Solving stiff dark matter equations via Jacobian Normalization with Physics-Informed Neural Networks

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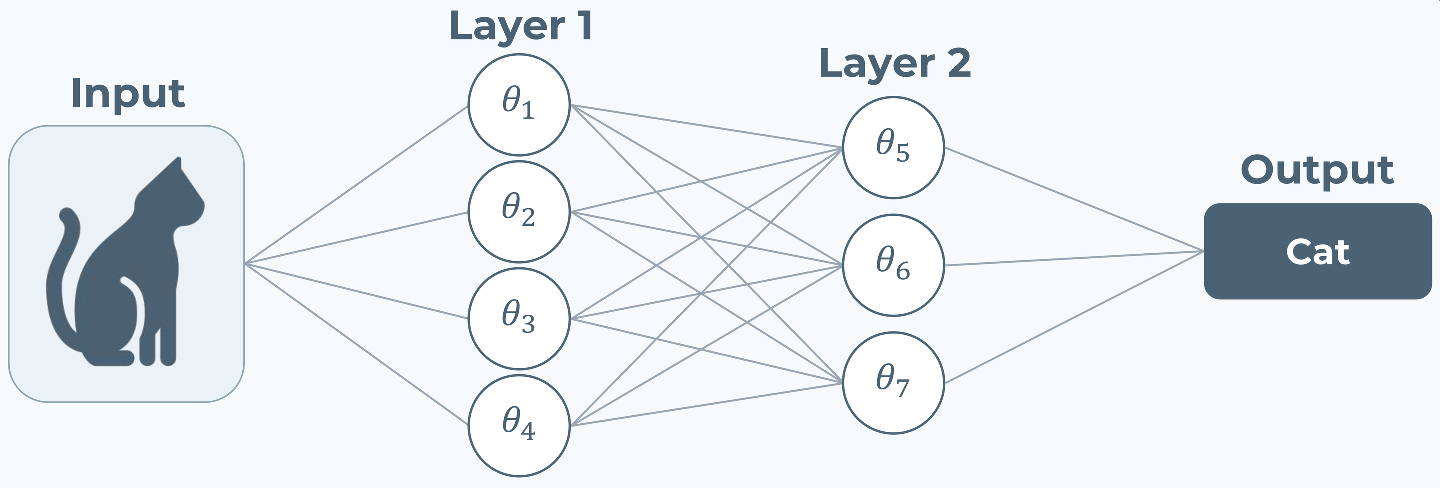
The problems

What is a neural network (NN)?

- It passes an input through layers of connected nodes, each carrying its own parameters θ
- Training tunes θ to shrink the gap between output and target
- Show it photos labelled "cat" or "not a cat" and it learns to spot cats it has never seen



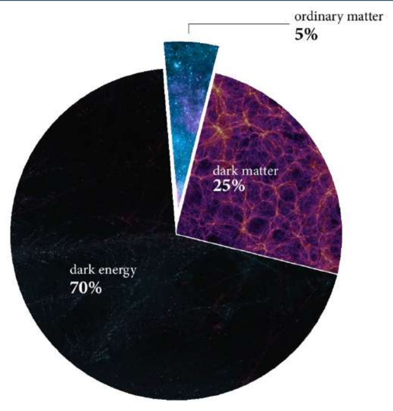
Train the NN to satisfy a differential equation instead of matching labels, and it is called a PINN

Stiffness breaks PINNs

For systems with widely separated time scales, gradients associated with fast-decaying modes dominate backpropagation, suppressing the learning of slower modes and frequently **leading to convergence failure**

Dark Matter (DM)

Initially in thermal equilibrium with the SM plasma, WIMPs undergo **thermal freeze-out** as the Universe expands and cools, and their yield evolves according to a stiff, nonlinear **Boltzmann equation**



