Flavour Invariants of the N Higgs Doublet Model

João C. Belas[1,2*], João P. Silva[1]

[1]Departamento de Física and CFTP, Instituto Superior Técnico

[2]Masters in Physics and Technological Engineering

*joao.belas@tecnico.ulisboa.pt

The N Higgs Doublet Model Yukawa Lagrangian

$$-\mathcal{L}_Y = \overline{Q_L}\Gamma_k\Phi_k n_R + \overline{Q_L}\Delta_k\tilde{\Phi}_k p_R + h.c., \qquad \overline{Q_L} = \begin{pmatrix} \overline{p_L} \\ \overline{n_L}\end{pmatrix}, \qquad k = \{1, \dots, N\},$$

Rotating the **scalar doublets** to the **Charged Higgs Basis,** by a matrix $\mathcal{S}$, they can be parametrised as

$$\begin{pmatrix}\mathcal{H}_0\\ \vdots \\ \mathcal{H}_{N-1}\end{pmatrix} = \mathcal{S}\begin{pmatrix}\Phi_1\\ \vdots \\ \Phi_N\end{pmatrix}, \qquad \begin{pmatrix}\mathrm{M_d}\\ \mathrm{N_{d1}}\\ \vdots \\ \mathrm{N_{dN-1}}\end{pmatrix} = v\mathcal{S}\begin{pmatrix}\Gamma_1\\ \Gamma_2\\ \vdots \\ \Gamma_N\end{pmatrix}, \qquad \begin{pmatrix}\mathrm{M_u}\\ \mathrm{N_{u1}}\\ \vdots \\ \mathrm{N_{uN-1}}\end{pmatrix} = v\mathcal{S}\begin{pmatrix}\Delta_1\\ \Delta_2\\ \vdots \\ \Delta_N\end{pmatrix},$$

$$\mathcal{H}_0 = \begin{pmatrix} \mathrm{G^+} \\ \frac{1}{\sqrt{2}}(\mathrm{v} + \mathrm{H^0} + \mathrm{iG^0})\end{pmatrix}, \qquad \mathcal{H}_k = \begin{pmatrix} \mathrm{H_k^+} \\ \frac{1}{\sqrt{2}}(\mathrm{R_k} + \mathrm{iI_k})\end{pmatrix}, \qquad k = \{1, \dots, N-1\}.$$

The **quark fields** can also be rotated into their **mass basis,**

$$\mathrm{n_L} = \mathrm{V_{dL}d_L}, \quad \mathrm{n_R} = \mathrm{V_{dR}d_R}, \quad \mathrm{p_L} = \mathrm{V_{uL}u_L}, \quad \mathrm{p_R} = \mathrm{V_{uR}u_R}, \quad V = \mathrm{V_{uL}^\dagger V_{dL}},$$
$$\mathrm{M_d} = \mathrm{V_{dL}D_dV_{dR}^\dagger}, \mathrm{D_d} = \mathrm{diag}\{\mathrm{m_d, m_s, m_b}\}, \qquad \mathrm{M_u} = \mathrm{V_{uL}D_uV_{uR}^\dagger}, \mathrm{D_u} = \mathrm{diag}\{\mathrm{m_u, m_c, m_t}\},$$

where V is the **CKM matrix,** such that the Lagrangian becomes[1]

