IST-PhysFront'25

Self-Similar Gravitational Collapse of Elastic Spheres

PhD in Physics

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In collaboration with Jorge Rocha

Motivation

Why:
-Perfect fluids have been historically used to simulate realistic matter distributions.
-But, they are simple and may not account for complex dynamics.
Elasticity in the context of general relativity aims to correct this but requires much work to be understood.
How:
-Self-similarity is the invariance with regards to scale.
-Results by Koike, Hara and Adachi [1] exist for perfect fluids, so we look to generalize them to elasticity.

Coordinates and metric

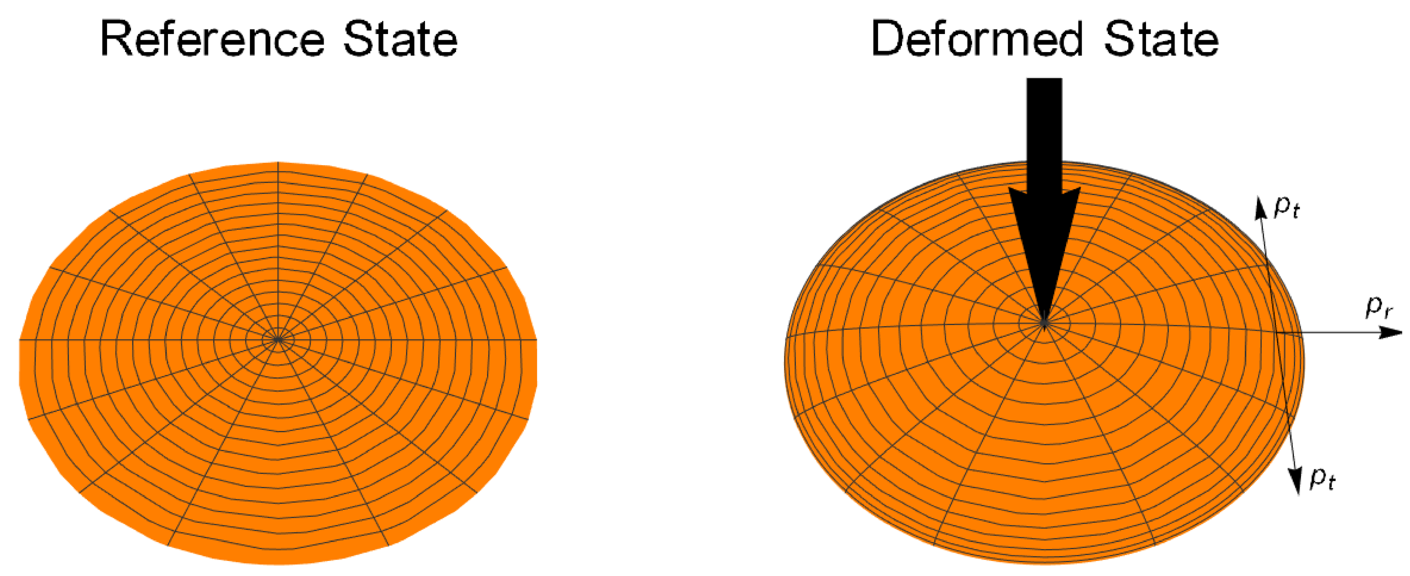
We consider self-similar coordinates and the metric:

$$\mathrm{e}^{x} = \frac{r}{-t}, \quad \mathrm{e}^{-\tau} = -t,$$

$$\mathrm{d}s^2 = \mathrm{e}^{-2\,\tau}\,\mathrm{e}^{2\,x}\,\left[-\left(N^2-1\right)\,A\,\mathrm{d}\tau^2 - 2\,A\,\mathrm{d}\tau\,\mathrm{d}x + A\,\mathrm{d}x^2 + \mathrm{d}\Omega_2^2\right].$$

Where the part in parenthesis depends only on the scale factor x. The simulation is done around a sonic point, set at x=0.

Geometry



Elasticity comes about as a comparison between the physical geometry and a reference (relaxed) geometry.

