Kardar-Parisi-Zhang in Anderson Localization

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I. Anderson Localization and KPZ Universality Class

Anderson Localization is a crucial result regarding the absence of diffusion in disordered systems [1]. For 1D, it has been proven exactly that, in the thermodynamic limit, localization occurs no matter how small the disorder in the system is. In 2D, RG approximations suggest that this is also the case, however, the localization might be very weak depending on the strength of the disorder [2].

In the strongly localized regime, it is possible to establish a relation between localized wave packets and the directed polymer (DP) problem [3]. It can be proven analytically that the DP belongs to the KPZ universality class. If our quantum system does belong to the KPZ universality class, we expect that the fluctuations of it's negative log-density obey:

$$\sigma\left(-\log|\psi(\mathbf{r})|^2\right) \propto r^{\beta},\ \beta = 1/3$$

β is called the growth exponent.

II. Model

Generalization of the Anderson Model:

$$H = \sum_i \varepsilon_i |i\rangle\langle i| + \sum_{i\neq j} \eta(R_{ij})|i\rangle\langle j|$$

With initial condition:

$$\psi_0(\mathbf{r}) = \delta(\mathbf{r}-\mathbf{r}_0)$$

And Schrödinger time evolution:

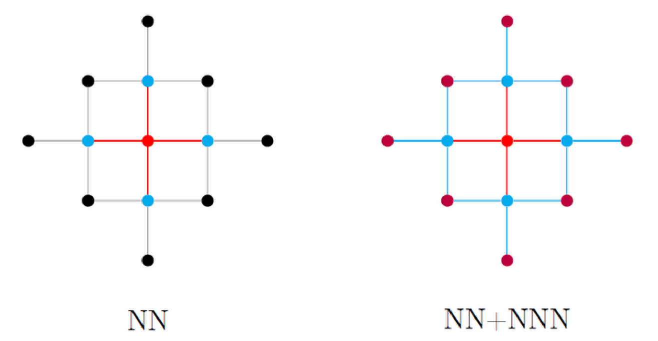
$$|\psi_t\rangle = U(t,t_0)|\psi_0\rangle = e^{-iHt}|\psi_0\rangle$$

NN+NNN:

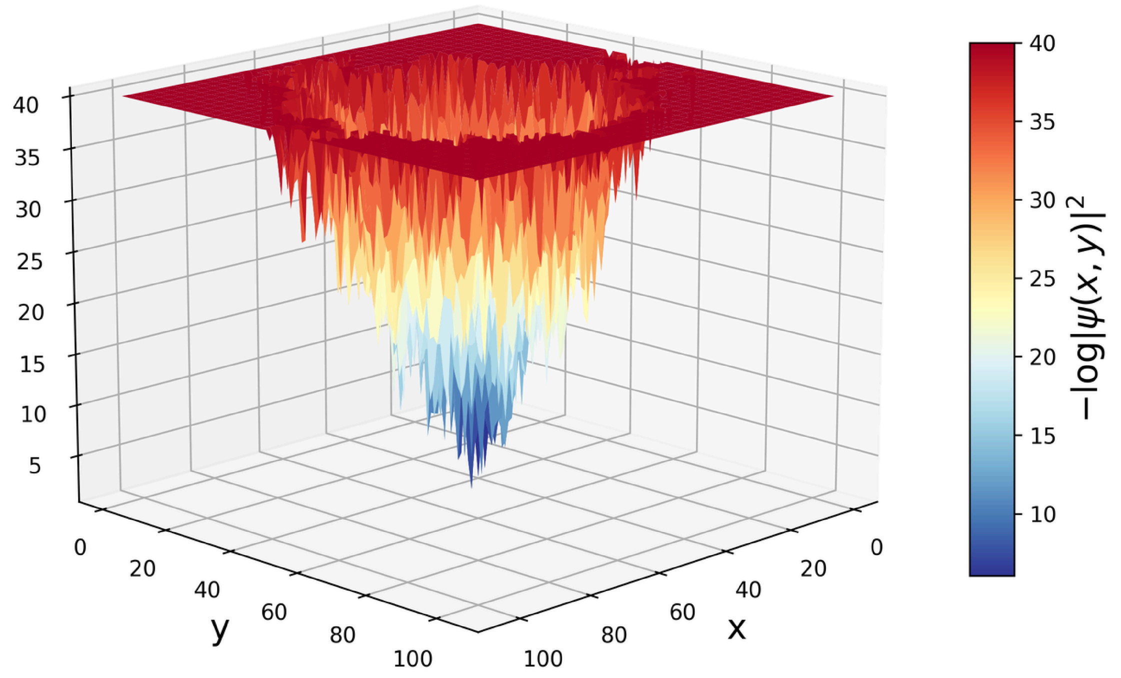
$$\eta(R_{ij}) = \Theta(R_{ij})\Theta(2\sqrt{2}-R_{ij})$$

Exponentially Decaying Model:

