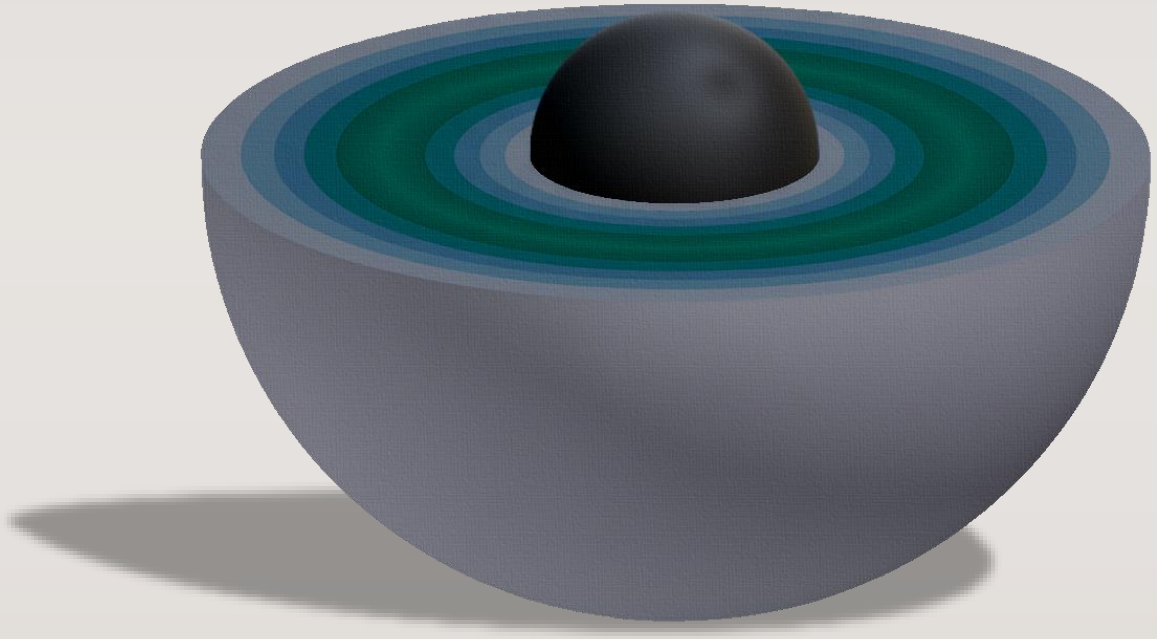
University of Aveiro, Portugal



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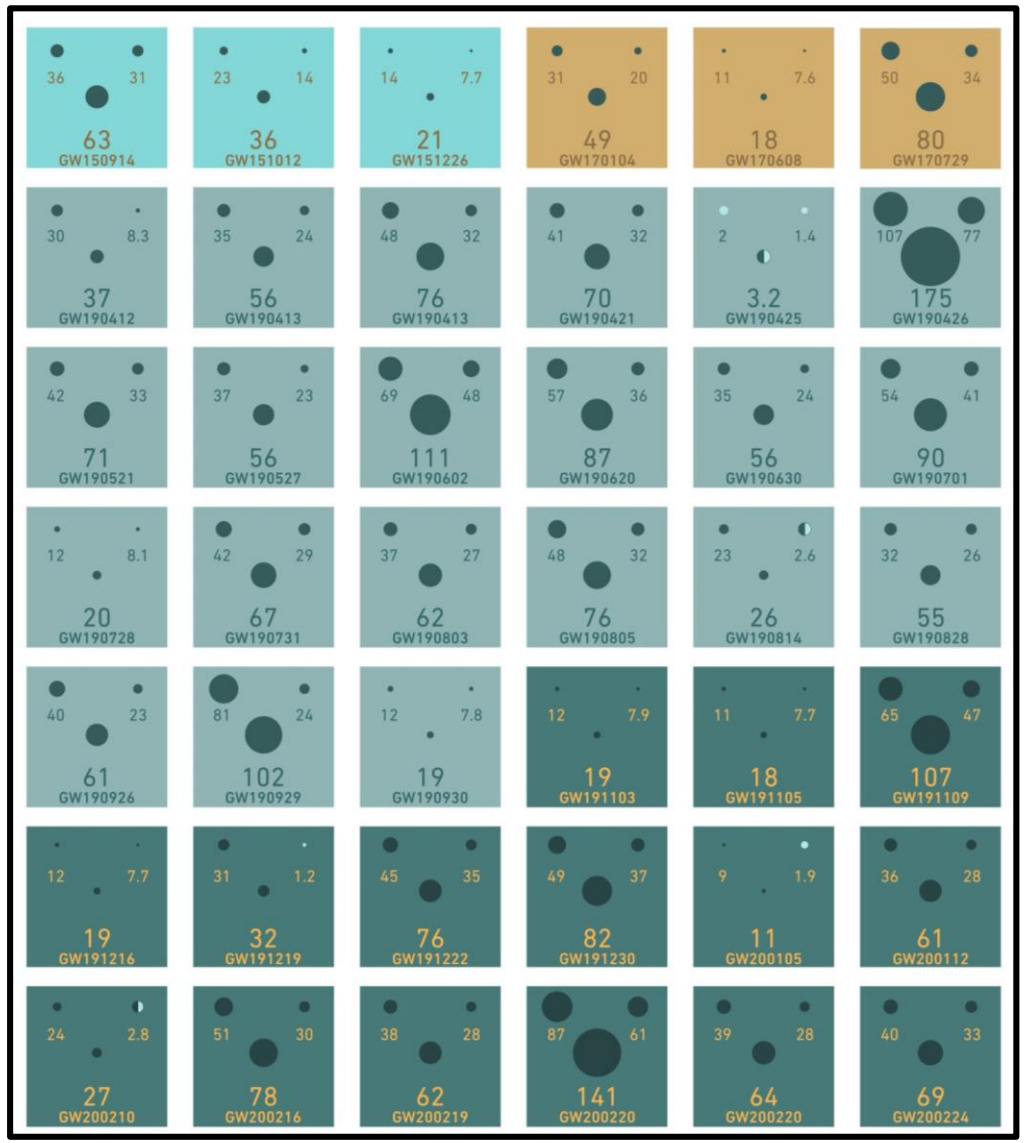

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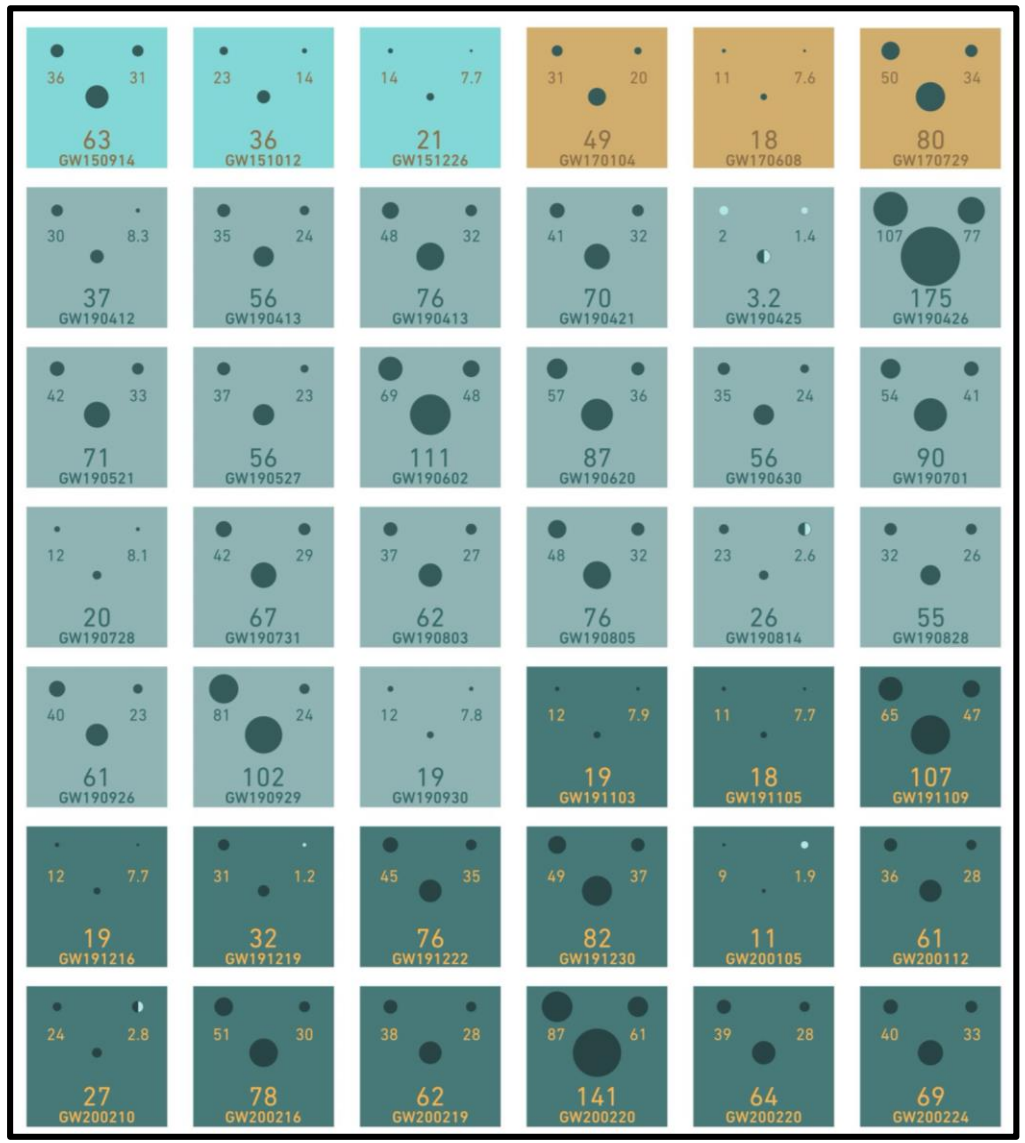

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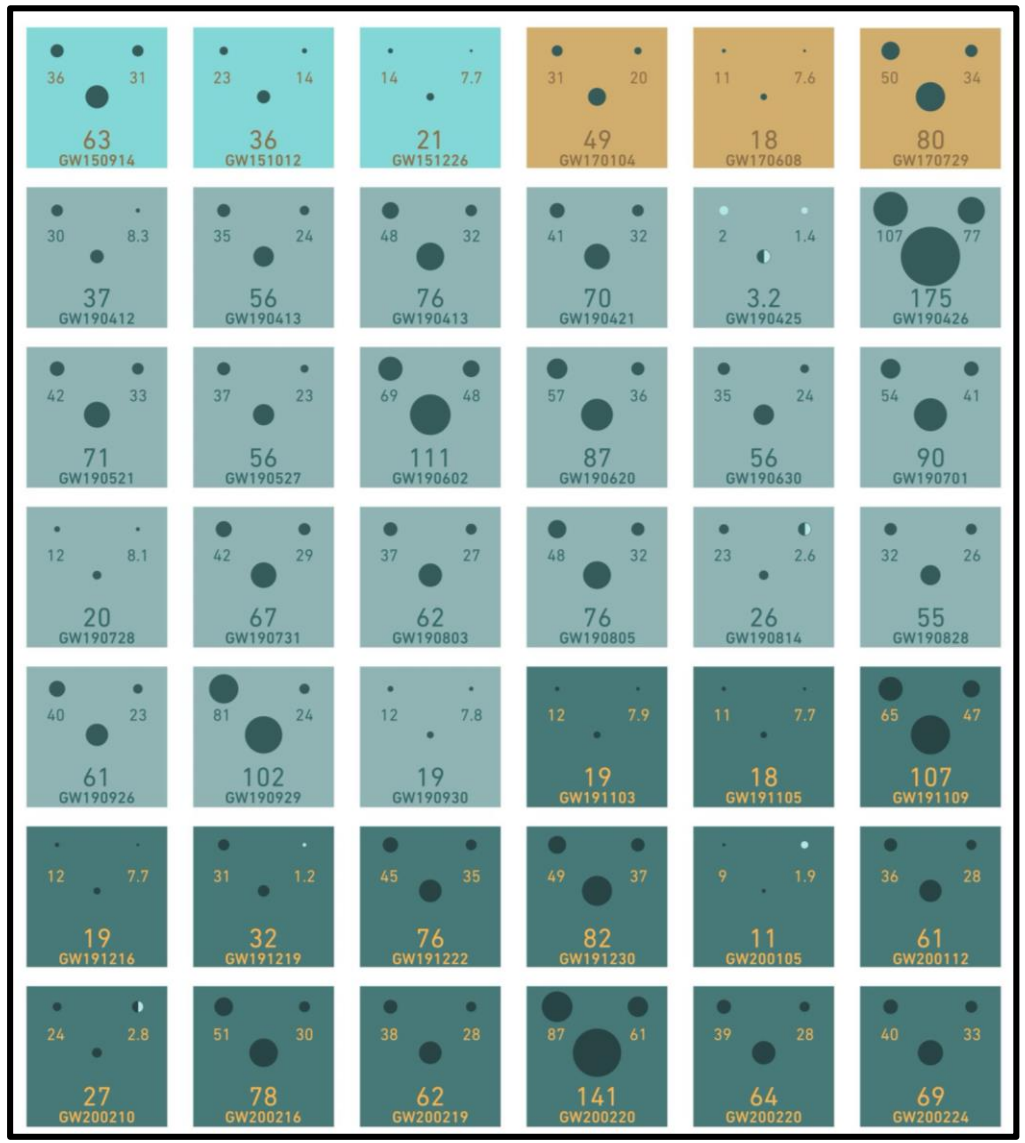