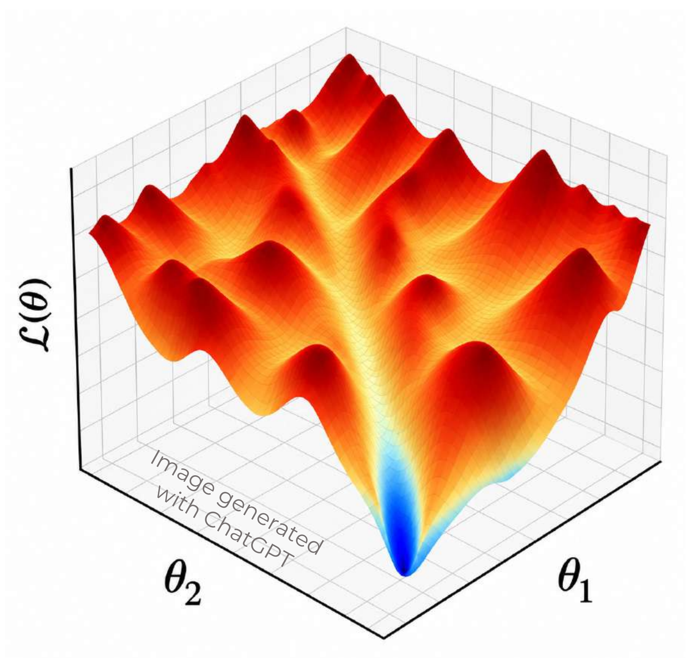
The solution

Jacobian Normalization of the Loss Residuals

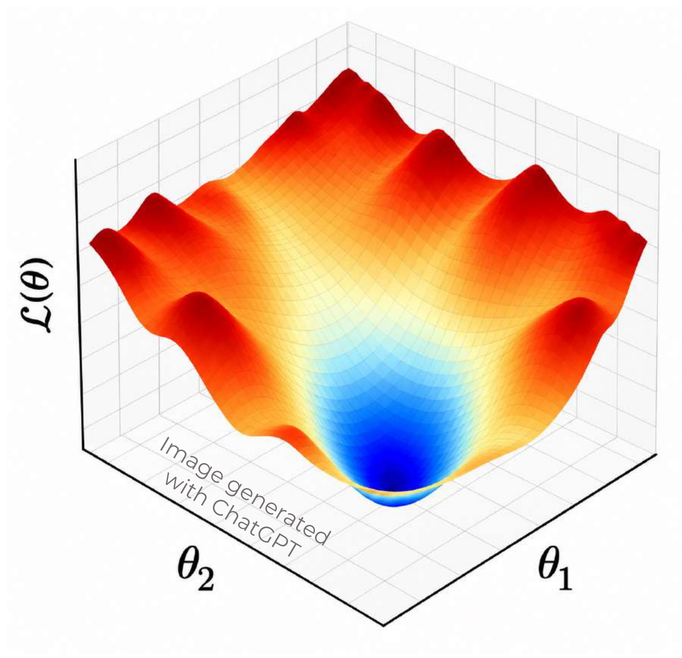
A simple rescaling removes the growth of the loss **Hessian eigenvalues** $|\lambda_{max}| \sim \Theta(J_y^{2+\alpha})$ and restores convergence:

$$\mathcal{L}_r = \frac{1}{m}\sum\left[\frac{\partial y}{\partial x} - f(y,x)\right]^2 \longrightarrow \boxed{\frac{1}{m}\sum\frac{\left[\frac{\partial y}{\partial x} - f(y,x)\right]^2}{1+J_y^2}}$$