$$-\mathcal{L}_Y = (\overline{\mathrm{d_L}}\mathrm{D_d d_R} + \overline{\mathrm{u_L}}\mathrm{D_u u_R}) + \frac{\mathrm{H^0}}{\mathrm{v}}(\overline{\mathrm{d_L}}\mathrm{D_d d_R} + \overline{\mathrm{u_L}}\mathrm{D_u u_R}) +$$
$$\frac{\mathrm{R_k}}{\mathrm{v}}(\overline{\mathrm{d_L}}\mathrm{V_{dL}^\dagger N_{dk} V_{dR} d_R} + \overline{\mathrm{u_L}}\mathrm{V_{uL}^\dagger N_{uk} V_{uR} u_R}) + i\frac{\mathrm{I_k}}{\mathrm{v}}(\overline{\mathrm{d_L}}\mathrm{V_{dL}^\dagger N_{dk} V_{dR} d_R} - \overline{\mathrm{u_L}}\mathrm{V_{uL}^\dagger N_{uk} V_{uR} u_R}) +$$
$$\sqrt{2}\frac{\mathrm{H_k^+}}{\mathrm{v}}(\overline{\mathrm{u_L}}\mathrm{V_{uL}^\dagger N_{dk} V_{dR} d_R} - \overline{\mathrm{u_R}}\mathrm{V_{uR}^\dagger N_{uk}^\dagger V_{dL} d_L}) + \mathrm{h.c.}.$$

Physical Parameters

Looking at the **decomposition** of the **mass matrix** $\mathrm{M_d}$ in detail,

$$\mathrm{M_d} = \mathrm{V_{dL}D_dV_{dR}^\dagger} =$$

$$\{1, \alpha_1^{\mathrm{dL}}, \alpha_2^{\mathrm{dL}}\}\boldsymbol{R}_{23}^{\mathrm{dL}}\{-\delta^{\mathrm{dL}}, 1,1\}\boldsymbol{R}_{13}^{\mathrm{dL}}\{\delta^{\mathrm{dL}}, 1,1\}\boldsymbol{R}_{12}^{\mathrm{dL}}\{\alpha_3^{\mathrm{dL}}, \alpha_4^{\mathrm{dL}}, \alpha_5^{\mathrm{dL}}\}\mathrm{diag}\{\mathrm{m_d, m_s, m_b}\}$$
$$\{-\alpha_3^{\mathrm{dR}}, -\alpha_4^{\mathrm{dR}}, -\alpha_5^{\mathrm{dR}}\}\boldsymbol{R}_{-23}^{\mathrm{dR}}\{-\delta^{\mathrm{dR}}, 1,1\}\boldsymbol{R}_{-13}^{\mathrm{dR}}\{\delta^{\mathrm{dR}}, 1,1\}\boldsymbol{R}_{-12}^{\mathrm{dR}}\{1, -\alpha_1^{\mathrm{dR}}, -\alpha_2^{\mathrm{dR}}\},$$

the phases α_3^{dL} and α_3^{dR} only appear in the **combination** $\alpha_3^{\mathrm{dL}} - \alpha_3^{\mathrm{dR}}$, and likewise for α_4 and α_5. Therefore, one of each can be chosen as **an arbitrary parameter**. It is convenient to choose the **left phases as arbitrary,** both in the **down and up sectors.** Then, looking at the CKM matrix in detail,

$$V = \mathrm{V_{uL}^\dagger V_{dL}} =$$

$$\{-\alpha_3^{\mathrm{dR}}, -\alpha_4^{\mathrm{dR}}, -\alpha_5^{\mathrm{dR}}\}\boldsymbol{R}_{-23}^{\mathrm{dR}}\{-\delta^{\mathrm{dR}}, 1,1\}\boldsymbol{R}_{-13}^{\mathrm{dR}}\{\delta^{\mathrm{dR}}, 1,1\}\boldsymbol{R}_{-12}^{\mathrm{dR}}\{1, -\alpha_1^{\mathrm{dR}}, -\alpha_2^{\mathrm{dR}}\}$$
$$\{1, \alpha_1^{\mathrm{dL}}, \alpha_2^{\mathrm{dL}}\}\boldsymbol{R}_{23}^{\mathrm{dL}}\{-\delta^{\mathrm{dL}}, 1,1\}\boldsymbol{R}_{13}^{\mathrm{dL}}\{\delta^{\mathrm{dL}}, 1,1\}\boldsymbol{R}_{12}^{\mathrm{dL}}\{\alpha_3^{\mathrm{dL}}, \alpha_4^{\mathrm{dL}}, \alpha_5^{\mathrm{dL}}\} =$$

$$\{-\alpha_3^{\mathrm{dR}}, -\alpha_4^{\mathrm{dR}}, -\alpha_5^{\mathrm{dR}}\}\{1, \alpha_1, \alpha_2\}\boldsymbol{R}_{23}\{-\delta, 1,1\}\boldsymbol{R}_{13}\{\delta, 1,1\}\boldsymbol{R}_{12}\{\alpha_3, \alpha_4, \alpha_5\}\{\alpha_3^{\mathrm{dL}}, \alpha_4^{\mathrm{dL}}, \alpha_5^{\mathrm{dL}}\}.$$