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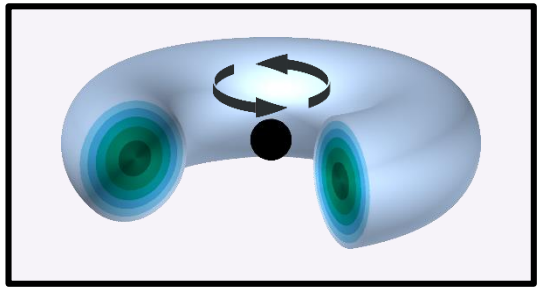

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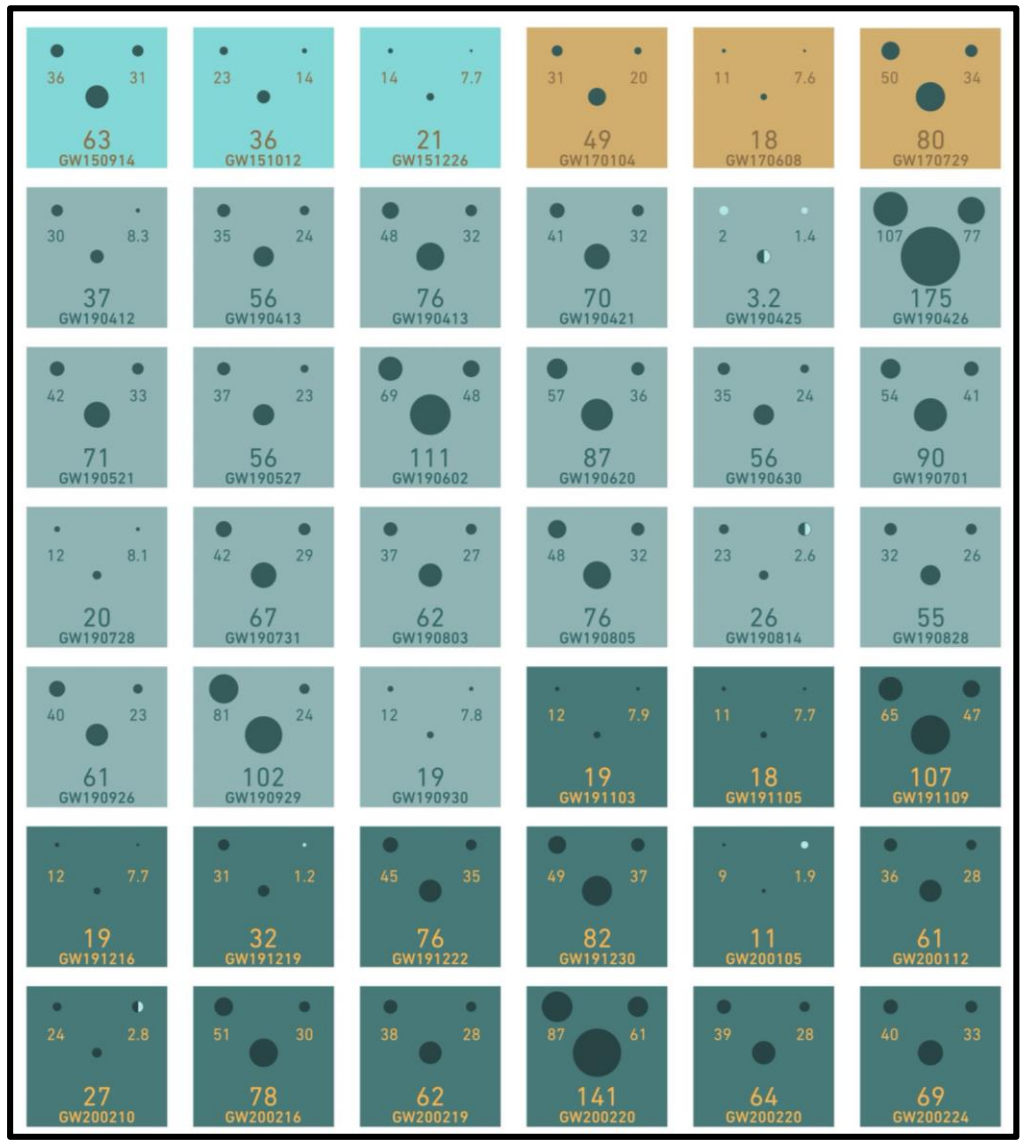

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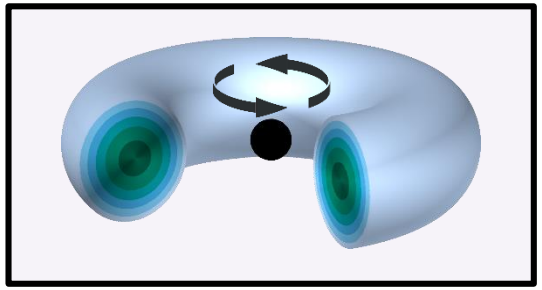

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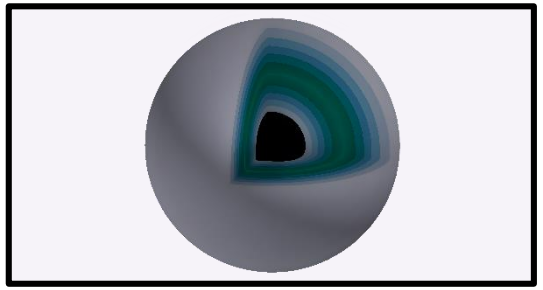

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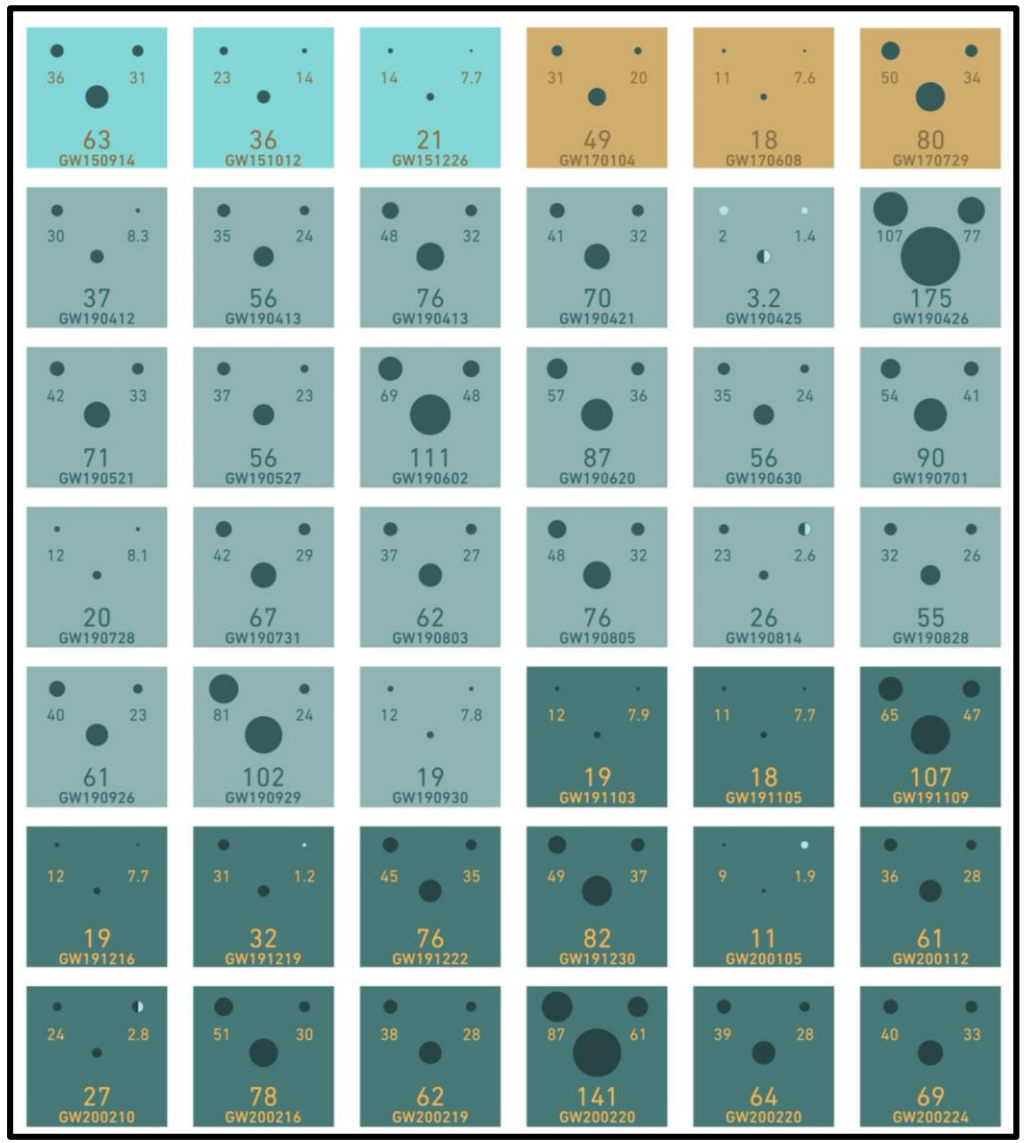

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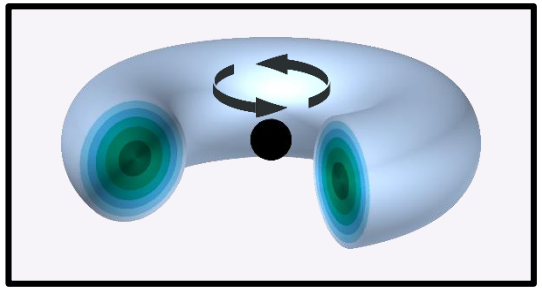

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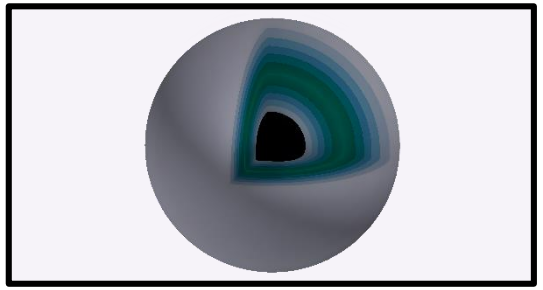

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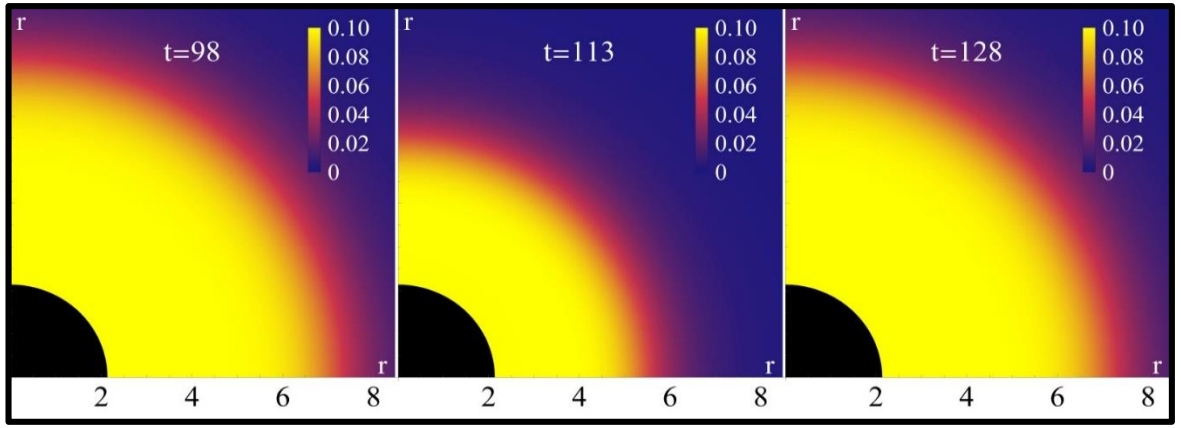

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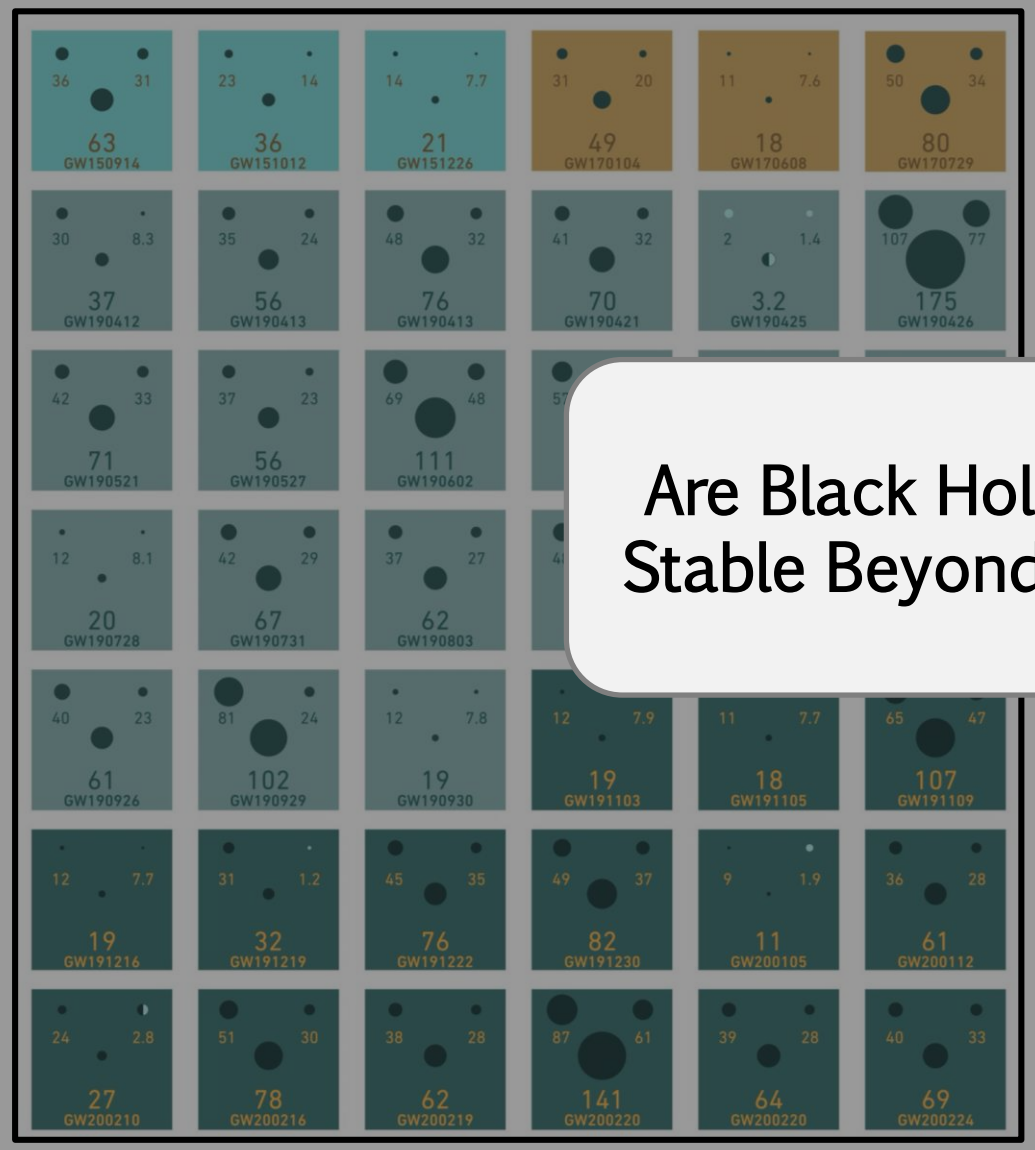

