Genuinely Quantum Steady-States of Open Quantum Systems

Introduction

- While closed quantum systems evolve accordingly to the Schrödinger Equation, open quantum systems evolve, most generally, according to the Lindblad Equation
- The effects of dissipation in a quantum system can change its behaviour dramatically
- Due to dissipation, most open quantum systems' steady states behave in an almost classical manner; by "classical" it is meant a state that has little quantum effects, an example of these states are the coherent spin states since they are product states, so they have no entanglement and they minimize the Heisenberg uncertainty
- One very interesting effect of specific environments is the ability to force a system into "quantumness"
- The aim of this work is to study the effects of Lindbladian dynamics in spin systems and how can they be driven to genuinely quantum states

The Lindblad Equation

the Lindblad Equation (1) is the most general evolution for a density matrix that preserves its trace and positivity. It is the sum of the Schrödinger Equation (2) with the dissipative term, that can be used to model an environment with "jump operators" (Lk).

$$\frac{d\rho}{dt} = -\frac{i}{\hbar}[H,\rho] + \sum_k \left[L_k \rho L_k^\dagger - \frac{1}{2}\left(L_k^\dagger L_k \rho + \rho L_k^\dagger L_k\right)\right] \quad (1)$$

$$\frac{d\rho}{dt} = -\frac{i}{\hbar}[H,\rho] \quad (2)$$

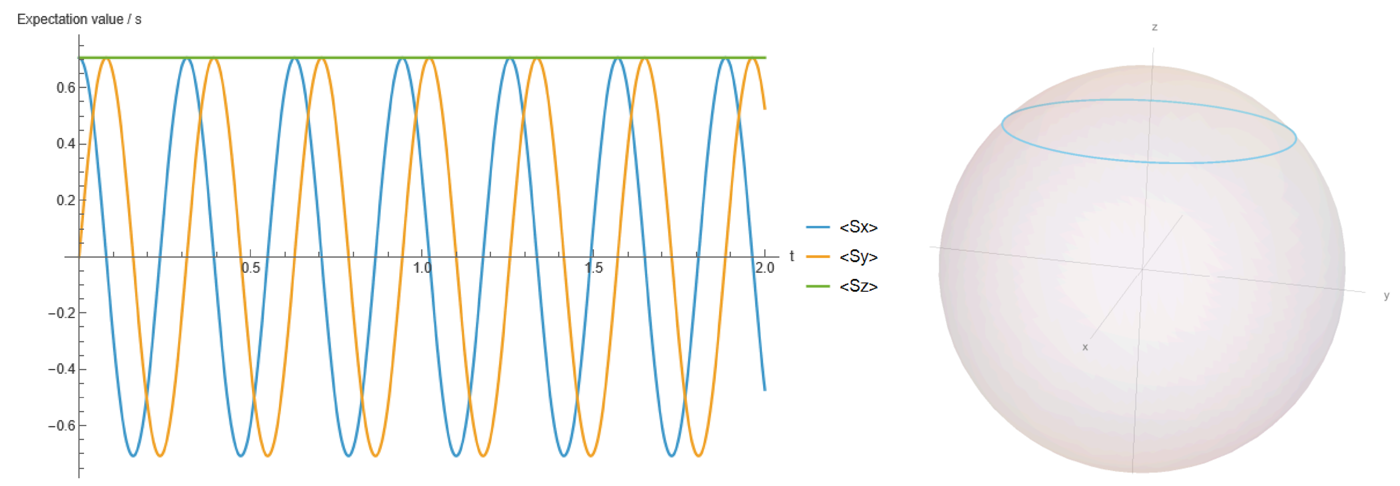
The Basic Effects of Dissipation

To better understand the effect of the dissipation term, we first consider a simple model, composed of N spin ½ non-interacting particles under a magnetic field, described by the Hamiltonian (3), where the **S** operators are the collective spin operators (4):

$$H = -hS_z \quad (3)$$

$$S_\alpha = \sum_i^N S_{\alpha_i} \quad (4)$$

Under Schrödinger Dynamics the system's expected values of **S** evolve as follows:



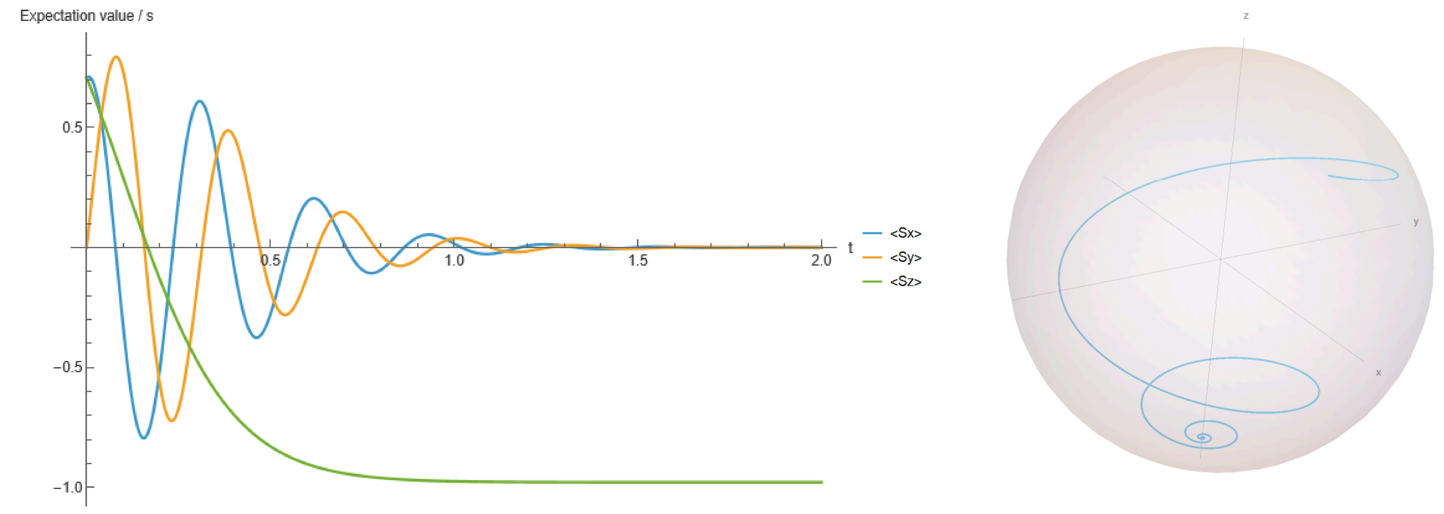
Unitary evolution of the system in Eq (3): h = 20, t = [0, 2]

The Basic Effects of Dissipation (cont.)

Using the same Hamiltonian (3), by choosing appropriate Jump Operators (5):

$$L_z = \sqrt{\gamma_z}S_z \quad L_+ = \sqrt{\gamma_+}S_+ \quad L_- = \sqrt{\gamma_-}S_- \quad (5)$$

Under Lindblad Dynamics the system's expected values of **S** evolve as follows:



LIndblad evolution of the system in Eq (3) with the operators (5): h =20, gz = 0.1, g+ = 0.1, g- = 1, t = [0, 2]

It is interesting to notice that the steady state of this dynamic is coherent and thus "classical-like". The effects of decoherence for Sx and Sy, and relaxation for Sz can be observed as well.

To be more precise, the state is considered classical or quantum according to its projection in the classical/quantum-like state (here for N=2):

$$\rho_{classical-like} = p_1|\alpha\rangle\langle\alpha| + p_2|\beta\rangle\langle\beta|$$

$$\rho_{quantum-like} = \frac{1}{2}(|\alpha\rangle + |\beta\rangle)(\langle\alpha| + \langle\beta|)$$

The Dissipative LMG Model

To study spin-spin interaction one can use the Lipkin-Meshkov-Glick model (6), where every particle interacts with each other and the **S** are given by the collective spin operators (4):

$$H = -hS_z - \frac{1}{2s}\left(\gamma_x S_x^2 + \gamma_y S_y^2\right) \quad (6)$$

By engineering some appropriate jump operators, it is possible to obtain a system that cannot be approximated classically even in the large spin limit (equivalent to system size).

$$L_+ = \sqrt{\frac{\Gamma}{(2s)^3}}S_+(S_z - m) \quad L_- = \sqrt{\frac{\Gamma}{(2s)^3}}S_-(S_z - m) \quad (7)$$

The Semi-Classical Approximation

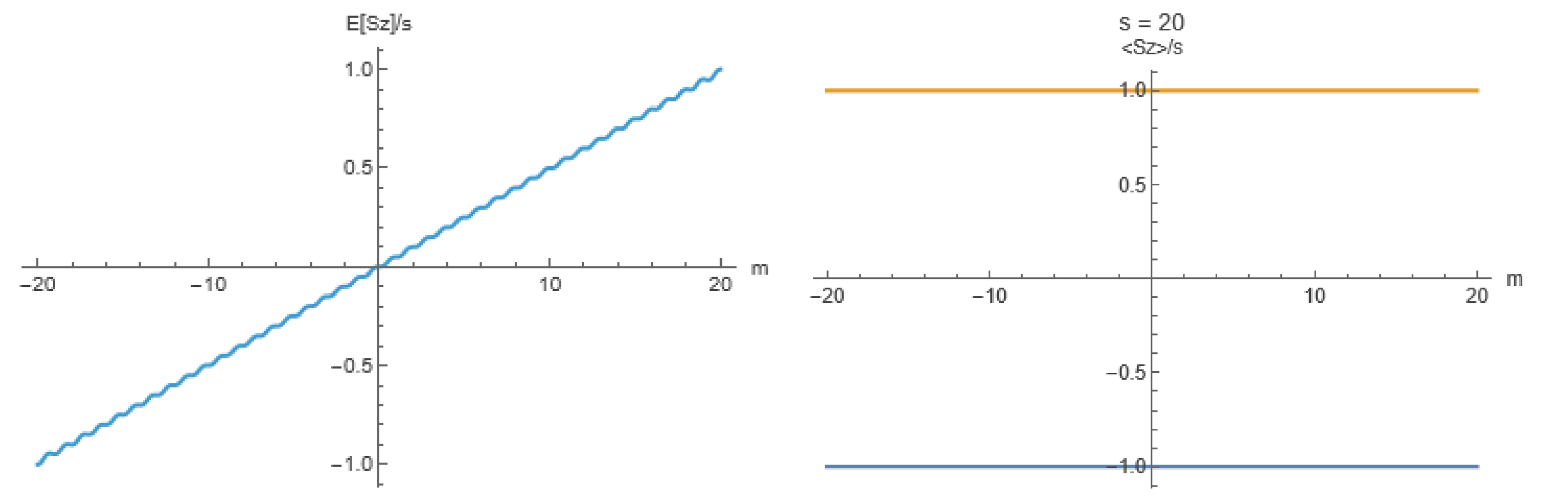
Because the Lindblad Equation is generally difficult to solve, a semi-classical approach can be employed. Specifically, we truncate the hierarchy of Lindblad Equations for the expected value of the spin operators.

