Non-equilibrium dynamics in a dissipative SYK model

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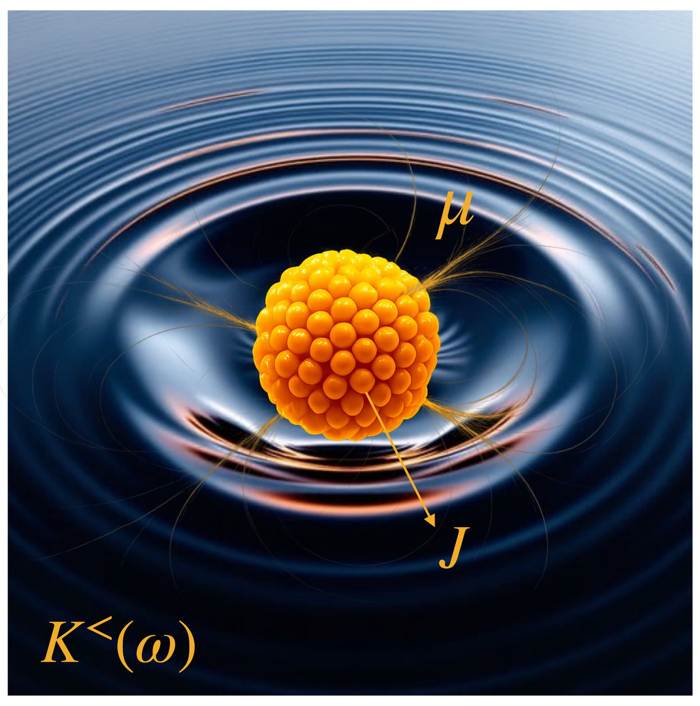
Introduction

- Tremendous progress has been made in controlling and manipulating individual quantum systems (e.g. superconducting qubits, ultracold atoms)
- However, no quantum system is truly isolated: interactions with the environment drive them towards a steady state with stationary properties

How do strongly correlated open quantum systems relax to the steady state?

How does this relaxation change when the system retains memory?



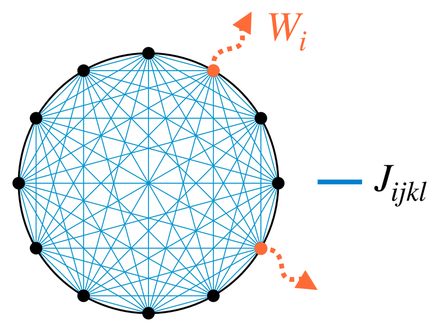
An artist's view of our model

- We explore these questions in a dissipative SYK model with a non-Markovian bath (i.e., that retains memory) and determine its relaxation properties

Dissipative Sachdev–Ye–Kitaev (SYK) model

- Studying many-body systems is inherently hard (non-perturbative and exponential complexity)
- The SYK model is a zero dimensional model of strongly interacting fermions, widely used as a tractable model for strongly correlated quantum matter [1,2]

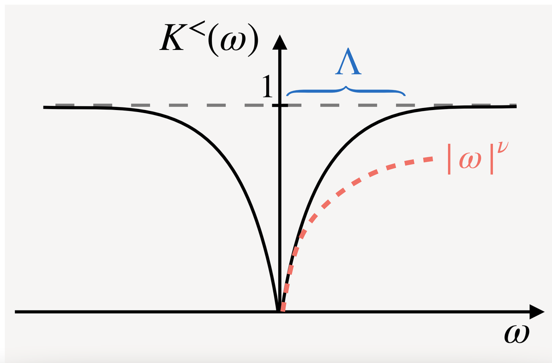
System: N Majorana fermions χ_i → $\chi_i^\dagger = \chi_i$, $\{\chi_i, \chi_j\} = \delta_{ij}$



Hamiltonian: $H = \sum_{i<j<k<l} J_{ijkl}\chi_i\chi_j\chi_k\chi_l$ → $\langle J_{ijkl}^2\rangle = \frac{6J^2}{N^3}$