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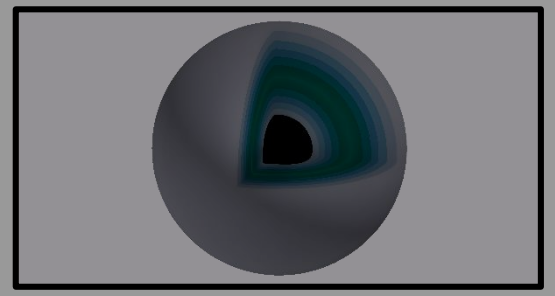
Are Black Holes with Resonant Hair Stable Beyond Spherical Symmetry?



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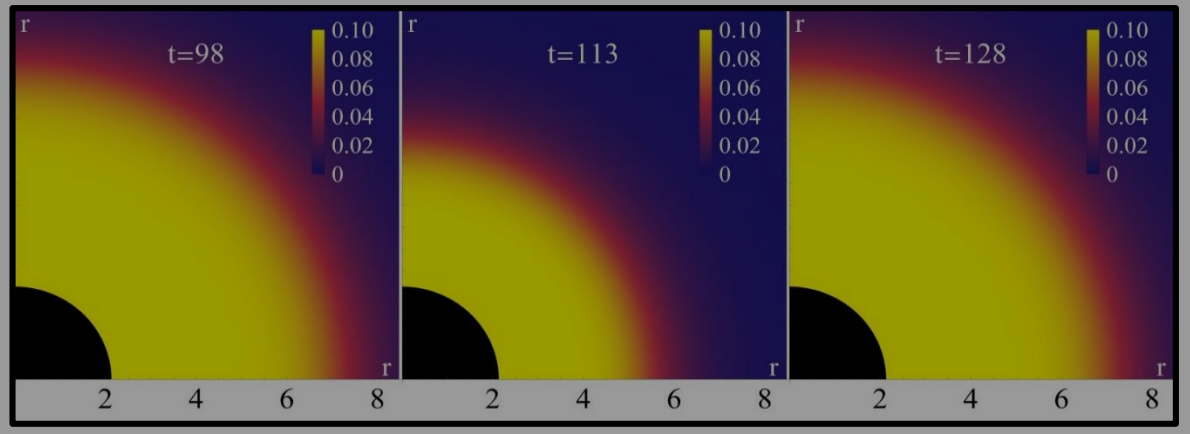

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$$S = \int \sqrt{-g} \left(\frac{R}{16\pi} - \frac{1}{4} F_{\alpha\beta} F^{\alpha\beta} - (D_\alpha \phi)^* D^\alpha \phi - V(|\phi|^2) \right) d^4x$$

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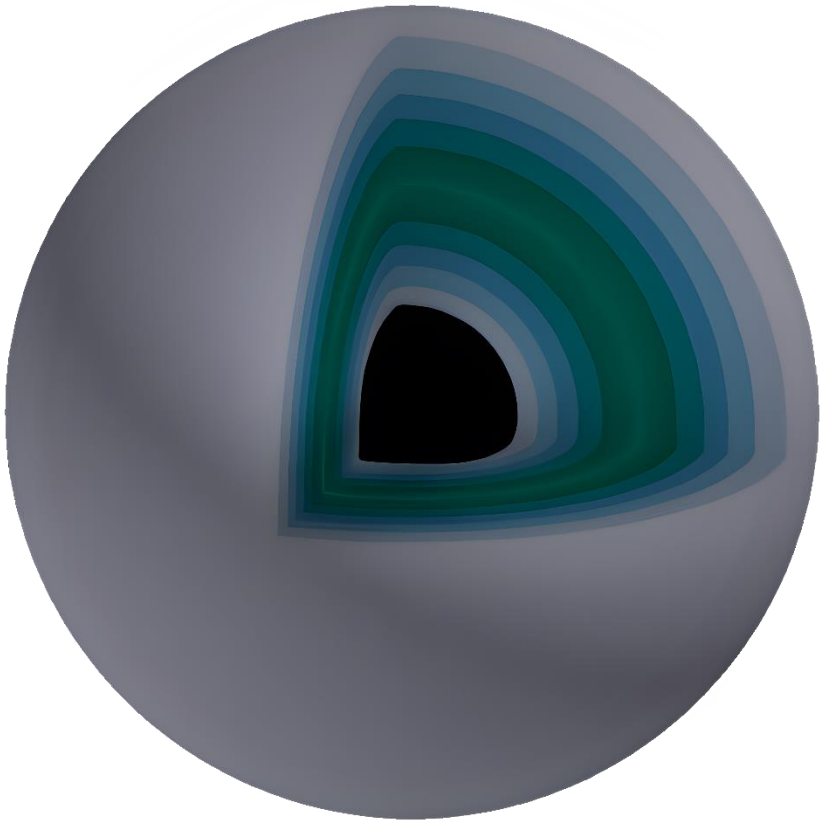

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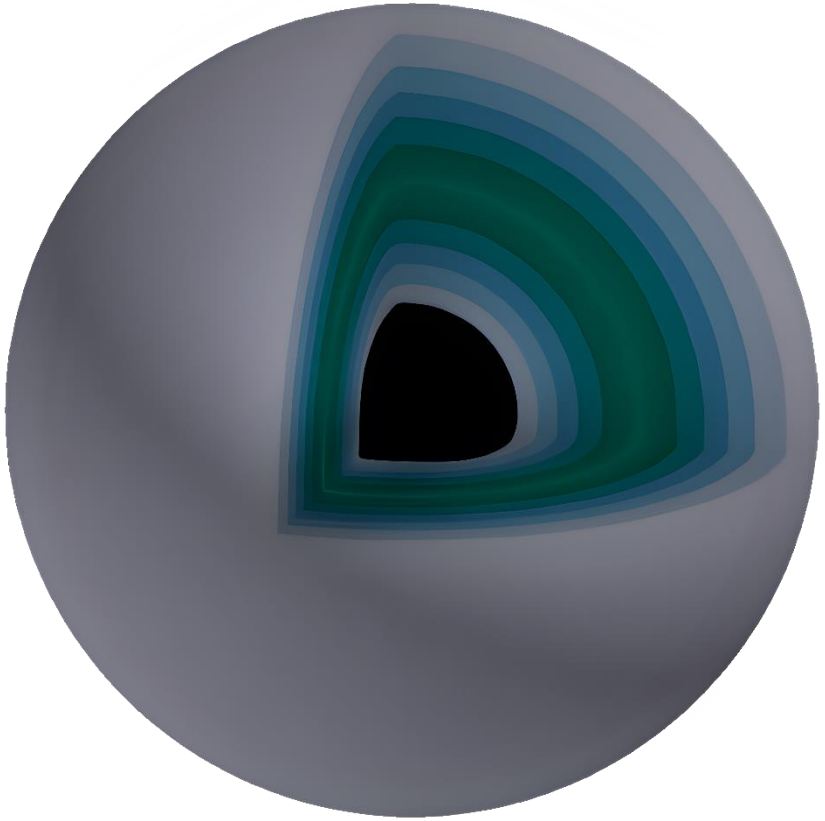

- Spherically symmetric

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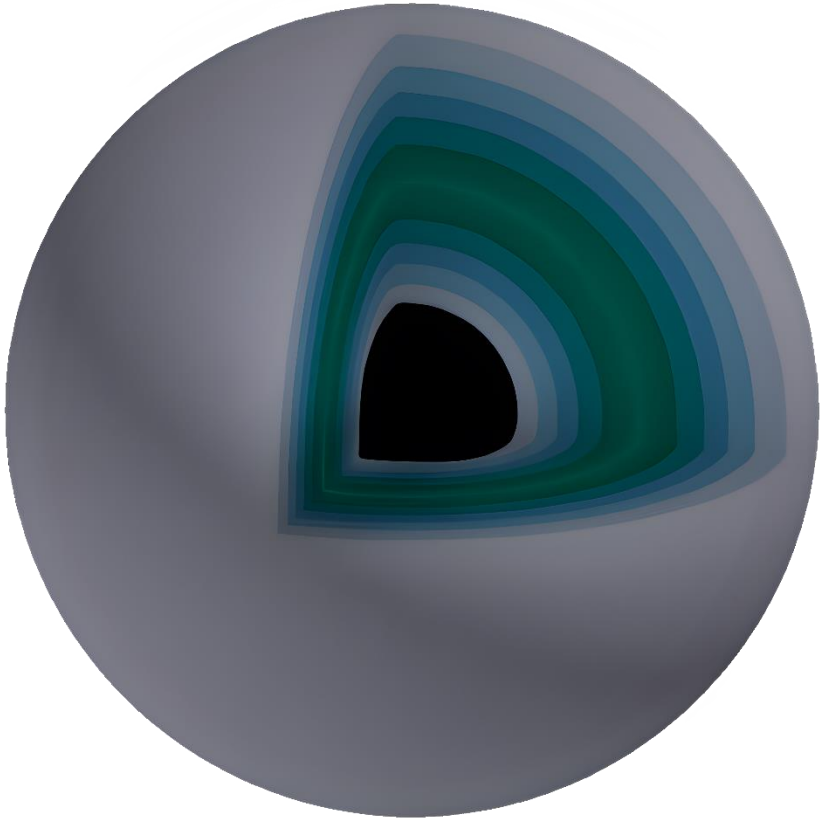

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- Static

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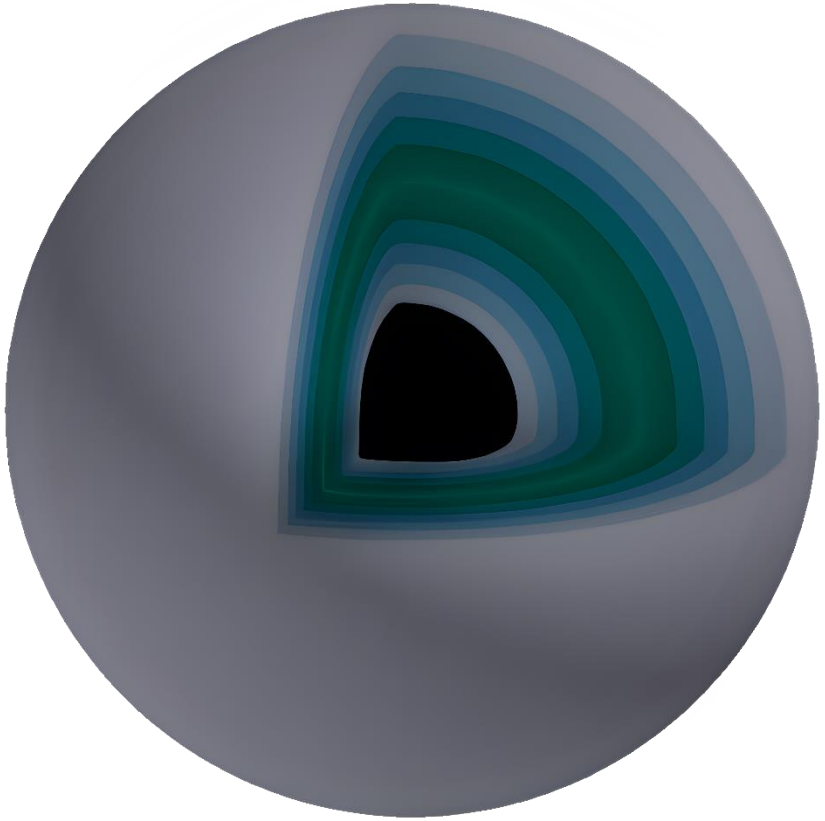

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- Static
- Two parameters:
  - $\boldsymbol{\omega}$, frequency of scalar field
  - $\boldsymbol{r_h}$, radius of the horizon