Matter

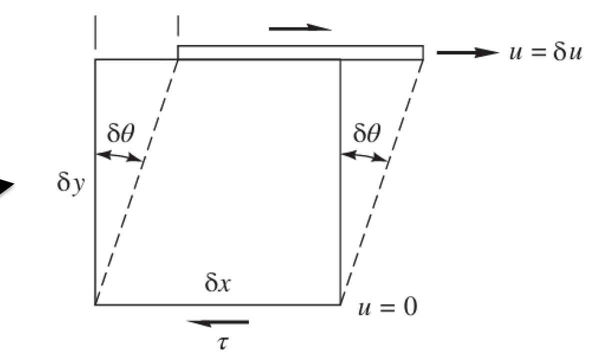
Self-similarity is made possible by specific matter models. Alho and collaborators found [2] self-similar elastic models follow:

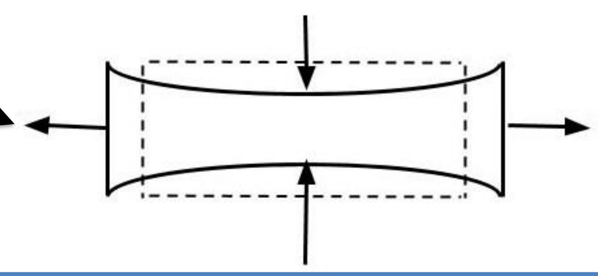
$$\rho = \frac{K}{\gamma\,(\gamma-1)}\,\eta^{\gamma}\left[1-\gamma\,(1-3\,A)\,\left(1-\frac{\delta}{\eta}\right) - 3\,\frac{\gamma}{\beta}\,A\,\left(1-\left(\frac{\delta}{\eta}\right)^{\beta}\right)\right],$$

$$A = \frac{\gamma-1}{\beta-1}\,\left(\frac{1-\nu}{1+\nu}\right)$$

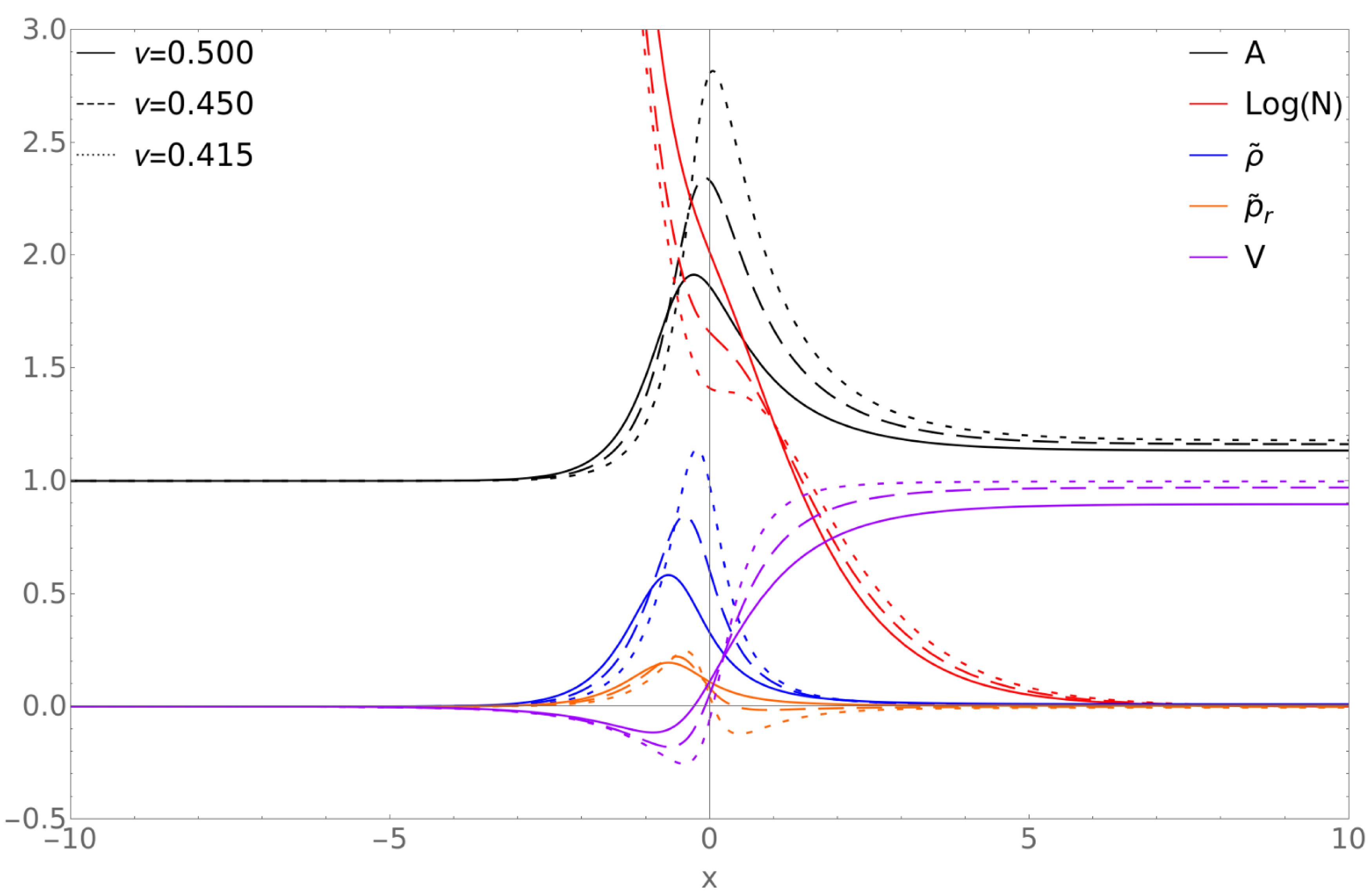
with K the matter magnitude and:

