SEARCHING FOR HIGHER PRECISION QUANTUM GRAVIMETERS

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Abstract

Quantum gravimeters estimate gravitational acceleration from the phase accumulated by freely falling atomic matter waves [1]. This work develops a density-matrix framework for modelling Raman-pulse atom interferometers with realistic noise, and uses Quantum Fisher Information to compare independent and entangled probes. For two atoms, Bell-state entanglement doubles the QFI relative to independent probes, reducing the estimator variance bound by a factor of two.

Why measure gravity with higher precision?

Precise measurements of the local gravitational acceleration (g) are relevant for:

- Geophysics: subsurface density variations, mapping natural resources and groundwater monitoring.
- Navigation: inertial sensing without external signals
- Fundamental physics: precision tests of gravity and quantum mechanics

Classical gravimeters are highly precise, but not absolute and suffer from drift requiring calibration [2].

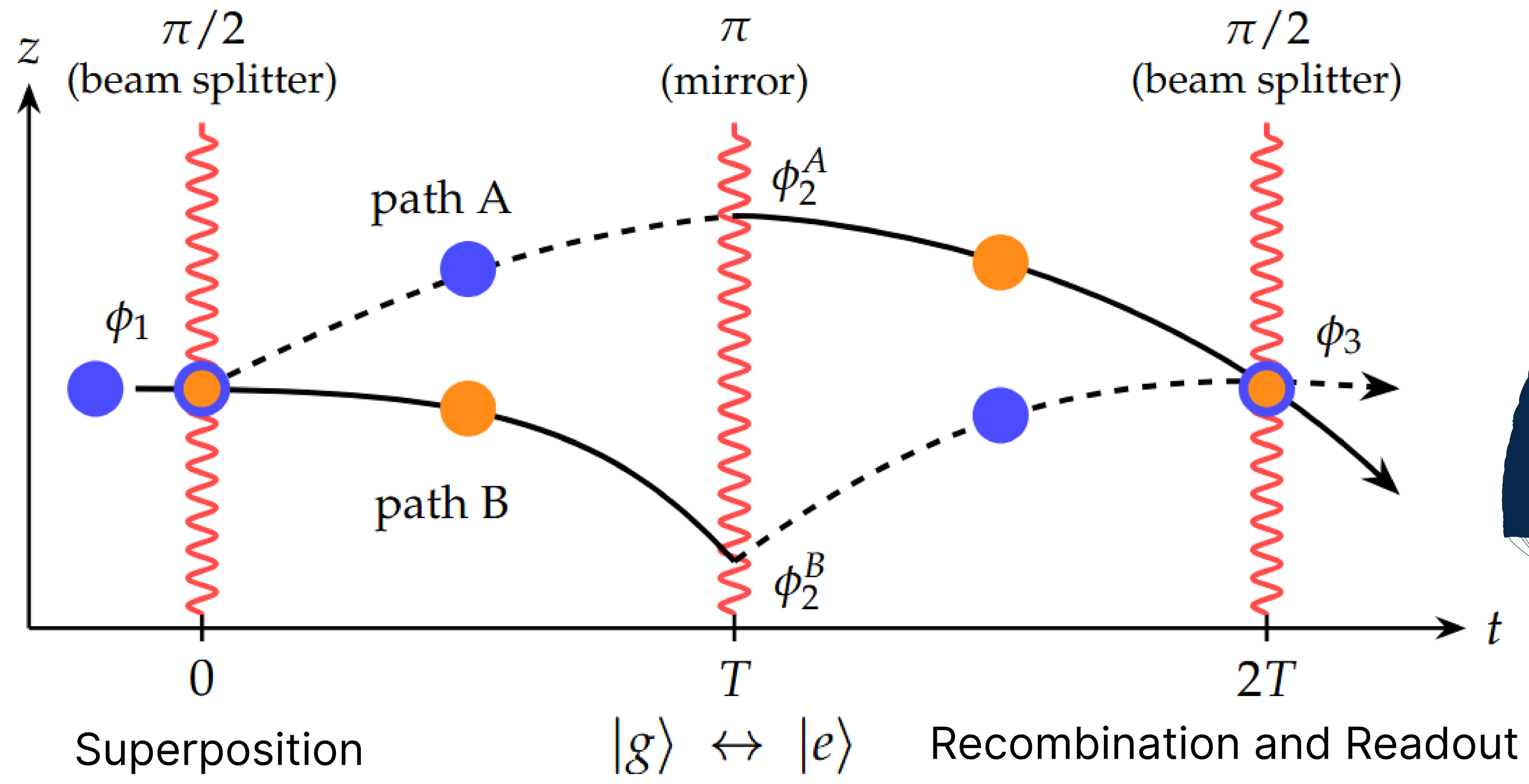
From classical test masses to matter waves

In an atom-interferometric gravimeter, ultracold atoms in free fall behave as coherent matter waves and are manipulated by Raman laser pulses. These form a Mach–Zehnder interferometer through a π/2 – π – π/2 pulse sequence [3].

The gravitational acceleration is encoded in the phase:

$$\Phi_g = k_{\text{eff}} g T^2$$

where k_{eff} is the effective Raman wavevector, g is the gravitational acceleration, and T is the time between pulses.



Raman pulses as quantum gates

The two hyperfine states of ^{87}Rb define an effective atomic qubit:

$|0\rangle \equiv |F=1, m_F=0; p\rangle$ and $|1\rangle \equiv |F=2, m_F=0; p+\hbar k_{eff}\rangle$

Raman pulses coherently drive transitions between these states [4] and implement unitary quantum gates.