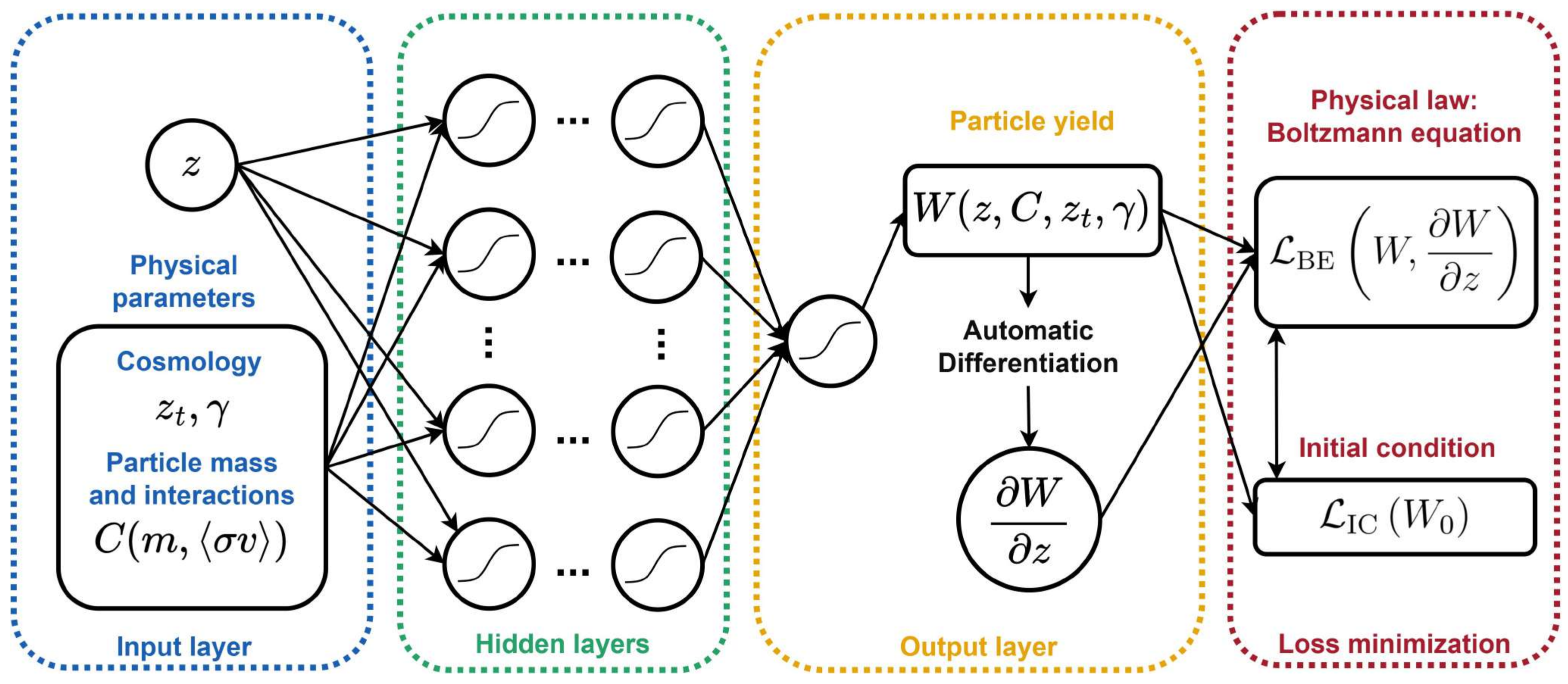
We introduce a principled normalization of loss residuals based on the Jacobian of the governing differential equation, mitigating stiffness without extra hyperparameters



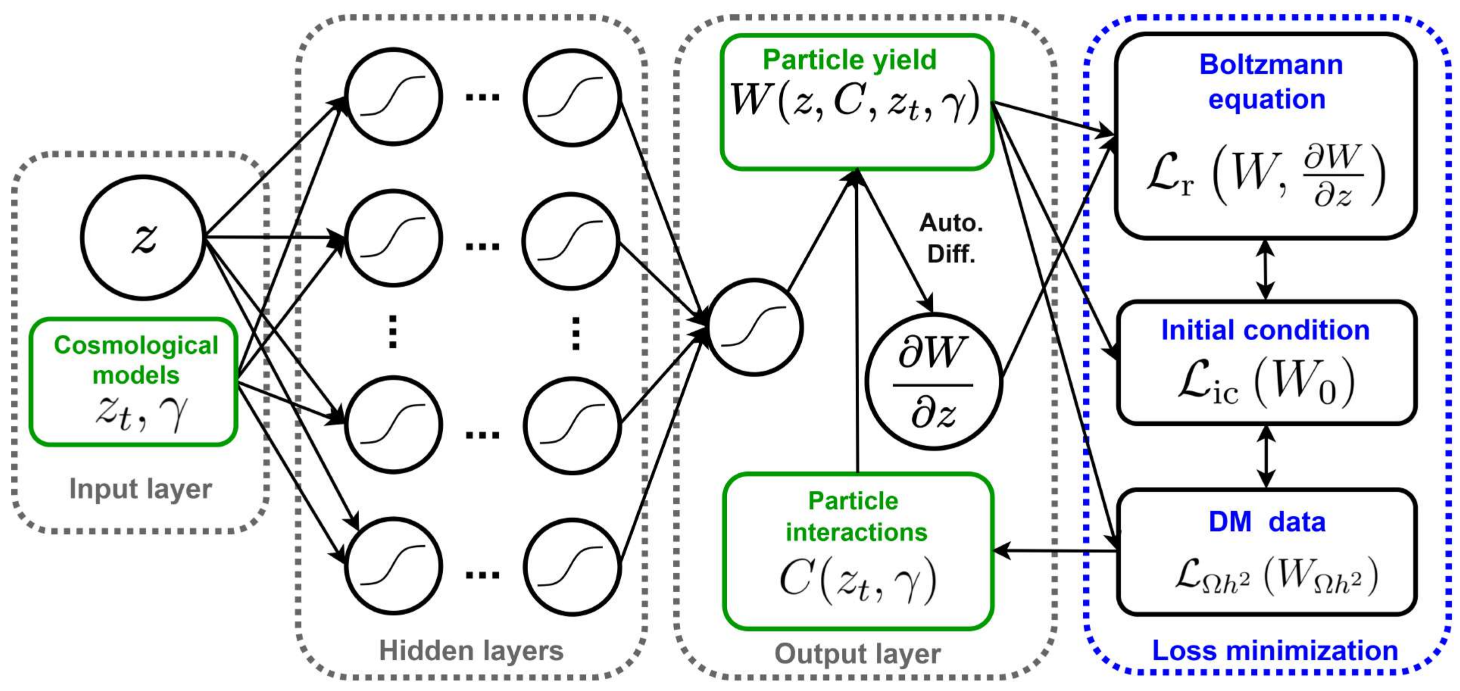
Jacobian normalization widens the minima, enabling **larger learning rates** and **stable convergence**