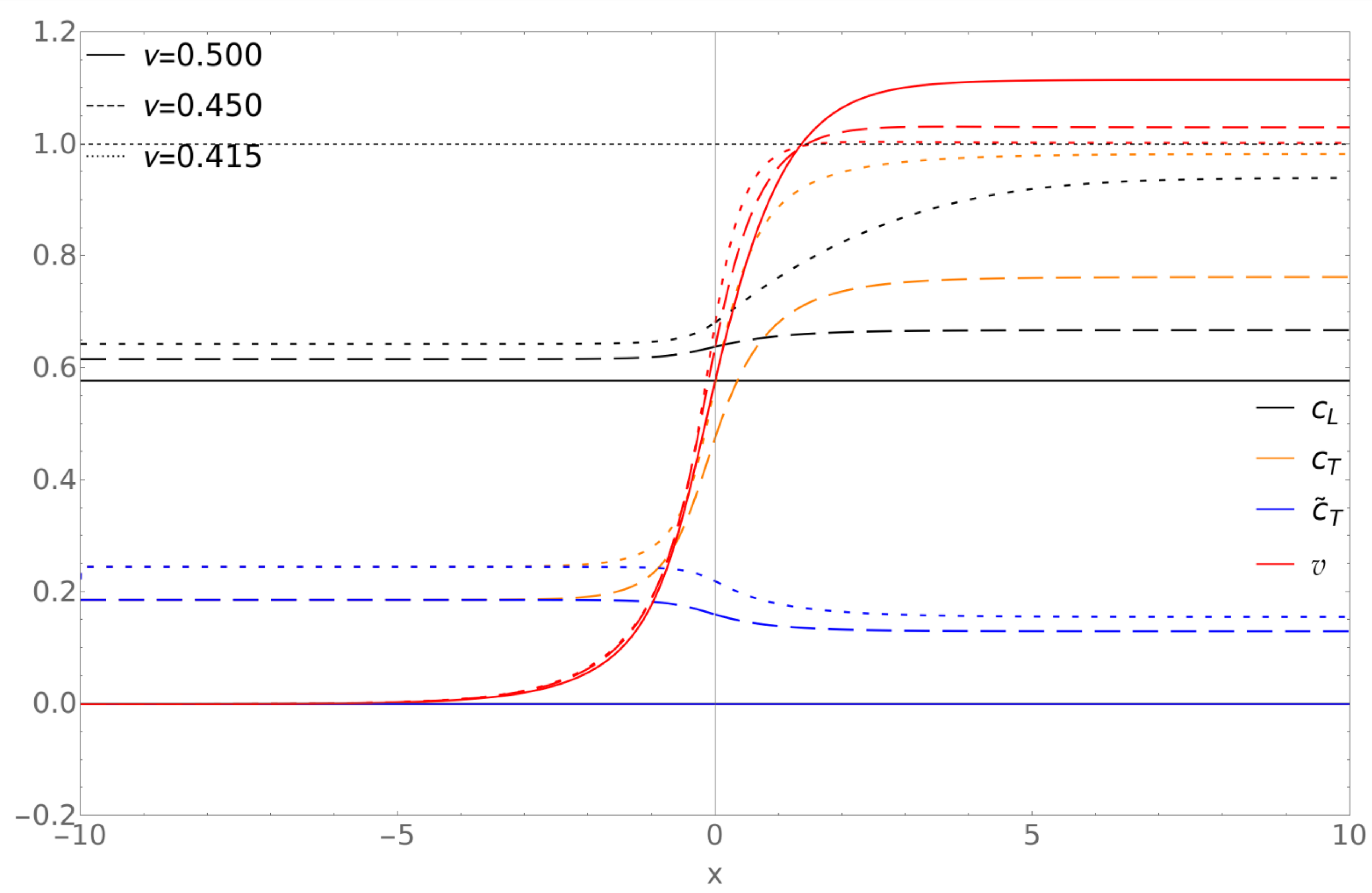
- $\gamma \rightarrow$ adiabatic index
- $\beta \rightarrow$ shear index
- $\nu \rightarrow$ Poisson ration
- $\delta \rightarrow$ linear radial density
- $\eta \rightarrow$ tangential linear density

