# Feynman Integrals at Spectral Speed

A fast, stable and adaptive approach to multi-scale calculations in particle physics


**Afonso Guerreiro (PhD in Physics) [1,2] · Samuel Abreu [3,4] · Ben Page [5]**

afonsojguerreiro@tecnico.ulisboa.pt

[1] LIP, Av. Prof. Gama Pinto, 2, P-1649-003 Lisboa, Portugal, [2] Instituto Superior Técnico (IST), Universidade de Lisboa, Av. Rovisco Pais 1, P-1049-001 Lisboa, Portugal, [3] CERN, Theoretical Physics Department, CH-1211 Geneva 23, Switzerland, [4] Higgs Centre for Theoretical Physics, School of Physics and Astronomy, The University of Edinburgh, Edinburgh EH9 3FD, Scotland, United Kingdom, [5] Department of Physics and Astronomy, Ghent University, 9000 Ghent, Belgium


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## 1 · INTRODUCTION

### Multi-loop integrals sharpen tests of the Standard Model

Higher-order quantum corrections are essential for precision collider predictions. Multi-loop Feynman integrals reduce theoretical uncertainty, enabling stringent tests of the Standard Model and increasing sensitivity to possible new physics.

## 2 · THE CHALLENGE

### Complexity grows with scales, systems and singularities

Phenomenology requires repeated evaluation of large master-integral systems across a singular, high-dimensional phase space.

1. **TRAVERSE PHASE SPACE QUICKLY**
   Low cost for repeated point-to-point evaluation.
2. **BE ROBUST IN THE PHYSICAL REGION**
   Handle physical thresholds and spurious alphabet singularities.
3. **HANDLE LARGE COUPLED SYSTEMS**
   Remain efficient as the number of master integrals grows.
4. **HANDLE MANY KINEMATIC SCALES**
   Remain practical as the number of independent invariants grows.
5. **REQUIRE MINIMAL ANALYTIC INPUT**
   Given the DEs, regional boundary values suffice—no closed-form solution is required

**Evaluate only the phase-space points the observable needs—quickly, accurately and robustly throughout the physical region.**

## SELECTED REFERENCES


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## 3 · METHOD

### Canonical equations transport the solution

Boundary data are evolved through kinematic space using a closed first-order system.

**THE USEFUL CASE — CANONICAL $\varepsilon$-FORM**

Choose a basis in which the regulator dependence factorises [1].

$$d\vec{I} = \varepsilon \sum_j A_j \, d\log W_j(s) \, \vec{I}$$

- **ε factor** — dimensional regulator
- **$A_j$ matrices** — constant rational entries
- **$W_j$ letters** — algebraic kinematic functions

## 4 · CENTRAL IDEA

### Why Chebyshev series?

1. **Fast convergence**
   Analytic regions are captured accurately with relatively few coefficients.
2. **One series per interval**
   A single approximation represents the function throughout each path segment.
3. **Cheap to reuse**
   Evaluation and integration become fast once the coefficients are known.

## 5 · ADAPTIVE ALGORITHM

### Coefficients guide the computation

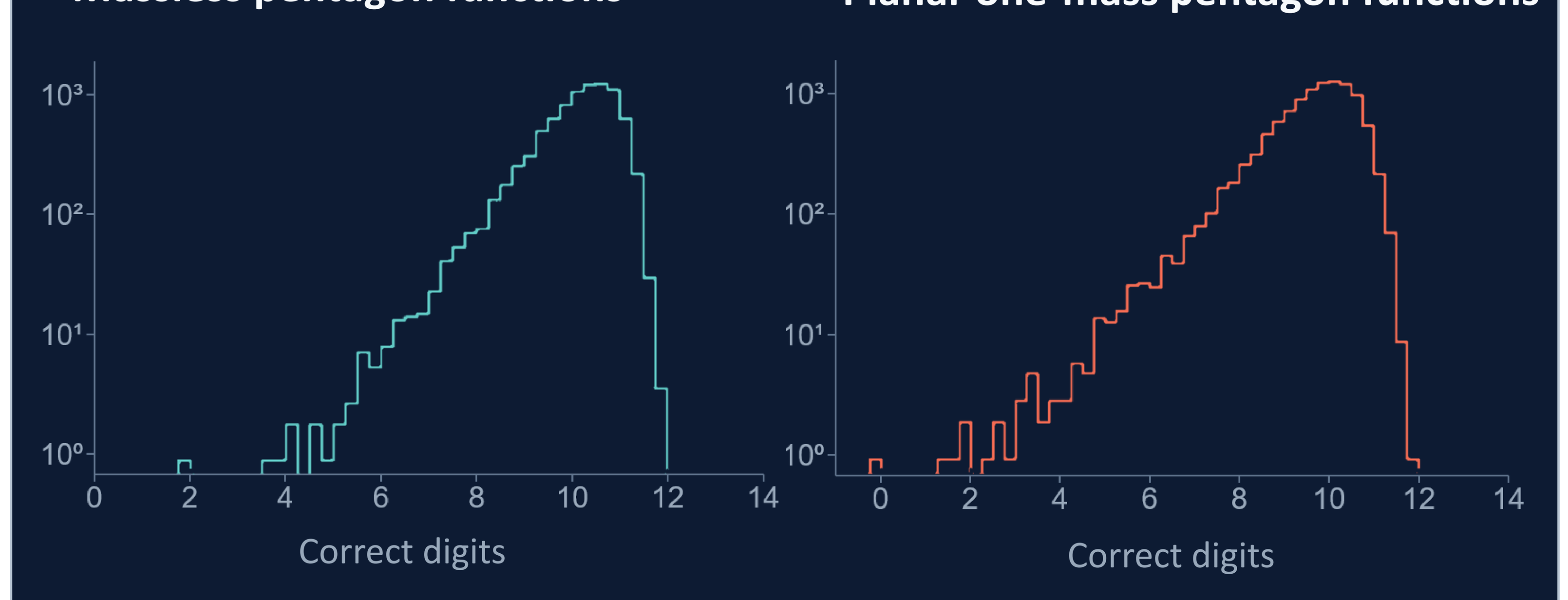
Adaptive sampling redirects computation toward regions with slower convergence [2-4].

START AT LOW ORDER → INSPECT COEFFICIENTS → CONVERGED? → YES: ACCEPT / NO: REFINE → BISECT/ INCREASE NUMBER OF NODES → INSPECT COEFFICIENTS

The approximation stays light in smooth regions and concentrates samples near difficult structures.

## 6 · RESULTS

### Agreement reaches about 10 correct digits




| Evaluation time per point | MEDIAN | MEAN | MAX |
|---|---|---|---|
| Massless pentagon | 0,35 s | 0,56 s | 6,5 s |
| Planar one-mass | 1,2 s | 1,45 s | 10,7 s |

- **≈10** correct digits
- **<2 s** median time
- **10 000** phase space points

## 7 · Conclusion and Future Work

**SCALABLE** — Fast, reusable approximations for large systems → **PHYSICAL** — Validated in physical relevant regions. → **COMPETITIVE** — Agreement with existing methods

Apply the framework to cutting-edge two-loop six-point massless integrals, building on recent results.

## TAKE-HOME MESSAGE

**Chebyshev series provide fast, reusable approximations.**

The coefficients diagnose convergence, allowing adaptivity to focus computation only where the analytic structure makes it necessary.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