Beam-splitter pulse:

$$U_{\pi/2}(\phi_L) = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -ie^{-i\phi_L} \\ -ie^{i\phi_L} & 1 \end{pmatrix}$$

Mirror pulse:

$$U_{\pi}(\phi_L) = \begin{pmatrix} 0 & -ie^{-i\phi_L} \\ -ie^{i\phi_L} & 0 \end{pmatrix}$$

After the full Mach–Zehnder sequence, the output populations oscillate with the total interferometric phase Φ_T as:

$$P_{g/e}(\phi_T) = \tfrac{1}{2}(1 \pm \cos\phi_T) \quad \text{with} \quad \Phi_T = k_{\text{eff}} g T^2 + (\phi_1 - 2\phi_2 + \phi_3)$$

Realistic modelling with density matrices

Environmental noise and imperfect operations transform pure states into mixed states, described by a density matrix ρ. Each operation is modelled as an ideal unitary followed by a noise channel: $\rho \mapsto \varepsilon(U\rho U^\dagger)$.

Noise channel	Physical origin	Effect
Dephasing	Laser phase noise	Suppresses coherences
Amplitude damping	Spontaneous decay	Transfers population from |e⟩ → |g⟩
Depolarizing noise	Pulse/readout errors	Models white noise behaviour of operations

Noise reduces the interferometric visibility (with 0≤V≤1):

$$P_g(\phi_T) = \tfrac{1}{2}(1 + \mathcal{V}\cos\phi_T) \text{ and } P_e(\phi_T) = \tfrac{1}{2}(1 - \mathcal{V}\cos\phi_T)$$

The gravimeter as a quantum sensor

A quantum sensing protocol follows three stages:

State preparation → Parameter encoding → Measurement

The parameter g is not measured directly. Instead, repeated measurements produce outcomes whose statistics depend on the gravitational phase, which allow to build an estimator: $\hat{g}$.

The relevant question is not only whether the estimator converges, but how fast it converges.

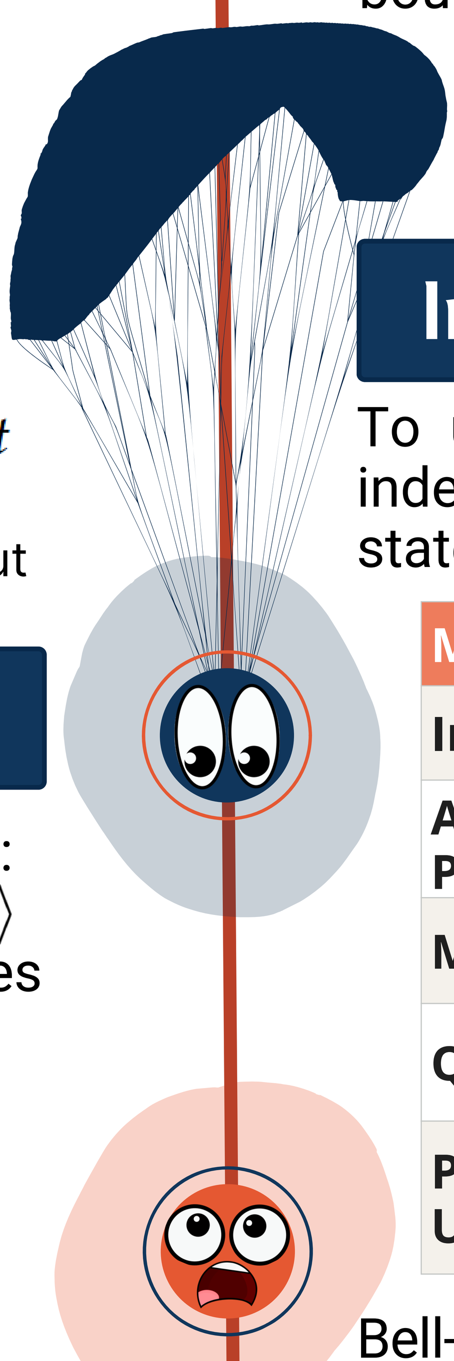
QFI as the precision metric

The Quantum Fisher Information (QFI) quantifies the maximum information that a quantum state can provide about a parameter, optimized over all possible measurements.

Given a number of samples *n*, it sets the quantum Cramér–Rao bound:

$$\text{Var}(\hat{\theta}) \geq \frac{1}{n\mathcal{F}_Q}$$

Higher QFI → Lower estimator variance → Higher precision



Independent vs. entangled probes

To understand the advantage of entanglement, we compare N independent atoms against N maximally entangled atoms (Bell states for N=2):

Metric / Property	Independent Probes	Entangled Probes (Bell)
Initial State	|+⟩ = (|0⟩ + |1⟩)/√2	|Φ+⟩ = (|00⟩+|11⟩)/√2
Accumulated Phase	Separately by each atom	Collectively by the pair
Measured Signal	2cos(θ)	cos(2θ)
QFI ($\mathcal{F}_Q$)	$\mathcal{F}_Q^{ind} = 2$	$\mathcal{F}_Q^{Bell} = 4 = 2\mathcal{F}_Q^{ind}$
Phase Uncertainty	Δϕ ∝ 1/√N **(SQL)**	Δϕ ∝ 1/N **(Heisenberg Limit)**

Bell-state entanglement doubles the QFI per sample, halves the estimator variance, and improves phase uncertainty by a factor of √2.

Conclusions and roadmap

Conclusions

- Atom interferometers encode gravitational acceleration as a quantum phase.
- Density-matrix framework to model different pulse sequences with quantum noise.
- For two probes, Bell-state entanglement doubles the QFI relative to independent probes.

Roadmap

- Study alternative pulse sequences and noise propagation with the developed framework.
- Find the noise threshold where entanglement stops being advantageous.
- Extend the QFI analysis to mixed states and larger entangled systems
- Assess entanglement-generation and joint-readout schemes.

References:

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