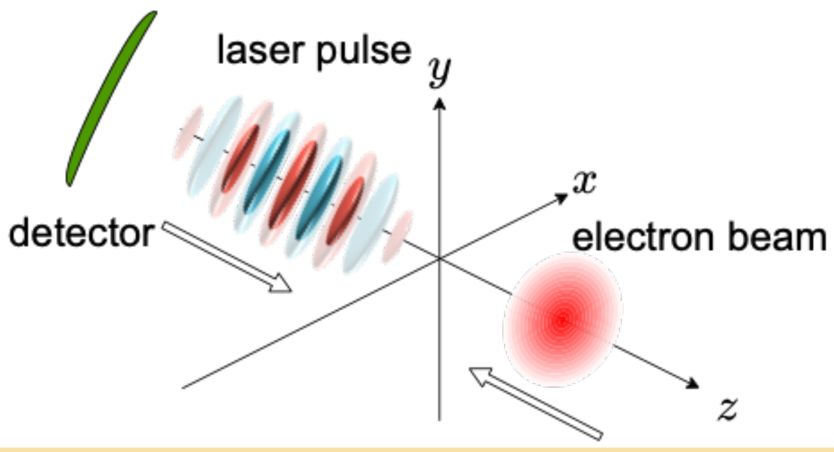
Structured Light from Generalised Thomson Scattering

R. Almeida[1*], John P. Palastro[2], J. Vieira[1]
[1] GoLP / Instituto de Plasmas e Fusão Nuclear, Instituto Superior Técnico, Lisbon, Portugal
[2] University of Rochester, Laboratory for Laser Energetics, Rochester, New York, 14623 USA
*rafaelrussoalmeida@tecnico.ulisboa.pt (Physics PhD Candidate)

Motivation

- **Structured light** is light whose degrees of freedom have been manipulated and coupled, for example, spatial and temporal. These spatiotemporal couplings were usually seen as a negative unwanted characteristic. However, accurate control of these couplings have brought a plethora of new applications such as:
  1. Particle acceleration with beams with angular momentum
  2. Brighter and more complex radiation sources
  3. High magnetic field generation
- On the other hand, **Thomson scattering** is a mechanism that can lead to great upshifts in the generated radiation frequency, but lacks spatiotemporal control



Here, we aim to use Thomson Scattering as an accurate and controllable light shaping tool.

Figure 1: Thomson scattering setup: relativistic particle beam colides with an intense laser, generating high frequency radiation in the beams's direction

Theoretical Model

- Starting from Maxwell's Equations for the electromagnetic fields, $\mathbf{E}$ and $\mathbf{B}$, that arise from a current distribution $\mathbf{J}$, using Fourier transforms as $\tilde{f}(\mathbf{k},\omega) = \iint f(\mathbf{r},t)\, e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)}\, d\mathbf{k}\, d\omega$ and assuming:
  1. the current given by $\mathbf{J}(\mathbf{r},t) \approx -Nq\, s(\mathbf{r}-c\boldsymbol{\beta}_0 t)\, \mathbf{v}(\mathbf{r},t)$, with $s(\mathbf{r})$ its constant shape
  2. the particles conserve their canonical momentum in the laser: $\mathbf{v}(\mathbf{r},t) \approx -q\mathbf{A}_\mathrm{L}(\mathbf{r},t)/m\gamma_0$
  3. Fourier transforms for both the laser field $\mathbf{A}_\mathrm{L}(\mathbf{r},t) \rightarrow \tilde{\mathbf{A}}_\mathrm{L}(\mathbf{k})$ and shape $s(\mathbf{r}) \rightarrow \tilde{s}(\mathbf{k})$

$$\tilde{\mathbf{E}}_\mathrm{rad}(\mathbf{k}) = \frac{-\pi\mu_0 N q^2}{2(2\pi)^{3/2} c m \gamma_0}\left(\mathbb{I} - \hat{\mathbf{k}}\otimes\hat{\mathbf{k}}\right)\cdot\int d\mathbf{q}\; \tilde{s}(\mathbf{k}-\mathbf{q})\; \tilde{\mathbf{A}}_\mathrm{L}(\mathbf{q})\; \delta[(\mathbf{k}-\mathbf{q})\cdot\boldsymbol{\beta}_0 + |\mathbf{q}| - |\mathbf{k}|]$$

"Wave mixing" of particle beam and laser spectra restricted to the Dirac-delta constraint