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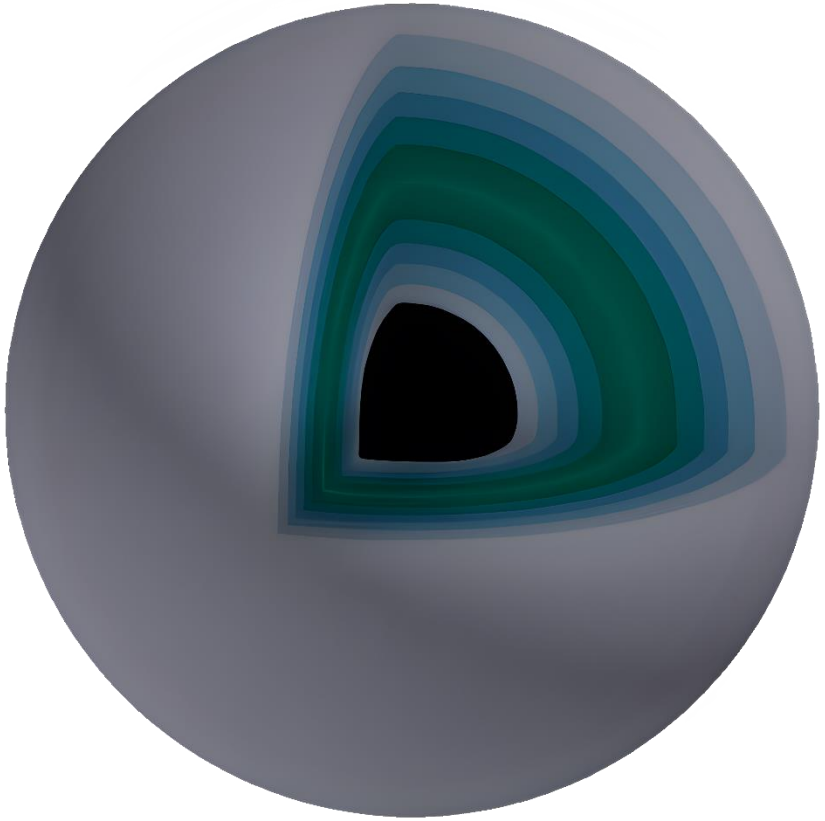

- Spherically symmetric
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- Two parameters:
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  - $\boldsymbol{r_h}$, radius of the horizon
- Branch-like structure

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Implementation

- 3+1 dimensions
- BSSN formalism
- Kreiss-Oliger Dissipation
- Symmetry on xy plane

Results

Results

Fission

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Fission

Absorption

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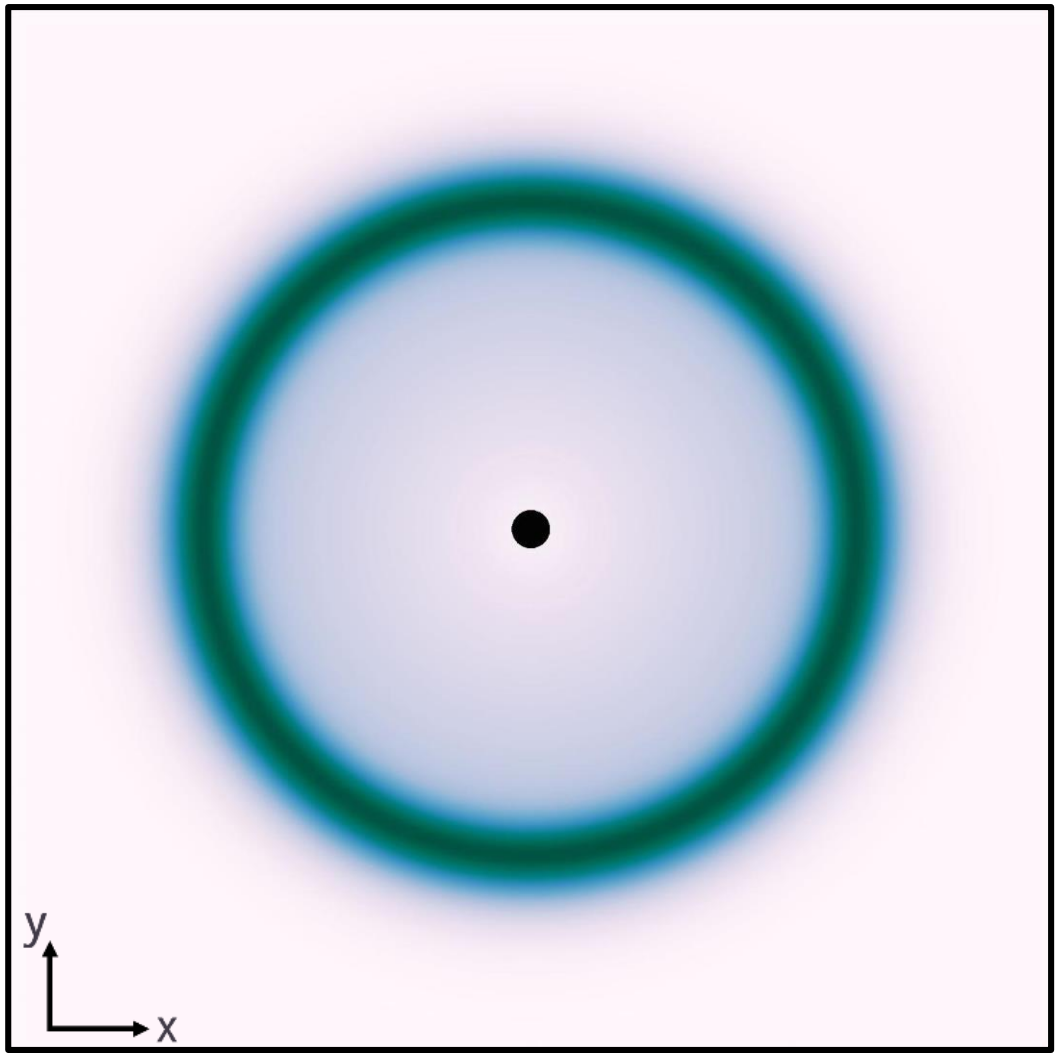
Fission



$r_h = 0.2\,, \omega = 0.7\,,$ upper branch

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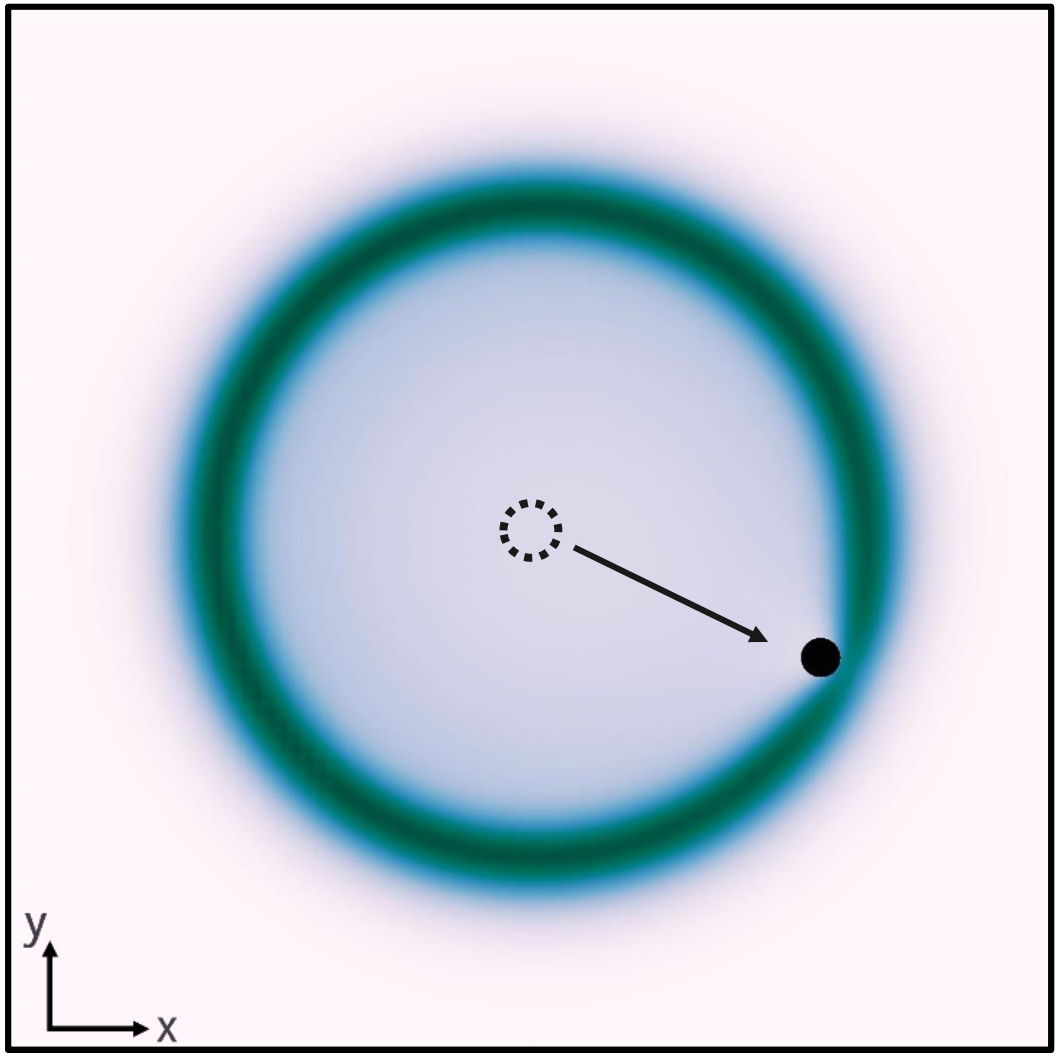

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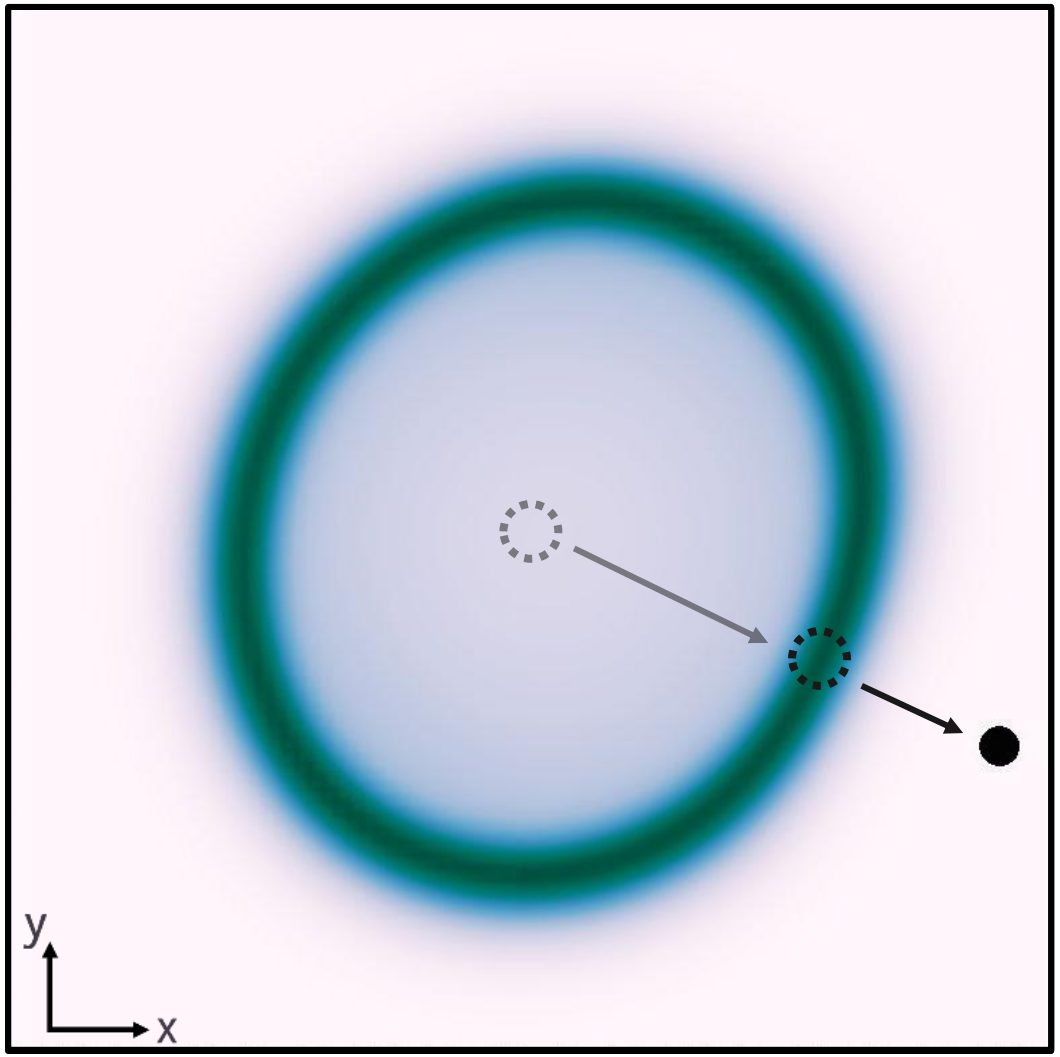

