QUANTUM FINANCE:
Path Integrals and Hamiltonians for Options Pricing

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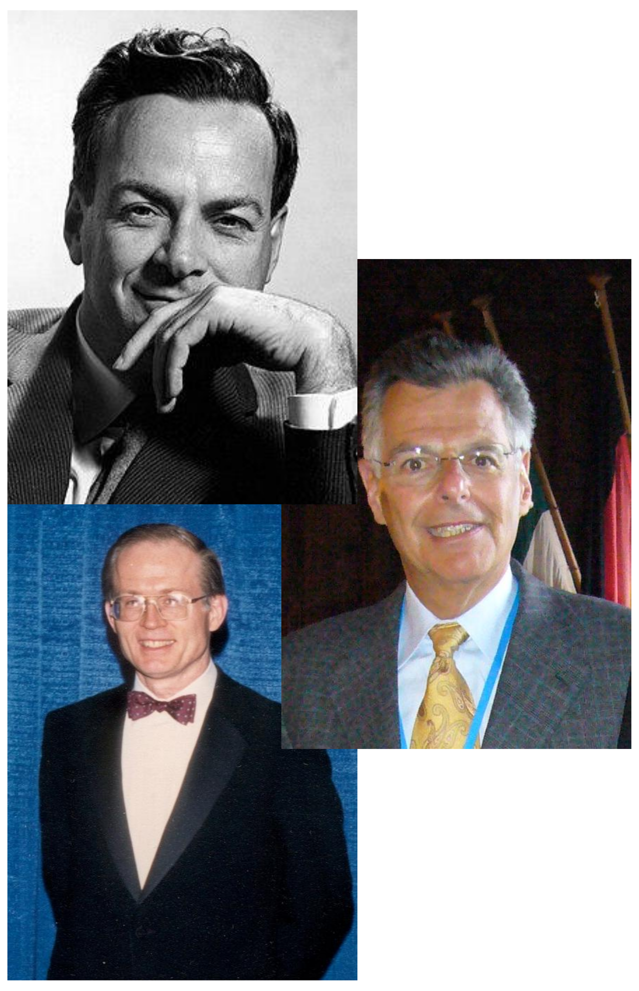
Introduction

- Explores option pricing through the lens of quantum mechanics.
- Investigates assesses the viability of this framework for modelling option markets beyond classical approaches.
- Demonstrates how these methods can be developed into real-world application.



Richard Feynman, Myron Scholes and Fisher Black
Sources: wikipedia.org

Methodology

- Reformulate classical option pricing models such as Black–Scholes and Merton–Garman through the lens of quantum mechanics, using path integrals and Hamiltonian operators to define a new pricing framework[1].

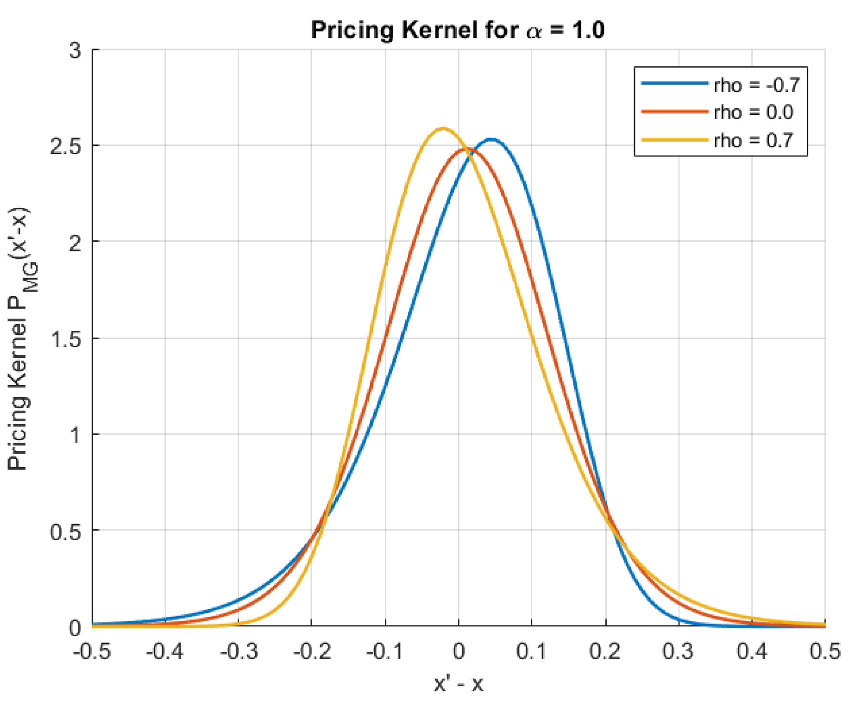
$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + rS\frac{\partial V}{\partial S} - rV = 0$$

$$\frac{\partial C}{\partial t} + rS\frac{\partial C}{\partial S} + (\lambda + \mu V)\frac{\partial C}{\partial V} + \frac{1}{2}VS^2\frac{\partial^2 C}{\partial S^2} + \rho\xi V^{1/2+\alpha} S\frac{\partial^2 C}{\partial S \partial V} + \xi^2 V^{2\alpha}\frac{\partial^2 C}{\partial V^2} = rC$$

$$\frac{\partial C}{\partial t} = H_{MG} C$$

$$p(x,\tau;x') = \langle x \,|\, e^{-\tau H} \,|\, x' \rangle \qquad C(t,x) = \int_{-\infty}^{+\infty} dx' \,\langle x | e^{-\tau H} | x' \rangle g(x')$$