The arbitrary phases can be used to **cancel** the outer phases, leaving the CKM matrix with **1 phase, δ.** The same reasoning can be applied to the **C_{dLk}, C_{dRk}, C_{uLk}, and C_{uRk}, matrices,** whose results are summarised in the following table.

Matrix	Magnitudes	Phases
D_u/D_d	3	0
E_{uk}/E_{dk}	3	0
V	3	1
C_{uLk}/C_{dLk}	3	3
C_{uRk}/C_{dRk}	3	6

A New Way of Looking at Flavour Physics

The same **decomposition** of the mass matrices $\mathrm{M_d, M_u}$, can be done to the other Yukawa matrices, $\mathrm{N_{dk}, N_{uk}}$,

$$\mathrm{N_{dk}} = \mathrm{U_{dLk}E_{dk}U_{dRk}^\dagger}, \qquad \mathrm{E_{dk}} = diag\{\mathrm{n_{d_1k}, n_{d_2k}, n_{d_3k}}\},$$
$$\mathrm{N_{uk}} = \mathrm{U_{uLk}E_{uk}U_{uRk}^\dagger}, \qquad \mathrm{E_{uk}} = diag\{\mathrm{n_{u_1k}, n_{u_2k}, n_{u_3k}}\}.$$

Relevant **mixing matrices,** akin to the CKM matrix, that encode the **flavour changing neutral couplings (FCNC)** of the model, can be computed

$$\mathrm{C_{dLk}} = \mathrm{V_{dL}^\dagger U_{dLk}}, \qquad \mathrm{C_{dRk}} = \mathrm{V_{dR}^\dagger U_{dRk}}, \qquad \mathrm{C_{uLk}} = \mathrm{V_{uL}^\dagger U_{uLk}}, \qquad \mathrm{C_{uRk}} = \mathrm{V_{uR}^\dagger U_{uRk}},$$

simplifying the Lagrangian

$$-\mathcal{L}_Y = (\overline{\mathrm{d_L}}\mathrm{D_d d_R} + \overline{\mathrm{u_L}}\mathrm{D_u u_R}) + \frac{\mathrm{H^0}}{\mathrm{v}}(\overline{\mathrm{d_L}}\mathrm{D_d d_R} + \overline{\mathrm{u_L}}\mathrm{D_u u_R}) +$$
$$\frac{\mathrm{R_k}}{\mathrm{v}}(\overline{\mathrm{d_L}}\boldsymbol{C_{dLk}E_{dk}C_{dRk}^\dagger}\mathrm{d_R} + \overline{\mathrm{u_L}}\boldsymbol{C_{uLk}E_{uk}C_{uRk}^\dagger}\mathrm{u_R})$$
$$+ i\frac{\mathrm{I_k}}{\mathrm{v}}(\overline{\mathrm{d_L}}\boldsymbol{C_{dLk}E_{dk}C_{dRk}^\dagger}\mathrm{d_R} - \overline{\mathrm{u_L}}\boldsymbol{C_{uLk}E_{uk}C_{uRk}^\dagger}\mathrm{u_R}) +$$
$$\sqrt{2}\frac{\mathrm{H_k^+}}{\mathrm{v}}(\overline{\mathrm{u_L}}V\boldsymbol{C_{dLk}E_{dk}C_{dRk}^\dagger}\mathrm{d_R} - \overline{\mathrm{u_R}}\boldsymbol{C_{uRk}E_{uk}C_{uLk}^\dagger}V\mathrm{d_L}) + \mathrm{h.c.},$$

and allowing for the **interpretation** of the interaction between the quarks and the $\mathcal{H}_k$ scalar doublets as a **series of bases** in which the interaction would be **diagonal,** and **unitary mixing matrices** $\mathrm{C_{dLk}, C_{dRk}, C_{uLk}, C_{uRk}}$ that go **from the mass basis to those diagonal basis.**