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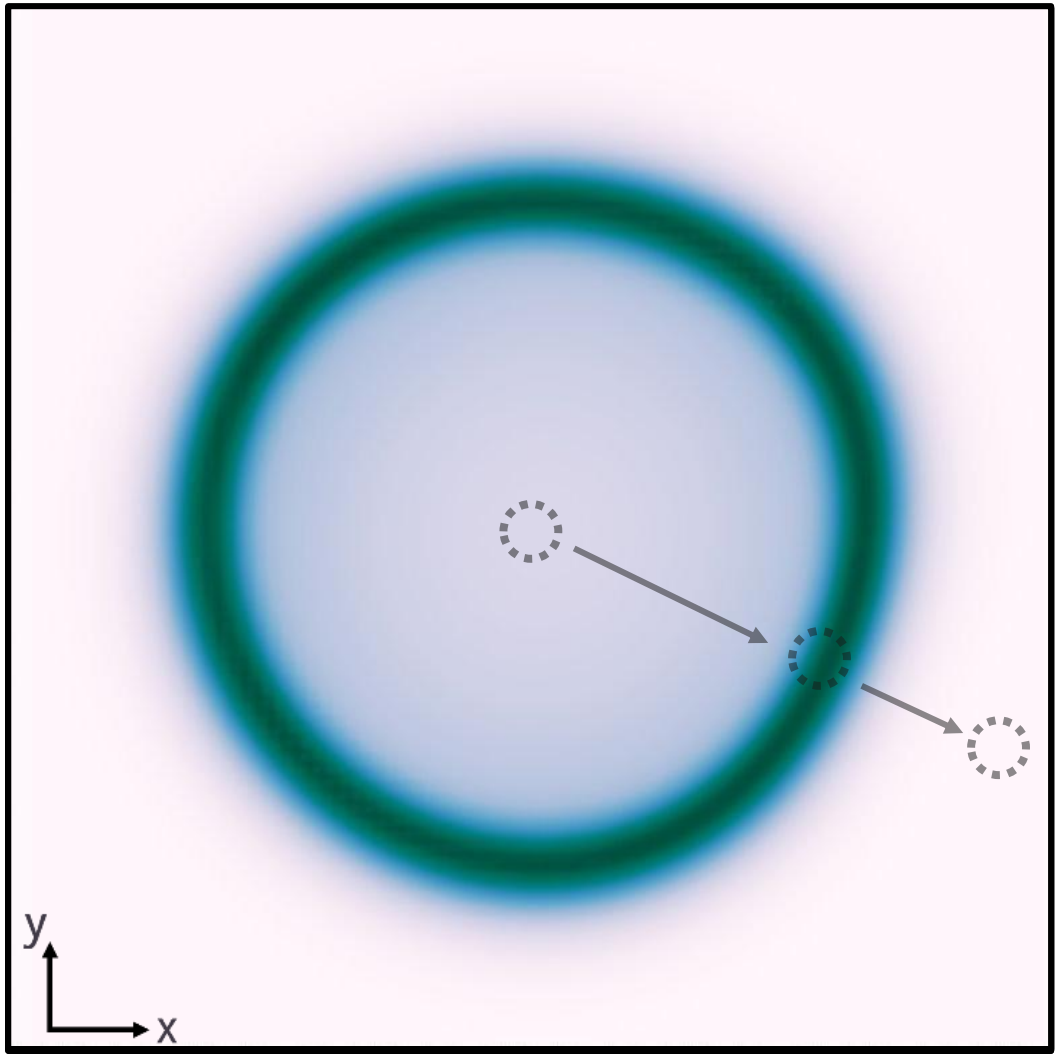

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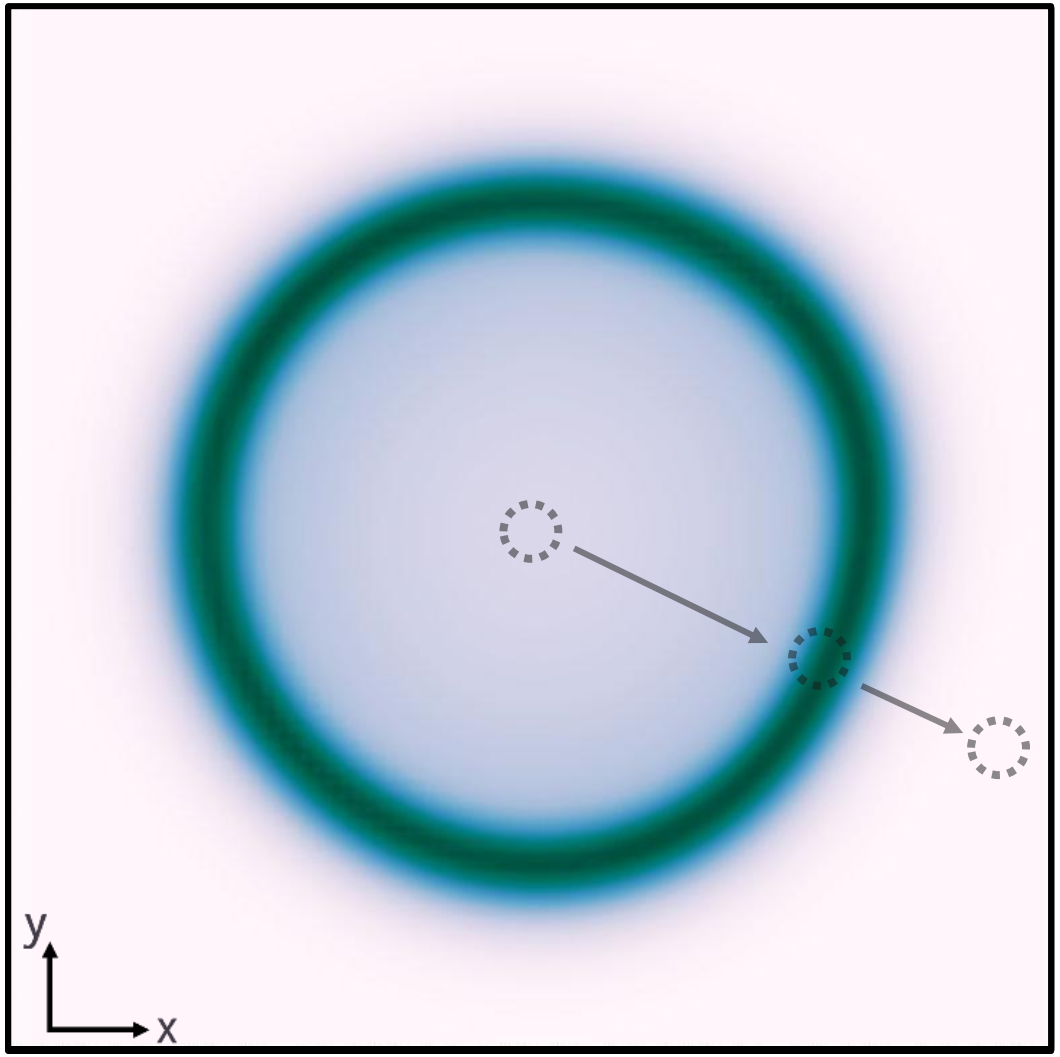

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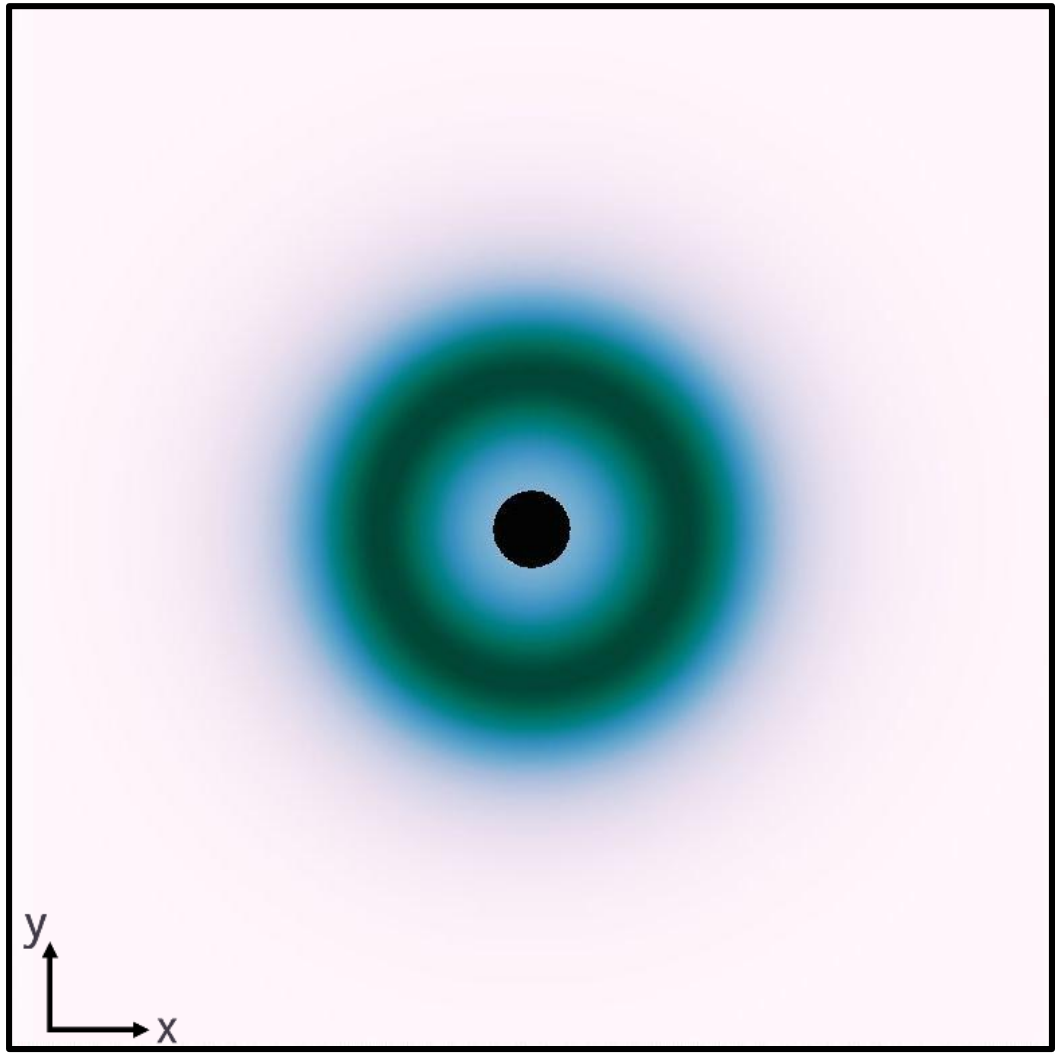

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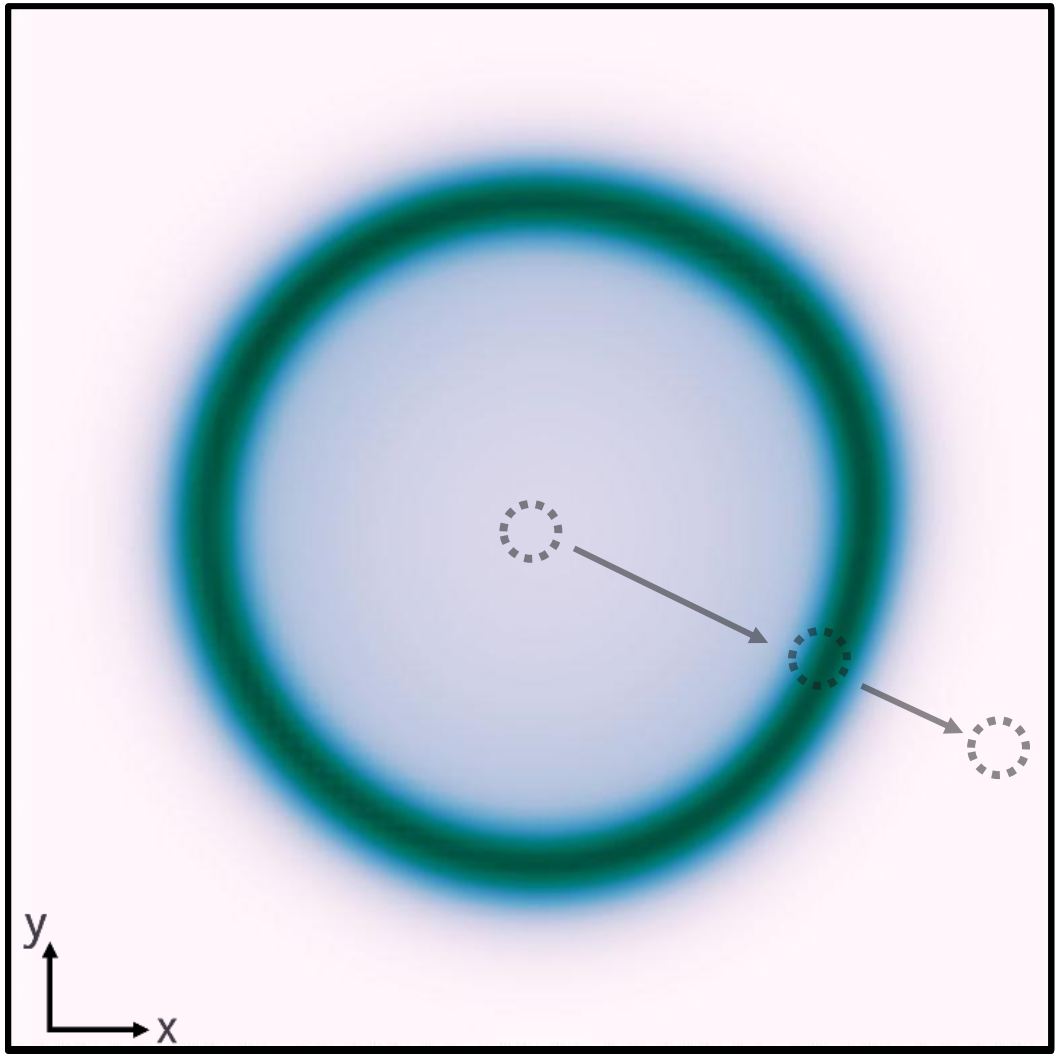