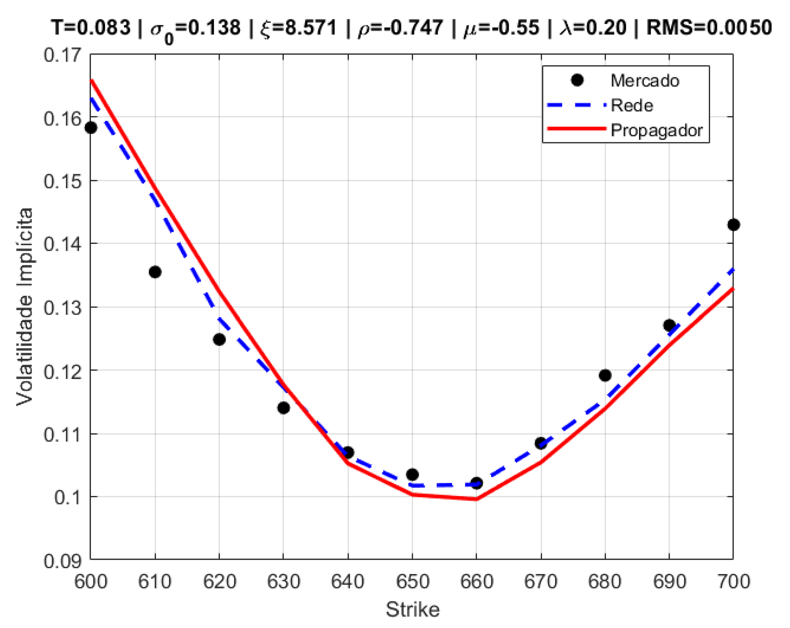
- Develop numerical algorithms to solve the reformulated problem and study the behaviour of the resulting equations[1,2].
- Compare the quantum-inspired approach with standard Monte Carlo (Euler) methods to benchmark performance.

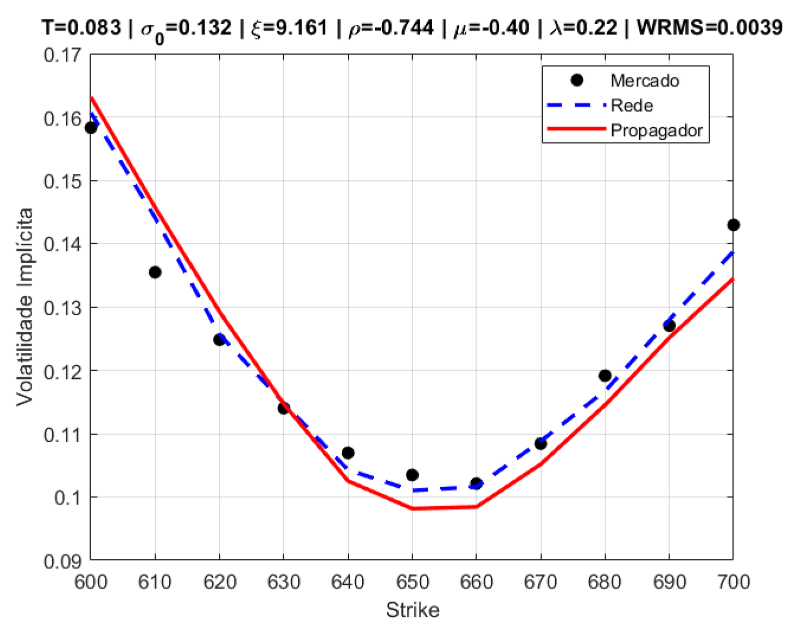


- Perform calibration with market data, estimating parameters through grid search and optimization, through Neural Networks, with the objective of minimizing RMSE and WRMSE between model and market implied volatilities[3].
- Validate the framework using SPY ETF option data, covering multiple strikes and maturities, to assess its real-world applicability.

Results

- The quantum-inspired reformulation successfully reproduces option prices and implied volatility curves comparable to classical model.
- Benchmarking showed that for Monte Carlo averaged 0.6 seconds and 12 800 kB of memory per run, while the path integral method required 3.6 seconds and 64.6 kB.
- Market calibration by minimizing RMSE and WRMSE, requiring several hours. Introducing a neural network push-forward reduced calibration time dramatically, from around 12 hours to about 30 minutes, while preserving accuracy.





Conclusions

- The Hamiltonian and path integral formalism reliably and accurately reproduces option prices and implied volatility curves.
- Compared to Monte Carlo, the propagator method requires more time but uses memory and computational resources more efficiently, making it a preferable framework in many settings.
- The longer runtime can be mitigated by using memory-oriented programming languages, allowing the propagator method to generate denser and more informative datasets.
- By combining propagator-generated data with a neural network push-forward, market calibration can be performed in practical time frames, enabling real-world financial applications.

References

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