Flavour Invariants

In the SM, **hermitian matrices** can be defined and the following **flavour invariants**

$$\mathrm{H_u} = \mathrm{M_uM_u^\dagger}, \qquad \mathrm{H_d} = \mathrm{M_dM_d^\dagger},$$
$$\mathrm{I_1^0} = \mathrm{Tr(H_u)}, \qquad \mathrm{I_2^0} = \mathrm{Tr(H_u^2)}, \qquad \mathrm{I_3^0} = \mathrm{Tr(H_u^3)},$$
$$\mathrm{I_4^0} = \mathrm{Tr(H_d)}, \qquad \mathrm{I_5^0} = \mathrm{Tr(H_d^2)}, \qquad \mathrm{I_6^0} = \mathrm{Tr(H_d^3)},$$
$$\mathrm{I_7^0} = \mathrm{Tr(H_uH_d)}, \qquad \mathrm{I_8^0} = \mathrm{Tr(H_u^2H_d)}, \qquad \mathrm{I_9^0} = \mathrm{Tr(H_uH_d^2)}, \qquad \mathrm{I_{10}^0} = \mathrm{Tr(H_u^2H_d^2)},$$

form a **basis** for the **parameter space** of the flavour sector[2]. In the NHDM it is useful to further define

$$\mathrm{J_{uk}} = \mathrm{N_{uk}N_{uk}^\dagger}, \quad \mathrm{J_{dk}} = \mathrm{N_{dk}N_{dk}^\dagger}, \quad \mathrm{A_u} = \mathrm{M_u^\dagger M_u}, \quad \mathrm{A_d} = \mathrm{M_d^\dagger M_d}, \quad \mathrm{B_{uk}} = \mathrm{N_{uk}^\dagger N_{uk}},$$
$$\mathrm{B_{dk}} = \mathrm{N_{dk}^\dagger N_{dk}}, \quad \mathrm{F_{uk}} = \mathrm{M_uN_{uk}^\dagger}, \quad \mathrm{F_{dk}} = \mathrm{M_dN_{dk}^\dagger}, \quad \mathrm{G_{uk}} = \mathrm{N_{uk}M_u^\dagger}, \quad \mathrm{G_{dk}} = \mathrm{N_{dk}M_d^\dagger},$$

and a **basis for the flavour sector of the NHDM** can be given by the SM flavour invariants, plus the **invariants given below,**