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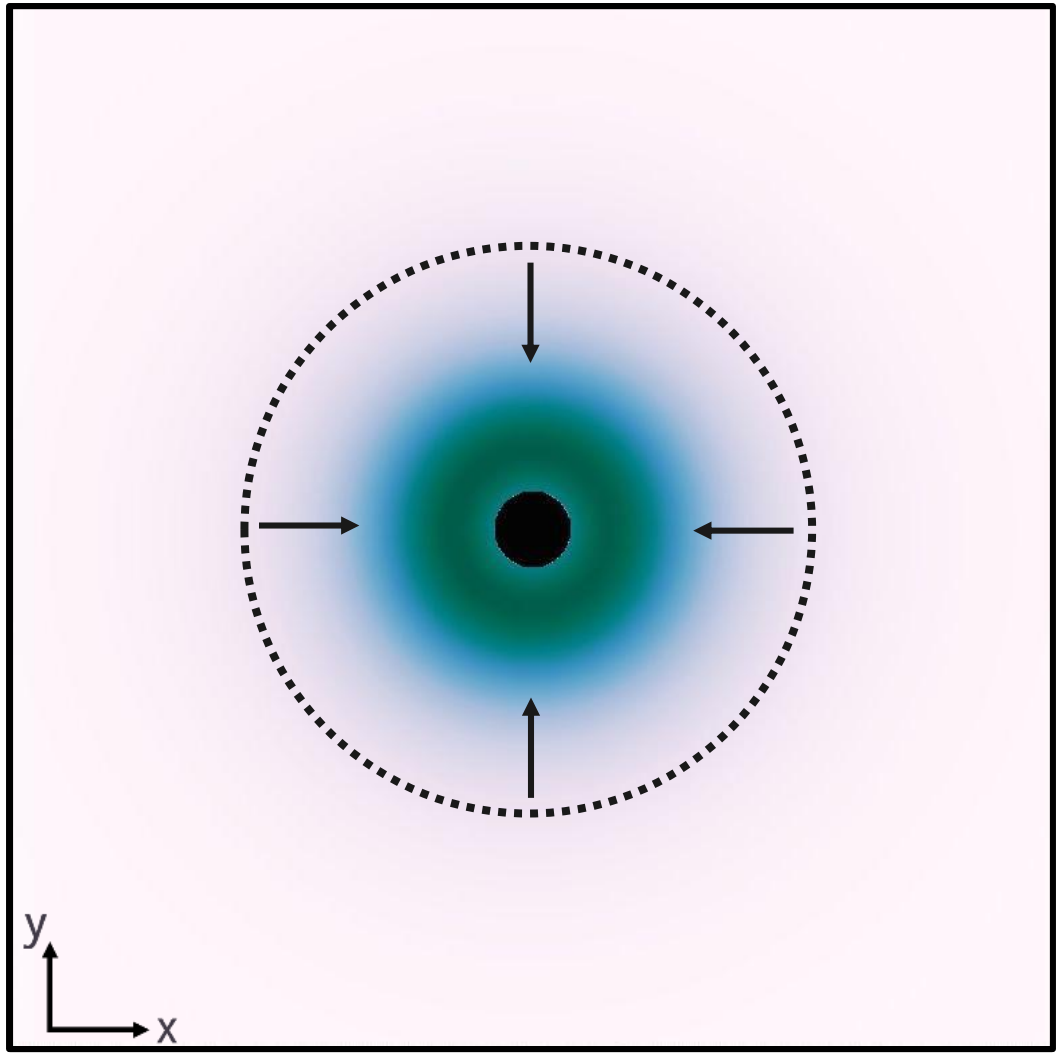

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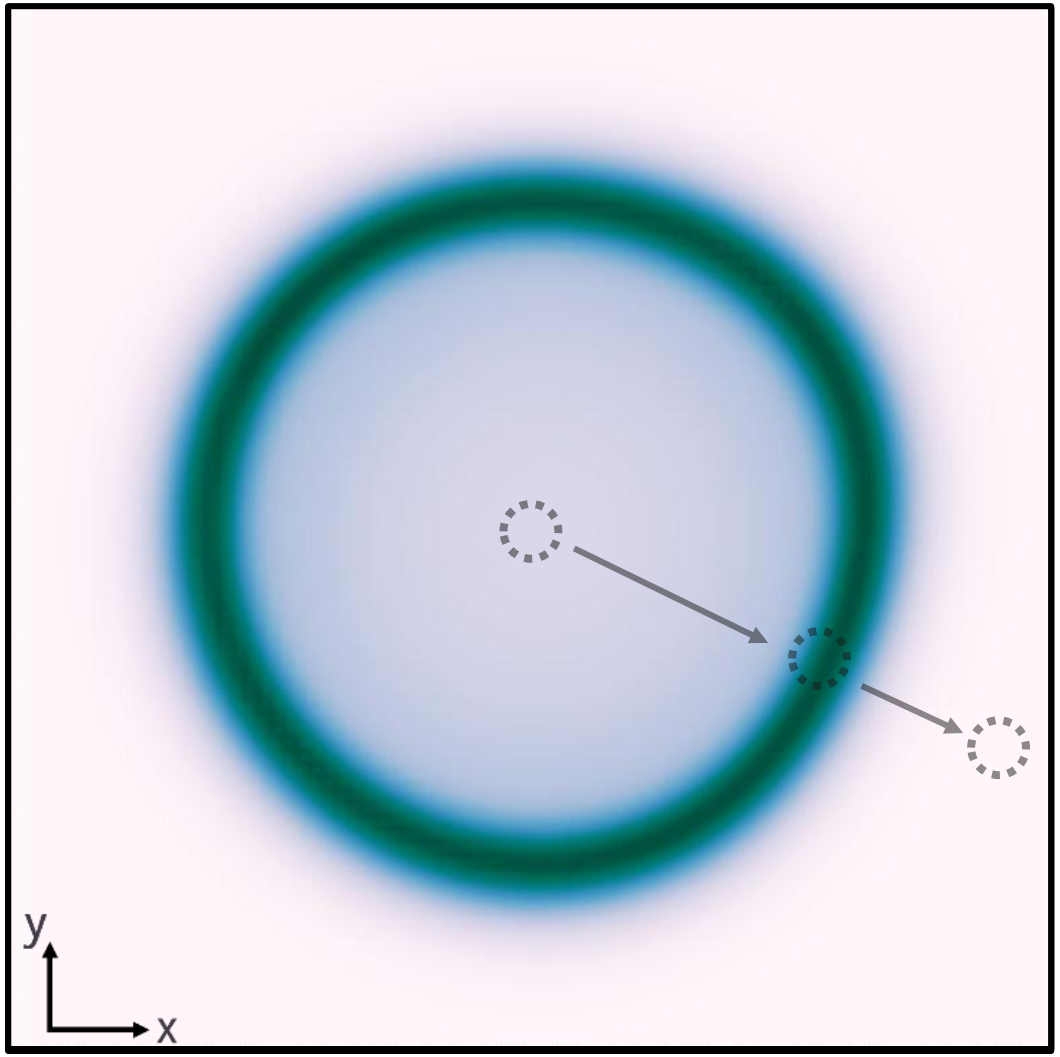

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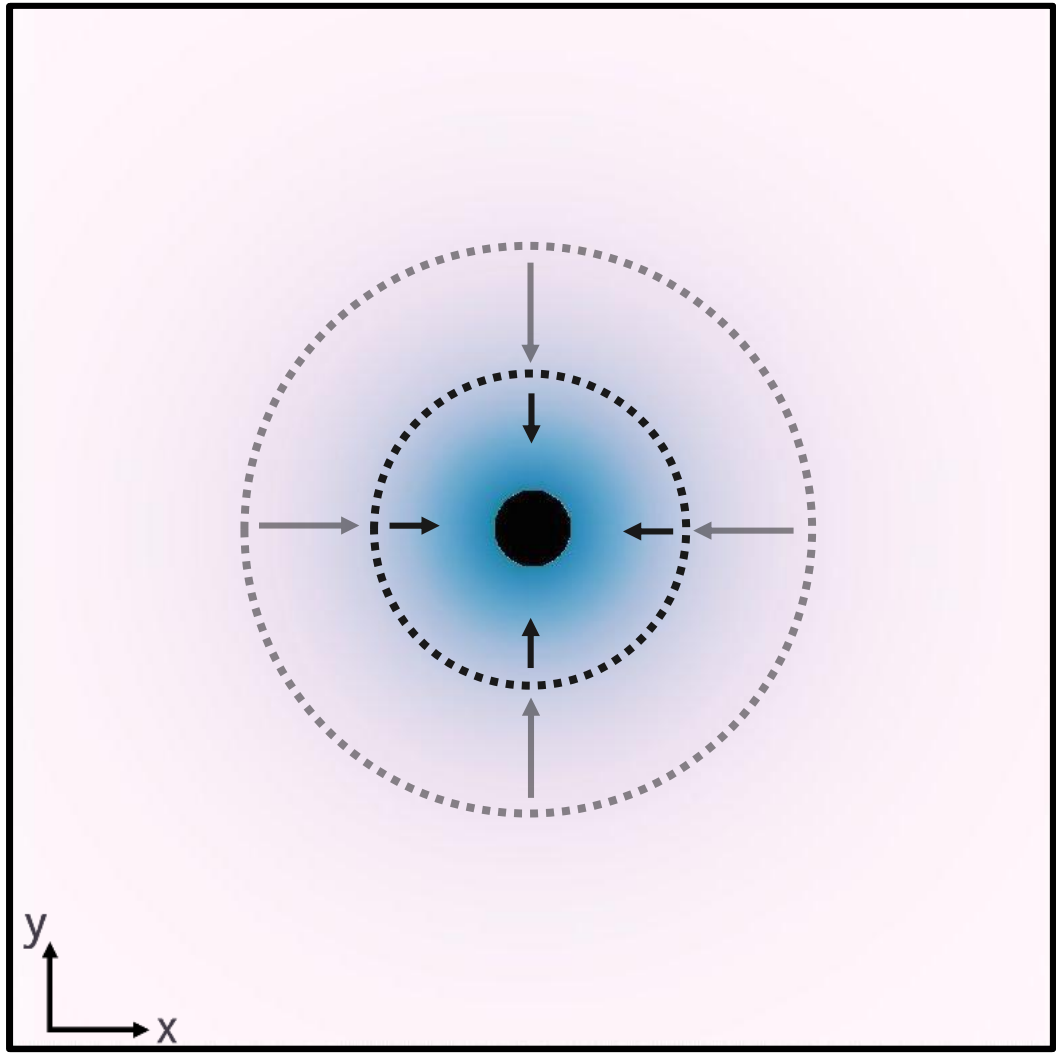

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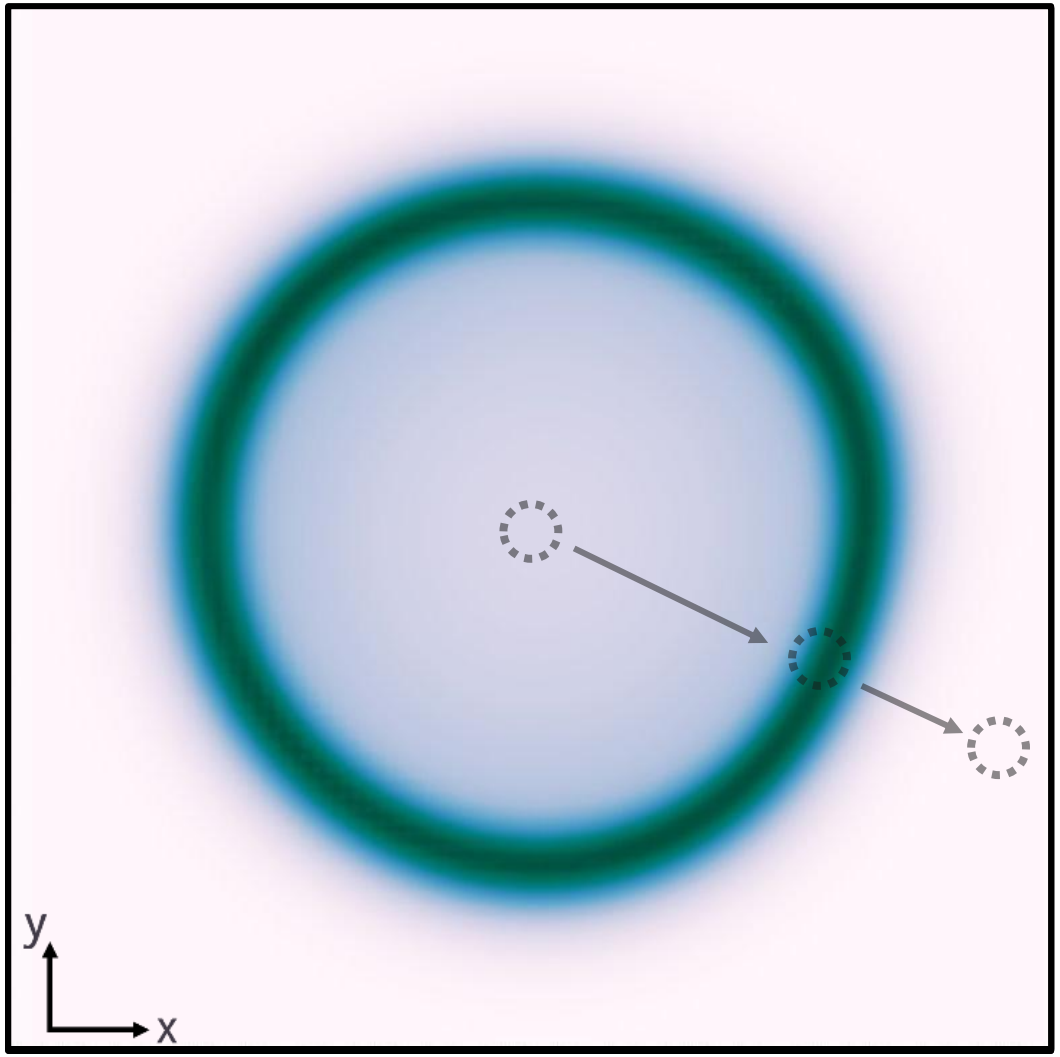