- Further assuming a "pancake" particle beam:
  1. **short** and **wide** beam: $s(\mathbf{r}) \approx \delta(x) \Rightarrow \tilde{s}(\mathbf{q}) = \delta(q_y)\delta(q_z)$, travelling with $\boldsymbol{\beta}_0 = \beta_0\hat{\mathbf{x}}$
  2. laser polarised in $\hat{\mathbf{y}}$ with radiation along $\hat{\mathbf{x}} \Rightarrow k_x \gg k_y, k_z$, thus $\mathbb{I} - \hat{\mathbf{k}}\otimes\hat{\mathbf{k}} \rightarrow \hat{\mathbf{y}} + \hat{\mathbf{z}}$

$$\tilde{\mathbf{E}}_\mathrm{rad}(\mathbf{k}) = \frac{-\pi\mu_0 N q^2}{2(2\pi)^{3/2} c m \gamma_0}\tilde{A}_y\left[\mathcal{Q}_\parallel(\mathbf{k},\beta_0)\hat{\mathbf{x}} + k_y\hat{\mathbf{y}} + k_z\hat{\mathbf{z}}\right]$$

<u>Remapping</u> the original laser beam spectrum ($\mathbf{k}_\mathrm{L}$) to a new set of wave vectors ($\mathbf{k}$)

$$k_y = k_y^L \text{ and } k_z = k_z^L \text{ and } k_x^L = \mathcal{Q}_\parallel(\mathbf{k},\beta_0) \Rightarrow k_x = \frac{k_x^L(\beta_0^2+1) \pm 2\beta_0|\mathbf{k}_\mathrm{L}|}{\beta_0^2 - 1}$$

Single Plane Wave Analysis

- Remapping of each laser plane wave with wave vector $\mathbf{k}_\mathrm{L}$ to a radiating plane wave with $\mathbf{k}$. Some examples:
  1. Head on collision recovers standard Thomson result $k_x \approx 4\gamma_0^2 k_L$
  2. Perpendicular collision has a slight lower upshift $k_x \approx 2\gamma_0^2 k_L$
  3. Any angle collision can be modelled
  4. Collision with realistic pulses (patches in k-space)

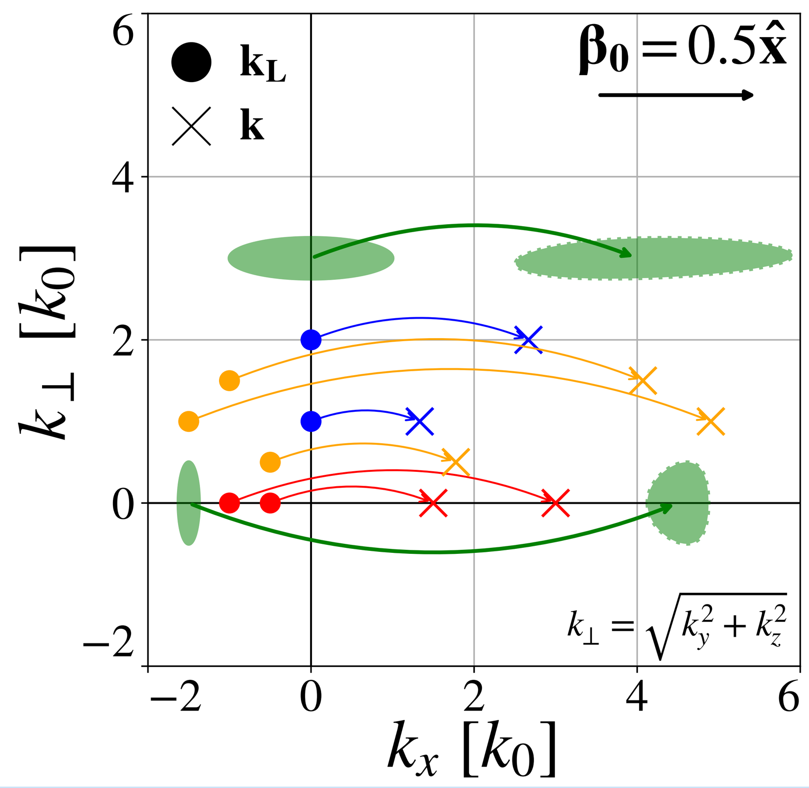
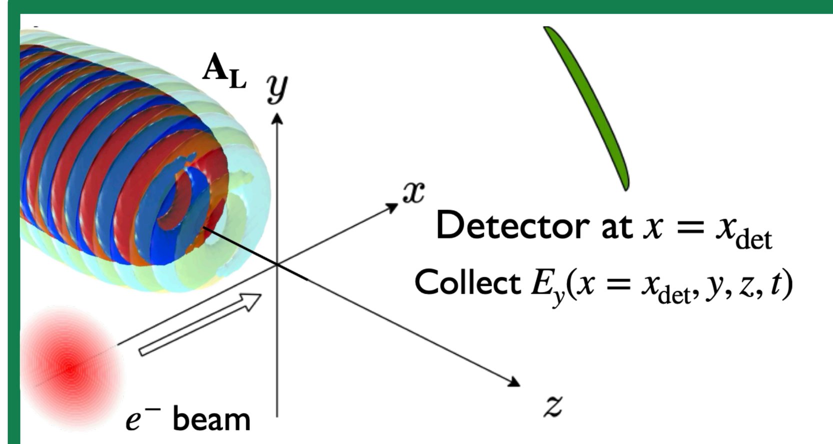