Jump operators: $W_i = \sqrt{\mu}\,\chi_i$

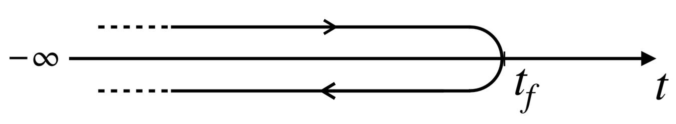
Bath density of states: $K^<(\omega) = \left(1 - e^{-\omega^2/\Lambda^2}\right)^{\nu/2}$



$\omega \ll \Lambda$: $K^<(\omega) \approx \left|\frac{\omega}{\Lambda}\right|^\nu$

$\omega \gg \Lambda$: $K^<(\omega) \approx 1$

Keldysh path-integral approach [3]



- The Keldysh formalism is a powerful tool for quantum systems out-of-equilibrium

$$Z = \mathrm{Tr}[\rho_{t_f}] = \int \prod_i \mathscr{D}a_i \exp\left(i\int_C dz\, \frac{1}{2}\sum_i a_i(z) i\partial_z a_i(z) - i\int_C dz \sum_{i<j<k<l} J_{ijkl} a_i(z)a_j(z)a_k(z)a_l(z) + \int dz dz' \mu K(z,z') \sum_i a_i(z)a_i(z') \right)$$

Propagator (first term); Coherent evolution (second term); Non-Markovian dissipation (third term)

- We can write a mean-field action for the collective field $G(z,z') = -\frac{i}{N}\sum_i a_i(z)a_i(z')$
- Taking the saddle point ($N \to \infty$), we have:

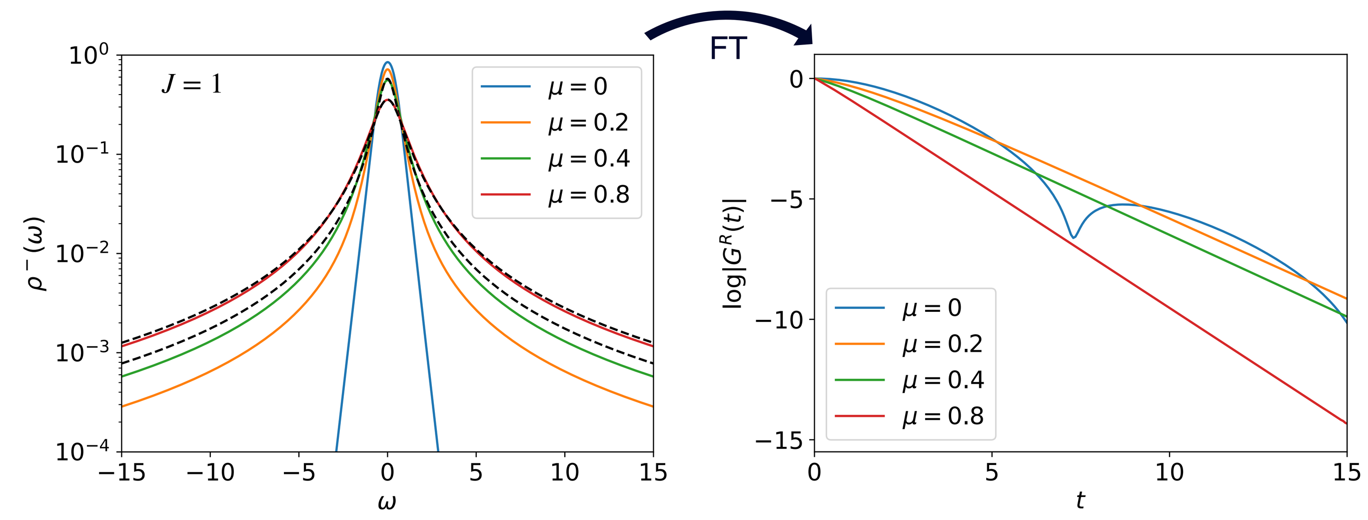
Schwinger-Dyson Equations

$$\sigma^-(\omega) = \frac{\mu}{\pi}K^<(\omega) + \frac{J^2}{4}(\rho \star \rho \star \rho)(\omega)$$