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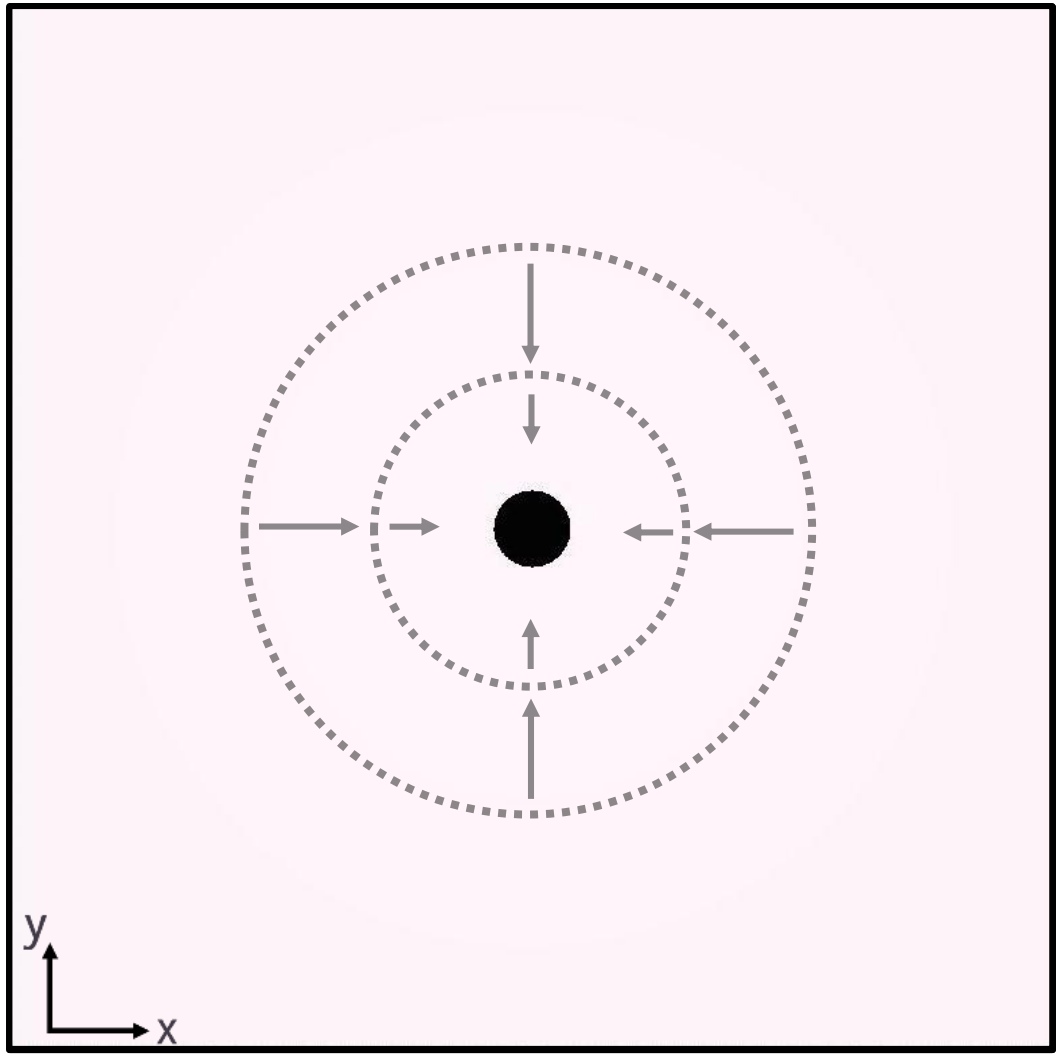

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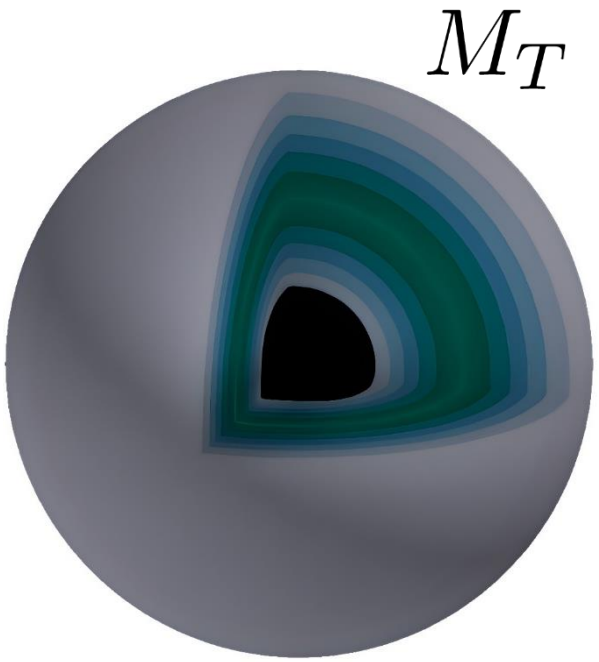

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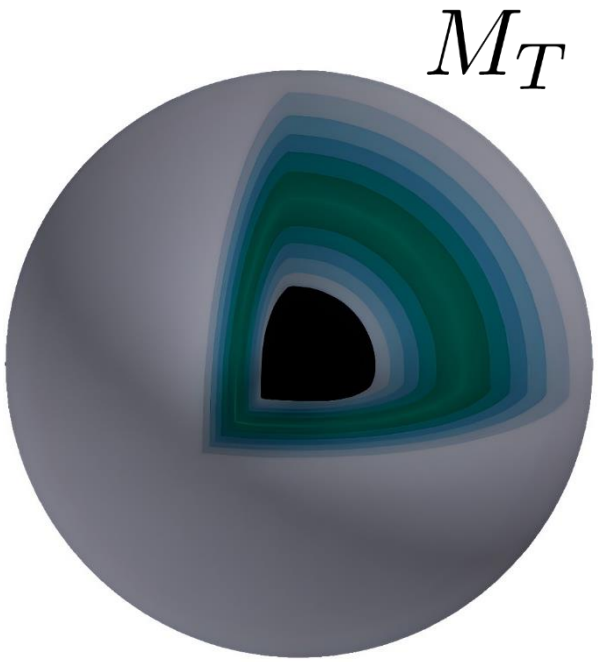

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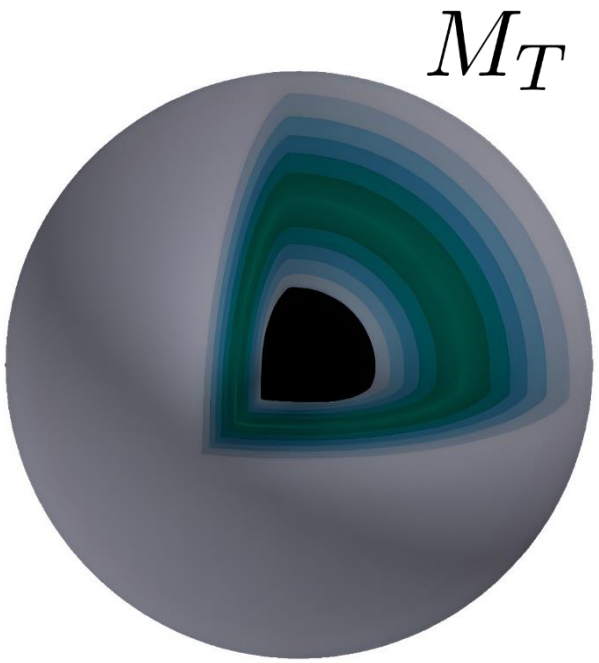

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$$M_T \approx M_\odot : \begin{cases} m \approx 10^{-12}\ eV/c^2 \\ \Delta\tau \approx 1\ ms \end{cases}$$

$$M_T \approx 10^9 M_\odot : \begin{cases} m \approx 10^{-21}\ eV/c^2 \\ \Delta\tau \approx 1\ week \end{cases}$$

Conclusions

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Thank you

Questions?

jpmferreira@ua.pt

[1] J. Nicoules, J. Ferreira, C. Herdeiro, E. Radu, and M. Zilhão. “Splitting the Gravitational Atom: Instabilities of Black Holes with Synchronized/Resonant Hair”. arXiv: 2509.20450 [gr-qc].

Extra Slides

Equations of Motion

Electromagnetic Field

$$\nabla_\alpha F^{\mu\alpha} = iq\left[(\widetilde{\nabla}^\mu\phi)^*\phi - \phi^*(\widetilde{\nabla}^\mu\phi)\right]$$

$$E^\mu \equiv \gamma^{\mu\alpha} n^\beta F_{\alpha\beta}$$

$$\mathcal{A}_\alpha \equiv \gamma_\alpha{}^\beta A_\beta \,,\; A_\phi \equiv -n^\alpha A_\alpha$$

$$\partial_t E^i = \alpha K E^i + \beta^j D_j E^i - E^j D_j \beta^i + D_j(\alpha(D^i\mathcal{A}^j - D^j\mathcal{A}^i)) - \alpha z^i$$

$$\partial_t \mathcal{A}^i = -\alpha E^i - D^i(A_\phi\alpha) + \beta^j D_j \mathcal{A}^i - \mathcal{A}^j D_j \beta^i$$

$$\partial_t A_\phi = \beta^i \partial_i A_\phi + \alpha K A_\phi - \alpha D_j \mathcal{A}^j - \mathcal{A}^j \partial_j \alpha$$

$$z^i = -2iq\left(\phi\partial^i\phi^* - \phi^*\partial^i\phi\right) - 2q^2\mathcal{A}^i\phi\phi^*$$

$$D_i E^i - 4\pi\rho_e = 0$$

$$\rho_e = -2iq(\phi K_\phi^* - \phi^* K_\phi) - 2q^2 A_\phi\phi\phi^*$$

Scalar Field

$$\widetilde{\nabla}_\alpha\widetilde{\nabla}^\alpha\phi = \frac{dV}{d\left|\phi\right|^2}\phi$$

$$\partial_t\phi = \beta^j\partial_j\phi - 2\alpha K_\phi$$

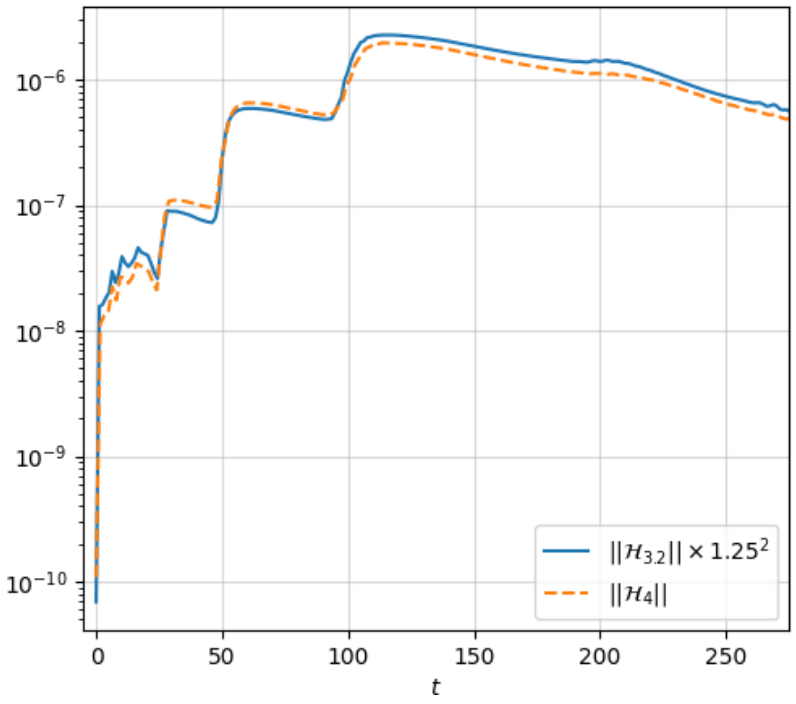
$$\partial_t K_\phi = \alpha K K_\phi + \beta^j\partial_j K_\phi - \frac{1}{2}D^i(\alpha\partial_i\phi) + \frac{1}{2}\alpha\frac{dV}{d\left|\phi\right|^2}\phi - i\alpha q\left(\mathcal{A}^i\partial_i\phi - 2K_\phi A_\phi\right) + \frac{\alpha q^2}{2}\left(\mathcal{A}^i\mathcal{A}_i - A_\phi^2\right)\phi$$