Forward PINN

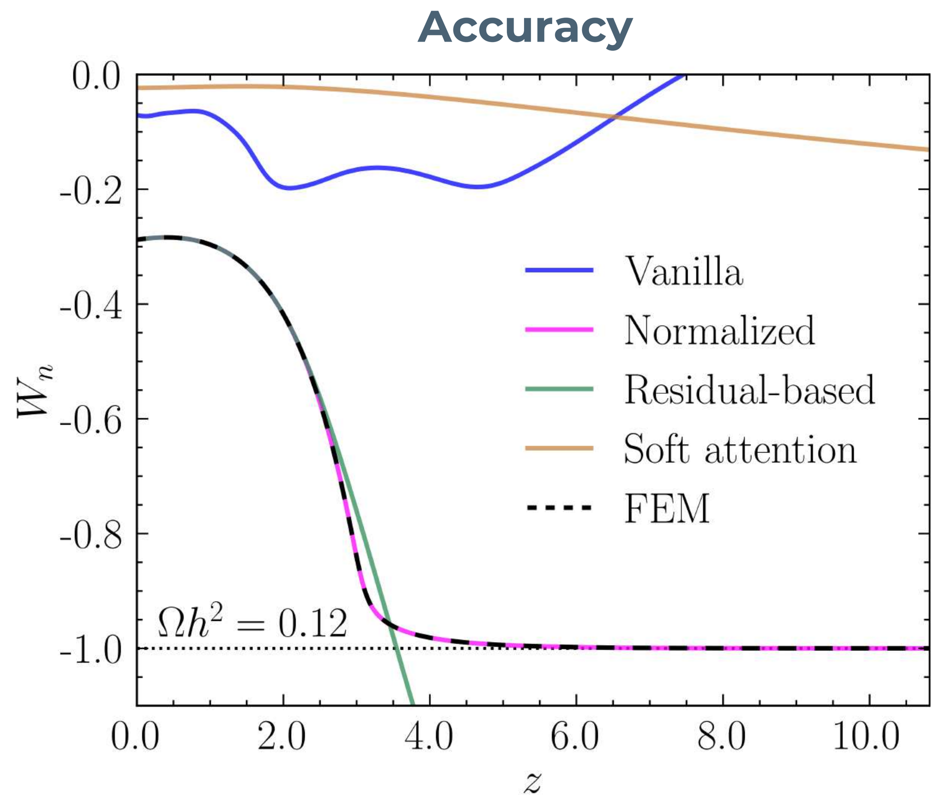


Inverse PINN



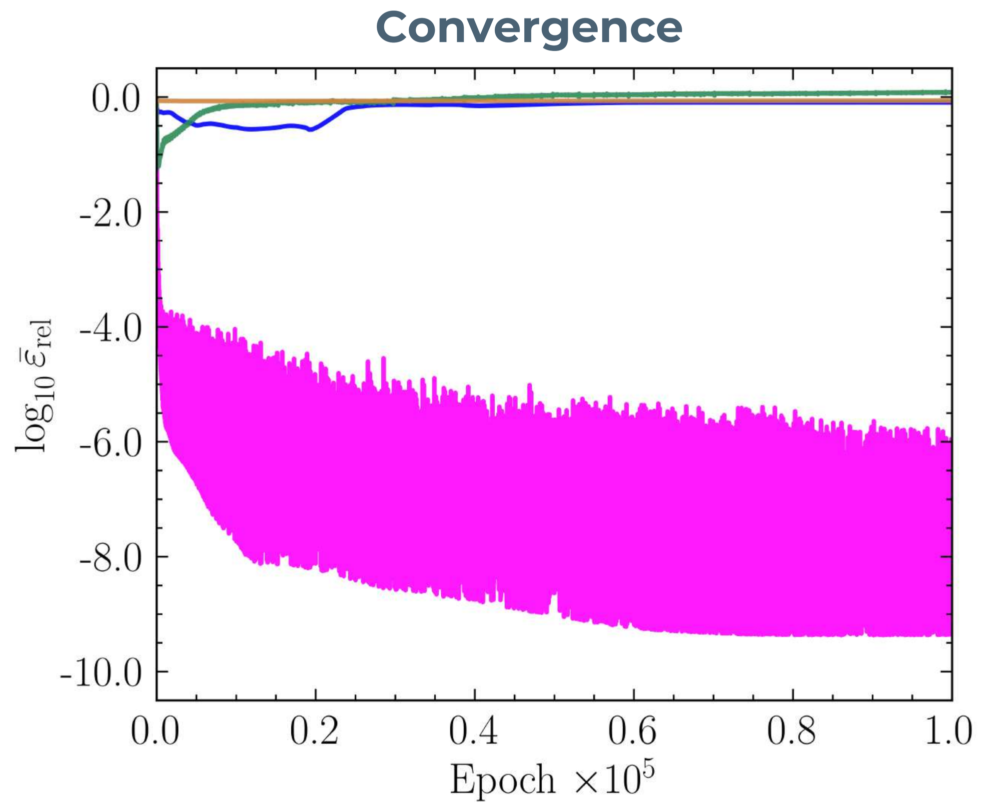
The results

Accuracy



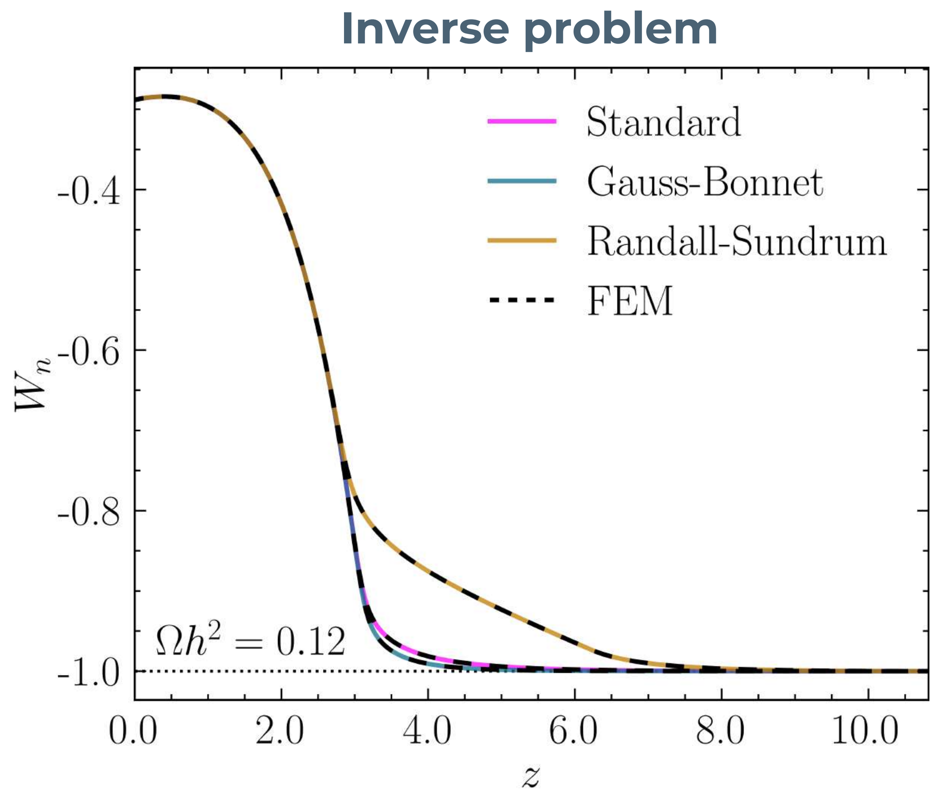
Only the Jacobian-normalized PINN reproduces the FEM result

Convergence



The relative error reaches $\bar{\varepsilon}_{rel} \sim 10^{-8}$ after only ~10^4 epochs

Inverse problem



Using only the observed DM relic density as input, the **network inferred** $\langle\sigma v\rangle$

Full paper

arXiv: 2602.21988

Jacobian-based normalization provides a simple way to enhance the robustness of PINNs in stiff regimes, paving the way for more accurate and efficient modeling of dynamical systems at the interface of particle physics and cosmology

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