$$\left\langle \frac{S_\alpha}{s}\frac{S_\beta}{s} \right\rangle \approx \left\langle \frac{S_\alpha}{s} \right\rangle \left\langle \frac{S_\beta}{s} \right\rangle + O\left(\frac{1}{s}\right) \quad (8)$$

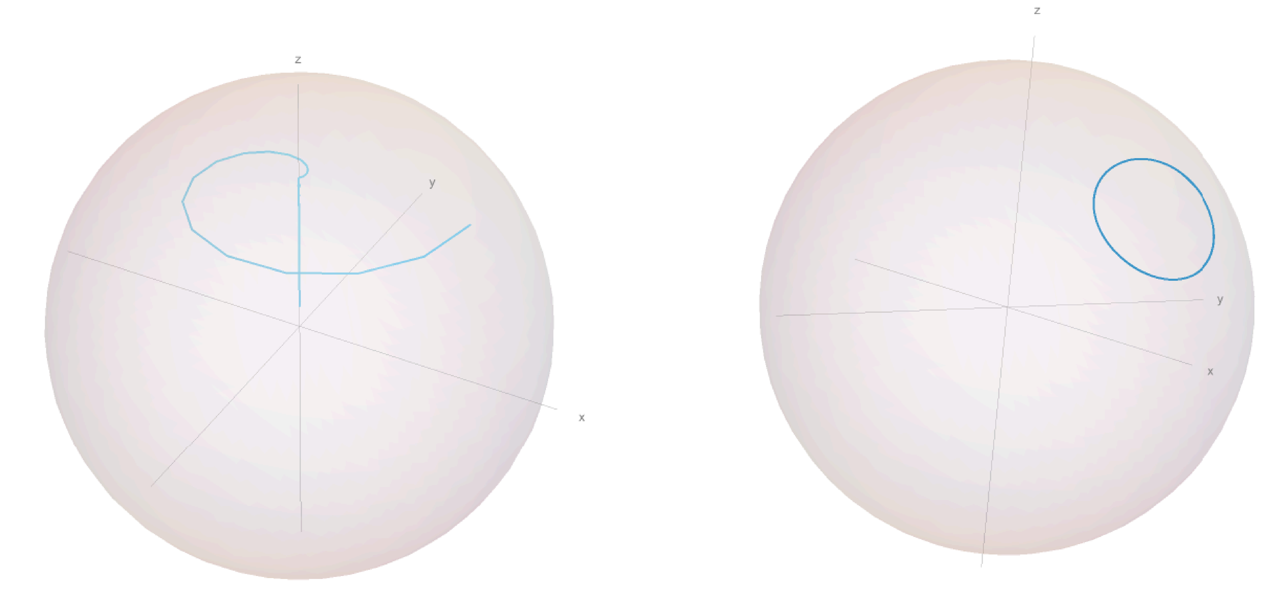
Which is valid in the large S limit for states with minimum uncertainty.

The Results

The steady state of the system (6) & (7) was obtained exactly and using the semi-classical approximation:



Steady state of the system varying with the value of m = [-s, s] (6) & (7): h = 1, gx = 1, gy = 1, s = 20, G = 1; Exact on the left, Approximate on the right



LIndblad evolution of the system: s = 15, m = 1, h = 10, t = [0, 600], gx = 10, gy = 10; Exact on the left; Approximate on the right

Conclusions

- It is clear that the usual unitary study of quantum dynamics is far from reality in most observed collective processes as the presence of an environment can drastically change the properties of a quantum system
- Specifically, the introduction of dissipation in a many-body system can make it have classical-like states, with low uncertainty and entanglement, or quantum-like states, with high uncertainty or entanglement
- This work can be extended to study the transitions between classical-like, coherent steady states and quantum-like steady states, as well as the failure of the semi-classical description in such systems, which is a major challenge in the area
- This study can be specially important for practical aplications in quantum metrology and nanotechnology

References

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