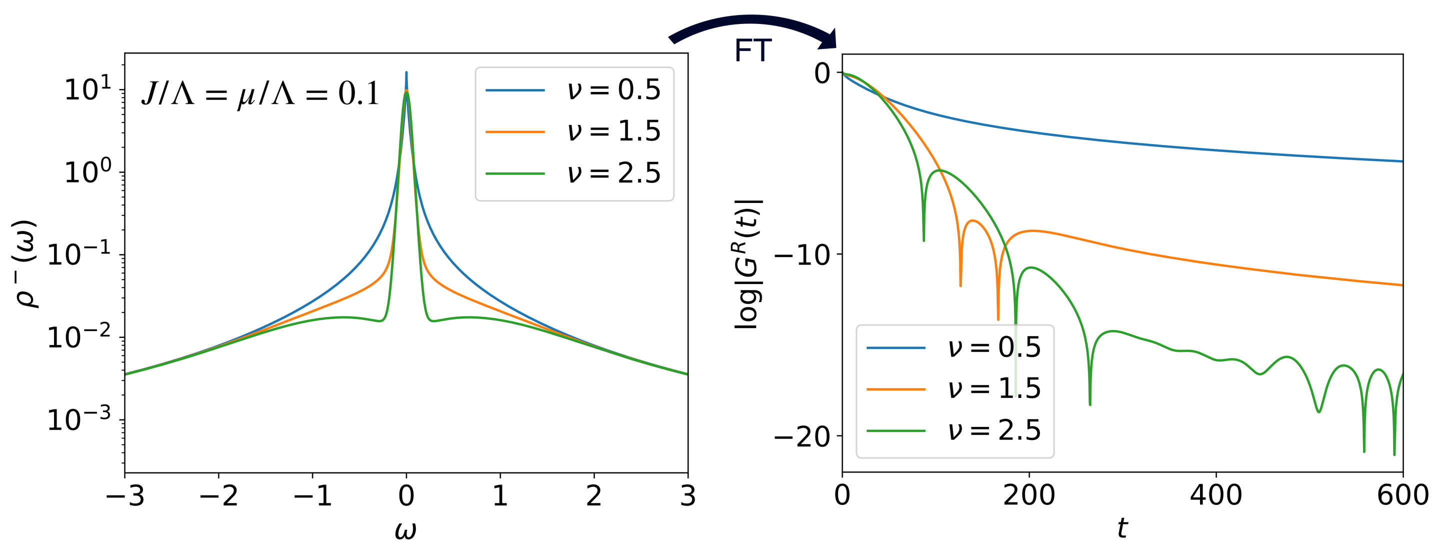
$$\rho^-(\omega) = \frac{\sigma^-(\omega)}{(\pi\sigma^H(\omega) + \omega)^2 + (\pi\sigma^-(\omega))^2}$$

Solutions of the SDE equations

- We numerically solve the equations on a frequency grid, by iterating until a fixed point is reached
- For $\nu = 0$ (Markovian limit), the solutions are known [4, 5]



- For $\nu > 0$ (non-Markovian regime), we can have slow relaxation



CONCLUSIONS

- We studied the **relaxation dynamics** of a dissipative SYK model with a non-Markovian bath
- Using **Keldysh formalism**, we could numerically study the thermodynamic limit of the system
- We found that the **non-Markovianity** can induce a **slow relaxation** (i.e., power-law instead of exponential)

OUTLOOK

How does the relaxation depend on the system parameters?

Are there any dynamical phase transition? If so, what is the phase diagram?

Is there some limit where can make analytical predictions?

References

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