$$\mathrm{I_1^k} = \mathrm{Tr(J_{uk})}, \qquad \mathrm{I_2^k} = \mathrm{Tr(J_{uk}^2)}, \qquad \mathrm{I_3^k} = \mathrm{Tr(J_{uk}^3)},$$
$$\mathrm{I_4^k} = \mathrm{Tr(J_{dk})}, \qquad \mathrm{I_5^k} = \mathrm{Tr(J_{dk}^2)}, \qquad \mathrm{I_6^k} = \mathrm{Tr(J_{dk}^3)},$$
$$\mathrm{I_7^k} = \mathrm{Tr(H_uJ_{uk})}, \quad \mathrm{I_8^k} = \mathrm{Tr(H_u^2J_{uk})}, \quad \mathrm{I_9^k} = \mathrm{Tr(H_uJ_{uk}^2)}, \quad \mathrm{I_{10}^k} = \mathrm{Tr(H_u^2J_{uk}^2)},$$
$$\mathrm{I_{11}^k} = \mathrm{Tr(H_dJ_{dk})}, \quad \mathrm{I_{12}^k} = \mathrm{Tr(H_d^2J_{dk})}, \quad \mathrm{I_{13}^k} = \mathrm{Tr(H_dJ_{dk}^2)}, \quad \mathrm{I_{14}^k} = \mathrm{Tr(H_d^2J_{dk}^2)},$$
$$\mathrm{I_{15}^k} = \mathrm{Tr(A_uB_{uk})}, \quad \mathrm{I_{16}^k} = \mathrm{Tr(A_u^2B_{uk})}, \quad \mathrm{I_{17}^k} = \mathrm{Tr(A_uB_{uk}^2)}, \quad \mathrm{I_{18}^k} = \mathrm{Tr(A_u^2B_{uk}^2)},$$
$$\mathrm{I_{19}^k} = \mathrm{Tr(A_dB_{dk})}, \quad \mathrm{I_{20}^k} = \mathrm{Tr(A_d^2B_{dk})}, \quad \mathrm{I_{21}^k} = \mathrm{Tr(A_dB_{dk}^2)}, \quad \mathrm{I_{22}^k} = \mathrm{Tr(A_d^2B_{dk}^2)},$$
$$\mathrm{I_{23}^k} = \mathrm{Tr(J_{uk}H_d)}, \quad \mathrm{I_{24}^k} = \mathrm{Tr(J_{uk}^2H_d)}, \quad \mathrm{I_{25}^k} = \mathrm{Tr(J_{dk}H_u)}, \quad \mathrm{I_{26}^k} = \mathrm{Tr(J_{dk}^2H_u)},$$
$$\mathrm{I_{27}^k} = \frac{1}{2}(\mathrm{Tr(F_{uk}F_{dk})} + \mathrm{Tr(G_{dk}G_{uk})}), \qquad \mathrm{I_{28}^k} = \frac{1}{2i}(\mathrm{Tr(F_{uk}F_{dk})} - \mathrm{Tr(G_{dk}G_{uk})}),$$
$$\mathrm{I_{29}^k} = \frac{1}{2}(\mathrm{Tr(H_uF_{uk}F_{dk})} + \mathrm{Tr(G_{dk}G_{uk}H_u)}), \qquad \mathrm{I_{30}^k} = \frac{1}{2i}(\mathrm{Tr(H_uF_{uk}F_{dk})} - \mathrm{Tr(G_{dk}G_{uk}H_u)}),$$
$$\mathrm{I_{31}^k} = \frac{1}{2}(\mathrm{Tr(H_dF_{dk}F_{uk})} + \mathrm{Tr(G_{uk}G_{dk}H_d)}), \qquad \mathrm{I_{32}^k} = \frac{1}{2i}(\mathrm{Tr(H_dF_{dk}F_{uk})} - \mathrm{Tr(G_{uk}G_{dk}H_d)}),$$
$$\mathrm{I_{33}^k} = \frac{1}{2}(\mathrm{Tr(F_{uk}J_{uk}F_{dk})} + \mathrm{Tr(G_{dk}J_{uk}G_{uk})}), \qquad \mathrm{I_{34}^k} = \frac{1}{2i}(\mathrm{Tr(F_{uk}J_{uk}F_{dk})} - \mathrm{Tr(G_{dk}J_{uk}G_{uk})}),$$
$$\mathrm{I_{35}^k} = \frac{1}{2}(\mathrm{Tr(F_{dk}J_{dk}F_{uk})} + \mathrm{Tr(G_{uk}J_{dk}G_{dk})}), \qquad \mathrm{I_{36}^k} = \frac{1}{2i}(\mathrm{Tr(F_{dk}J_{dk}F_{uk})} - \mathrm{Tr(G_{uk}J_{dk}G_{dk})}).$$

Conclusions

The **flavour sector of the NHDM** can be parametrised in terms of **diagonal eigenvalue and unitary mixing matrices.**

The number of **physical parameters** is **consistent** with previous ways of looking at flavour physics.

Flavour Invariants can be used to describe the flavour sector of the NHDM in a **basis invariant way.**

References

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