Metric

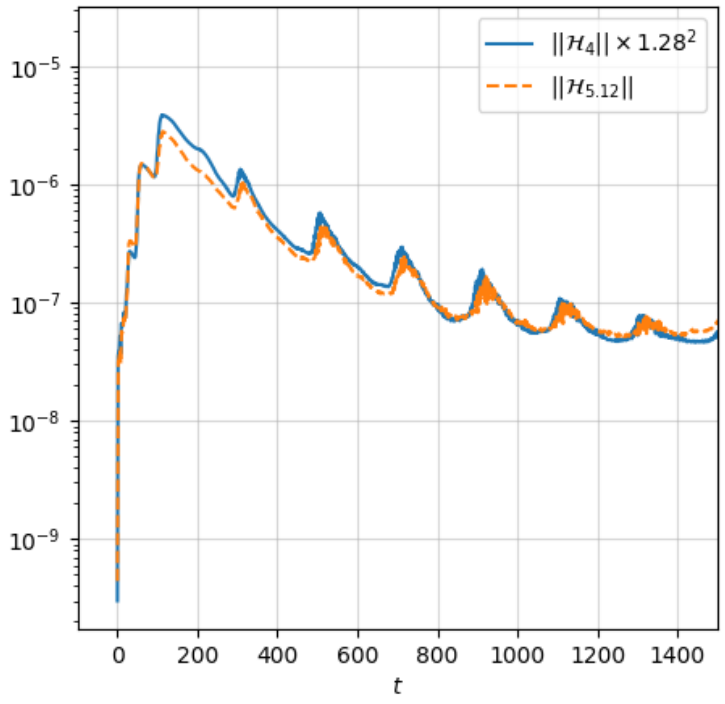
BSSN

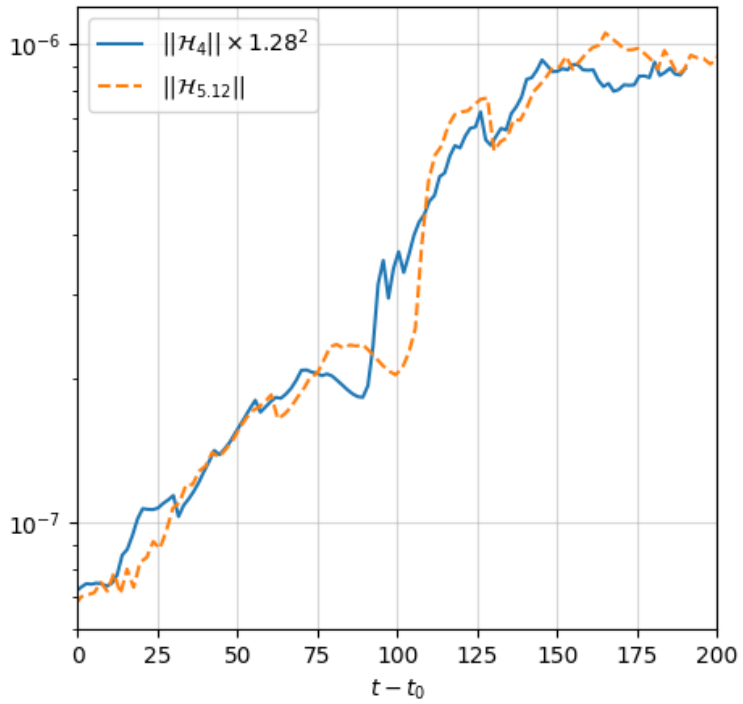
Convergence

Absorption



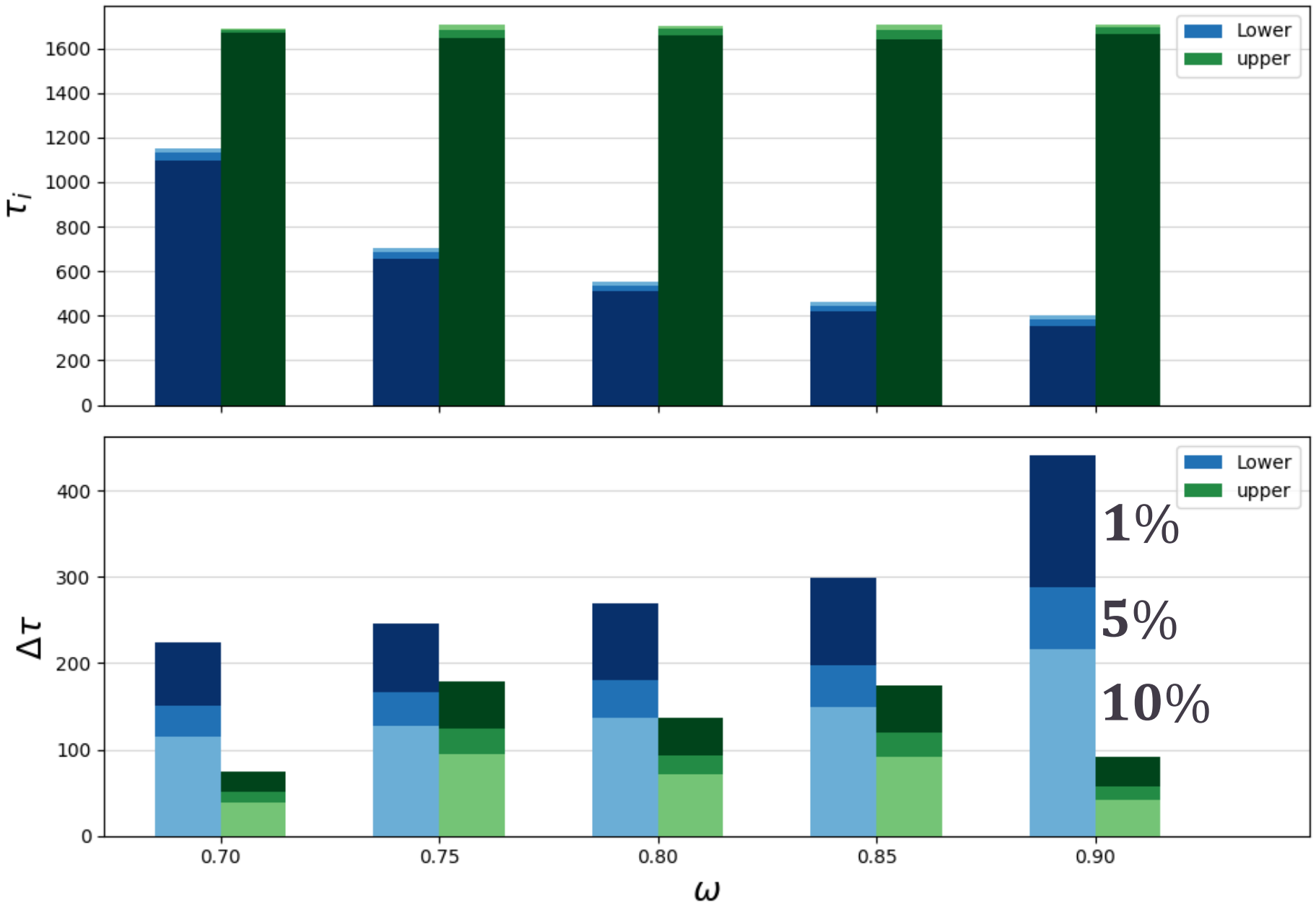
Fission





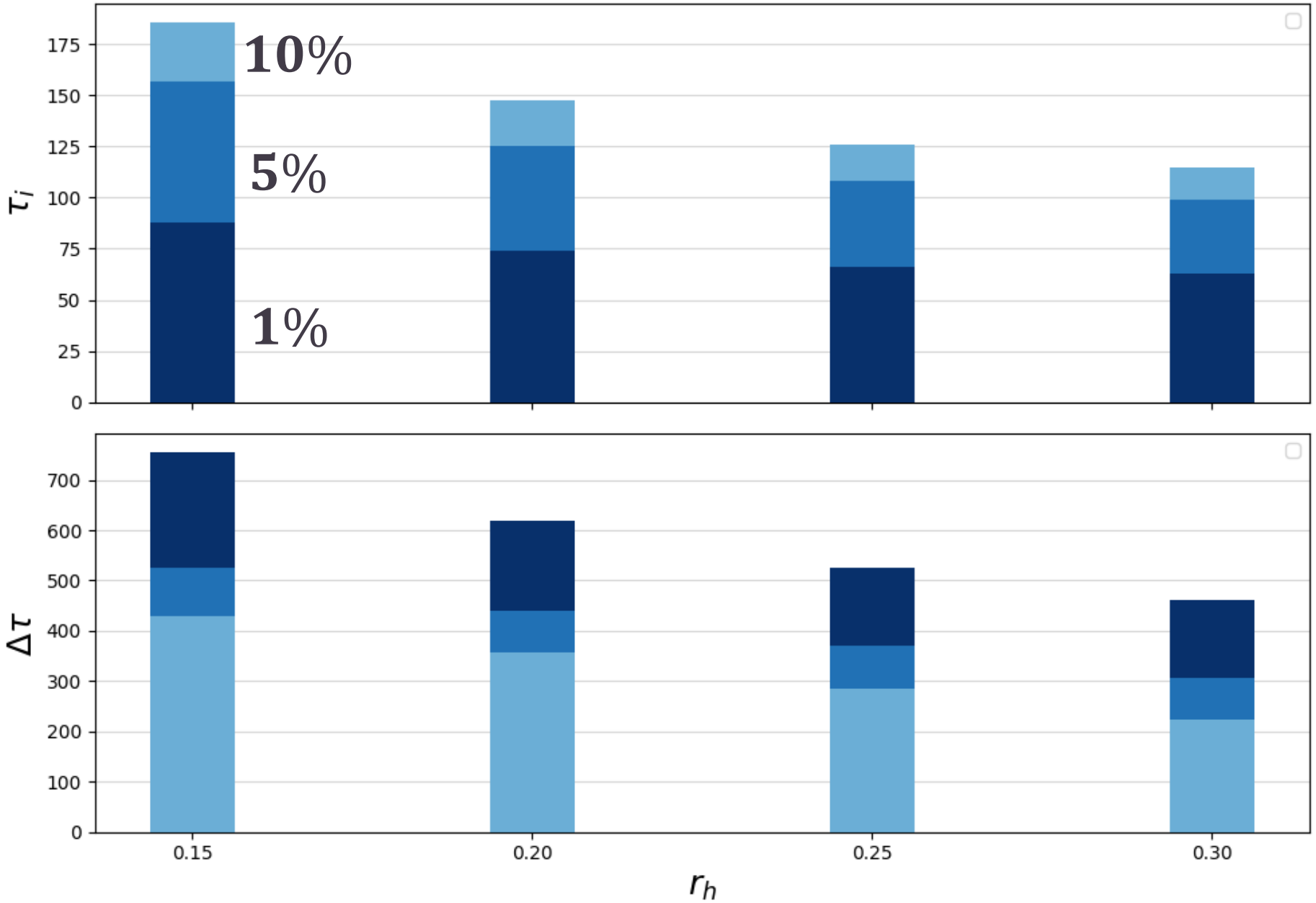
Time Scales for Different %

Fission
$(r_h = 0.2)$



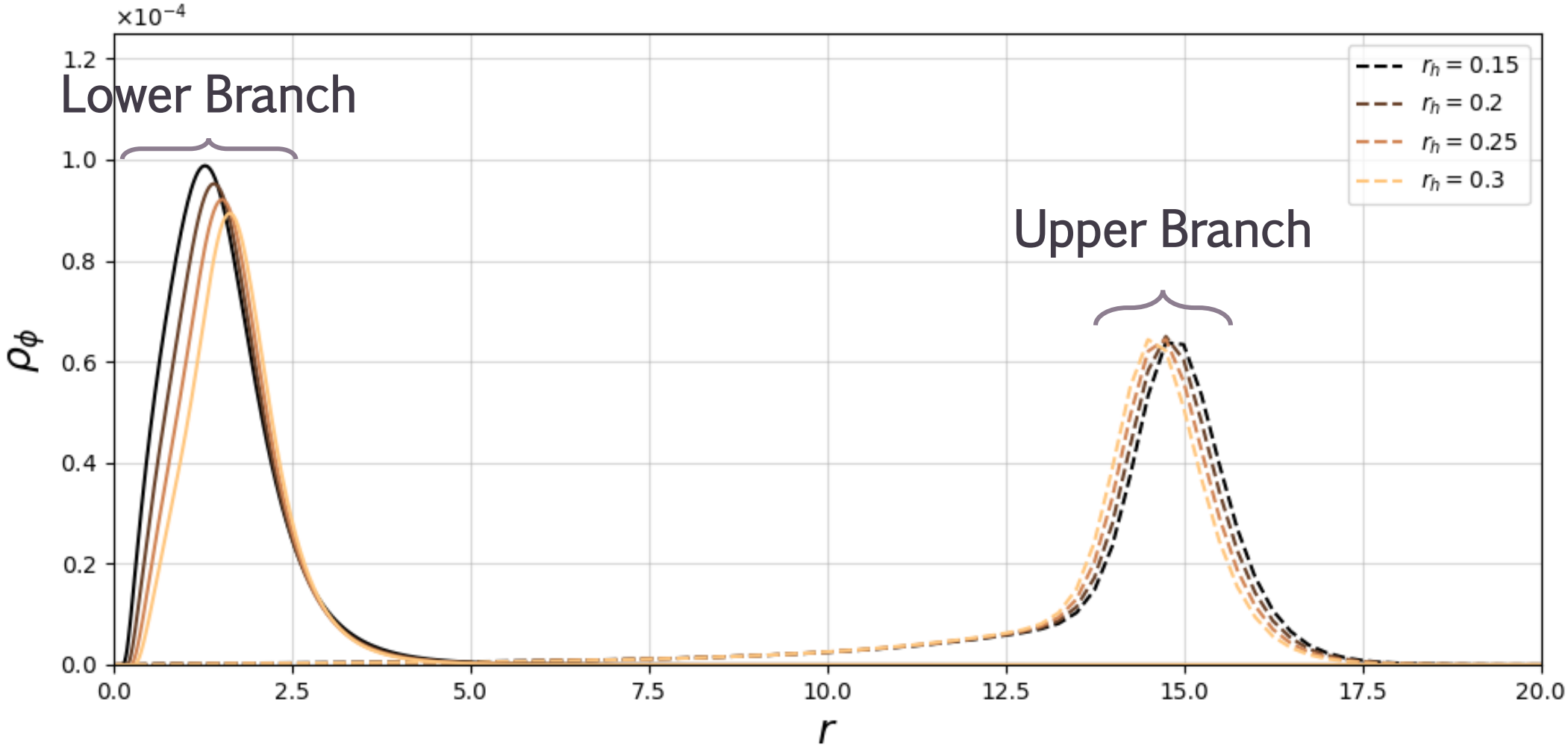
Time Scales for Different %

Absorption
$(\omega = 0.95)$



Hair Compactness for fixed ω

$$\omega = 0.95$$



Offset for Different Objects