Figure 2: Each dot represents a laser plane wave with the corresponding radiated plane wave as a cross in k-space. A patch is a physical wave packet.

An illustrative example: perpendicular collision with a Laguerre Gaussian Beam



- To exemplify this phenomenon, consider the setup on the side:
  - **Laguerre Gaussian beam** [1] polarised in the $\hat{\mathbf{y}}$ direction and propagating along $\mathbf{z}$ that carries orbital angular momentum (OAM)
  - **Particle beam** travelling along $\mathbf{x}$, colliding with the laser beam at the origin, radiating along its propagation direction
- The simulation was performed using the particle-in-cell code Osiris [2] coupled with the Radiation Diagnostic Tool RaDiO [3] to capture the radiation in a far field detector. The laser beam was injected using a custom-made exact plane wave injector [4]

Figure 3: Simulation setup: electric field of the laser beam in red/blue isosurfaces, electron beam in red and detector in the far-field along the positive $\mathbf{x}$ axis.

Laguerre Gaussian (LG) Beams have OAM parallel to their propagation direction ($\mathbf{k} \parallel \mathbf{L}$)
(Electric field looks like a double helix moving forward)

Interaction **shifts** and **stretches** the laser's plane wave spectrum along the particles' propagation direction

Spatiotemporal Optical Vortices (STOV) exhibit OAM perpendicular to their propagation direction ($\mathbf{k} \perp \mathbf{L}$)
(Electric field looks like a donut moving sideways)

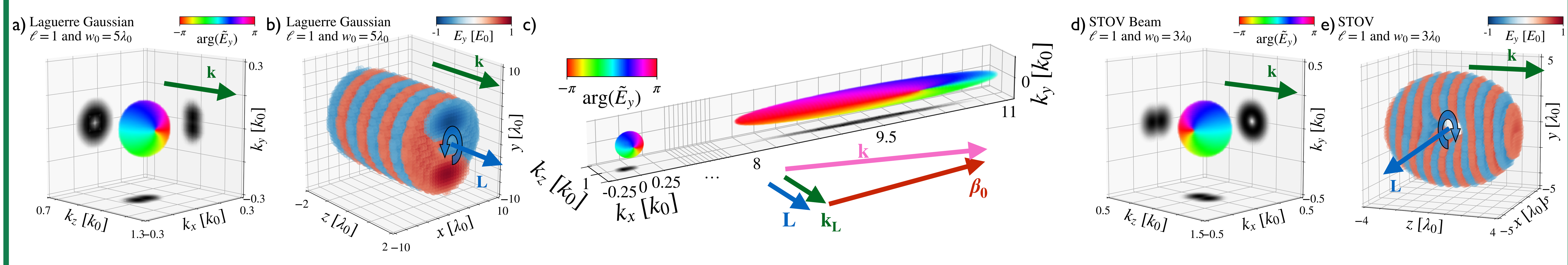


Figure 4: a) LG beam in k-space with color representing the phase of each plane wave. **b)** Electric field of a LG beam in real space. **c)** Representation of the interaction between the laser and the particle beams, portraying both the laser and radiation plane wave support in k-space. The arrows represent the main directions in the problem. **d)** STOV beam in k-space with color representing the phase of each plane wave. **e)** Electric field of a STOV beam in real space.

Conclusions

- **Thomson Scattering can be used a light shaping tool**, accurately manipulating the spectrum of the resulting radiation while upshifting it
- Generalised Radiation Formula from Laser - Beam Interaction that arises from Fourier Transforms
- **Spatiotemporal Optical Vortices can be generated from Laguerre Gaussian Beams**, converting normal longitudinal orbital angular momentum to transverse

References & Acknowledgements

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