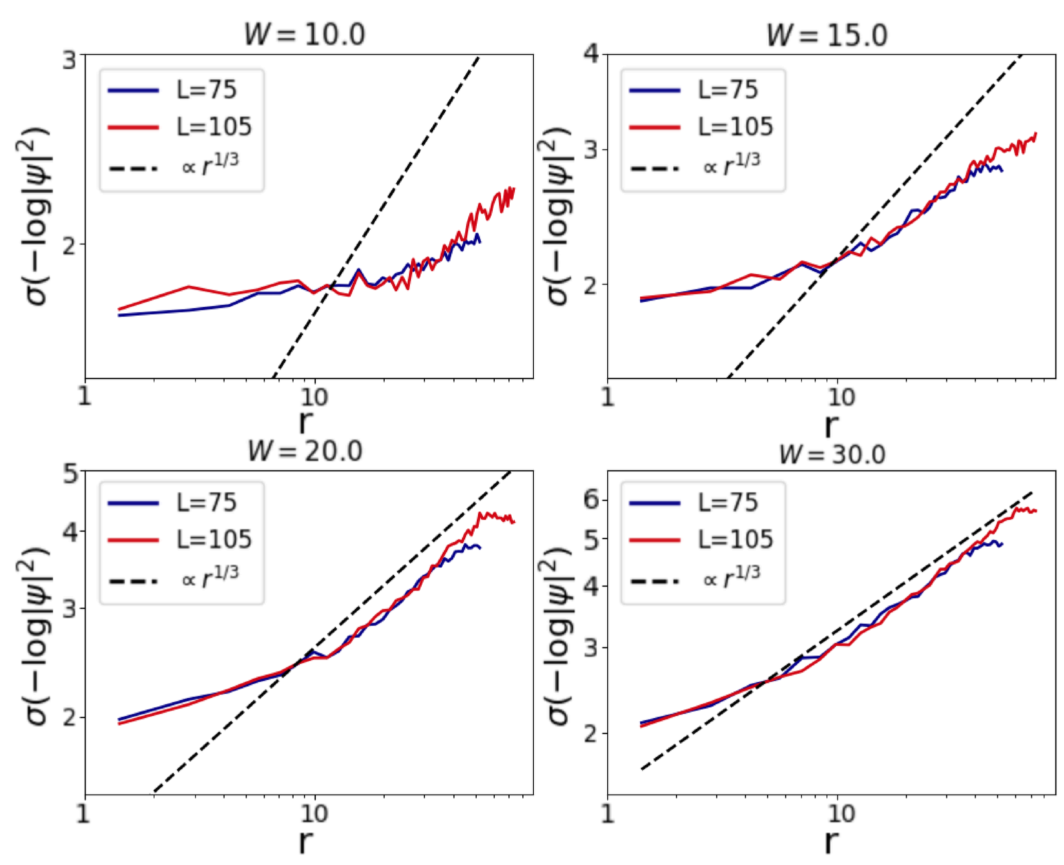
$$\eta(R_{ij}) = e^{-\alpha R_{ij}}$$



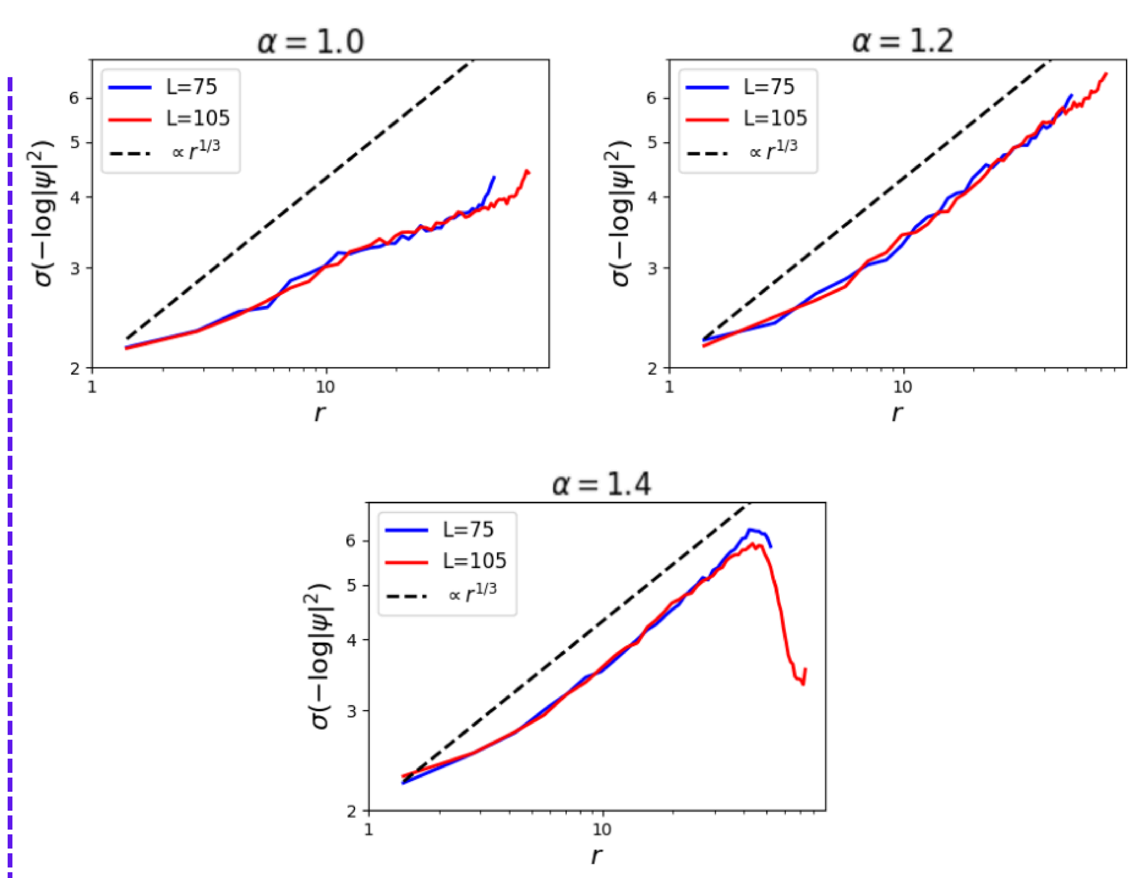
III. Fluctuations of the Wave Packet



Spatial structure of the wave packet intensity at t = 2000 for the NN+NNN model.

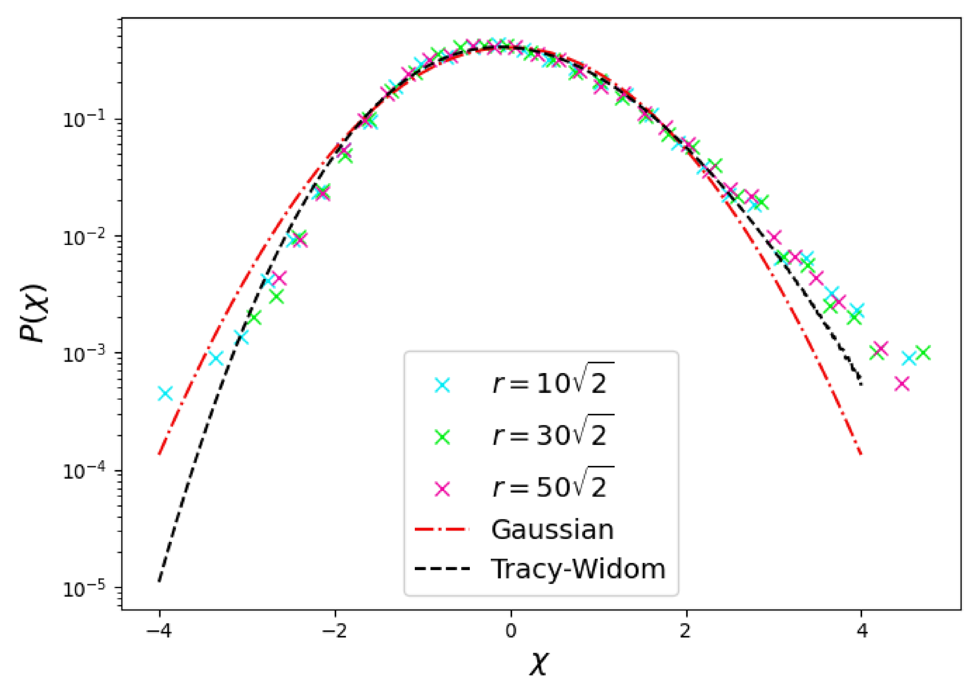


Standard deviation of the negative log-density for the NN+NNN model, for multiple W. The dashed lines correspond to β =1/3.



Standard deviation of the negative log-density for the Exponentially Decaying Hopping model, for W = 10 and different values of α.

IV. Probability Distribution



$$\chi(r) = \frac{\log|\psi(r)|^2 - \langle\log|\psi(r)|^2\rangle}{\sigma(\log|\psi(r)|^2)}$$

For high disorder the probability distribution follows the GUE TW distribution, a strong indicator for the regime to belong to the KPZ universality class (considering the droplet initial condition)

V. Conclusions

The highly (exponential) localized regimes demonstrate a growth exponent of β ≈ 1/3, in agreement with KPZ. The less localized regimes (stretched-exponential profile) display smaller growth exponents, while no longer belonging to the KPZ universality class.

The analysis of the probability distribution associated with each packet shows that for a regime where exponential localization and β ≈ 1/3 was observed, the probability distribution of the log density shows great agreement with the Tracy-Widom distribution.

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References:
[1] P. W. Anderson, Phys. Rev. 109, 1492 (1958)
[2] E. Abrahams, P. W. Anderson, D. C. Licciardello, and T. V. Ramakrishnan, Phys. Rev. Lett. 42, 673 (1979)
[3] S. Mu, J. Gong, and G. Lemarie, Phys. Rev. Lett. 132, 046301 (2024), arXiv:2306.08272