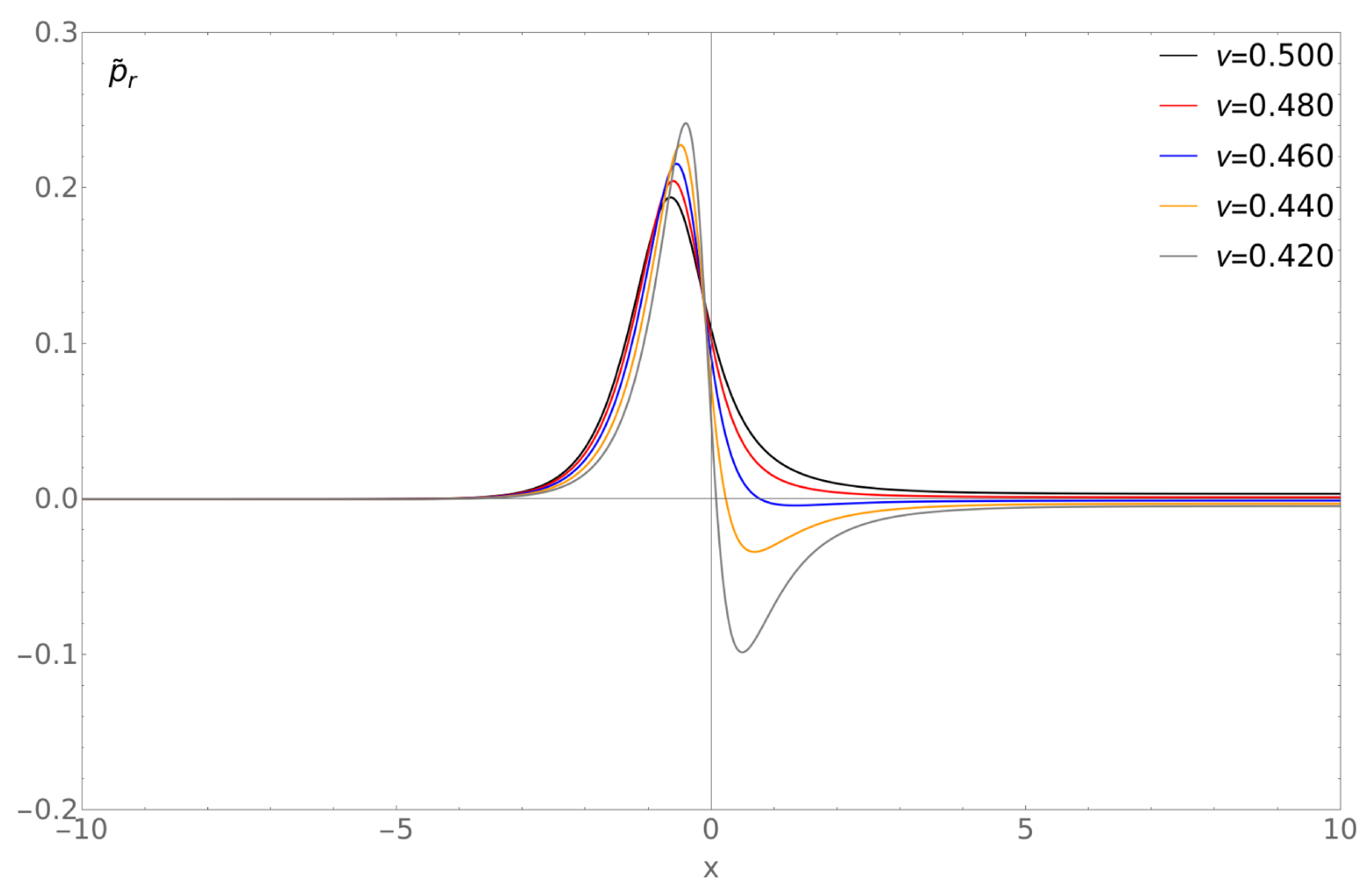




Results (with $\gamma = \beta = 4/3$ and variable ν)







Top: Metric, matter and velocity functions for different values of the Poisson ratio
Top right: Velocities, for sound, in black, blue and orange, and for particles registered by a fixed observer, in red.
Right: Pressure profile for different Poisson ratio and how it can become negative, unlike what is observed in PF models.

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https://doi.org/10.54499/2022.13617.BD

References: [1] Koike, Hara, Adachi, Phys. Rev. Lett. 74, 5170 (1995);
[2] Alho, Natário, Pani, Raposo, Phys. Rev. D 109, 064037 (2024)